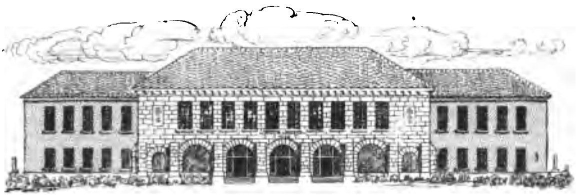
VANCED ARITHMETIC

THE NORMAL COURSE IN NUMBER

COOK AND CROPSEY

SILVER, BURDETT AND COMPANY



The Normal Course in Number

THE NEW ADVANCED ARITHMETIC

BY

JOHN W. COOK

PRESIDENT ILLINOIS STATE NORMAL UNIVERSITY

AND

MISS N. CROPSEY

ASSISTANT SUPERINTENDENT CITY SCHOOLS
INDIANAPOLIS, INDIANA

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PREFACE.

IT has seemed to the authors of the NORMAL COURSE IN NUMBER that there is room for another series of Arithmetics, notwithstanding the fact that there are many admirable books on the subject already in the field.

The ELEMENTARY ARITHMETIC is the result of the experience of a supervisor of primary schools in a leading American city. Finding it quite impossible to secure satisfactory results by the use of such elementary arithmetics as were available, she began the experiment of supplying supplementary material. An effort was made to prepare problems that should be in the highest degree practical, that should develop the subject systematically, and that should appeal constantly to the child's ability to think. Believing that abundant practice is a prime necessity to the acquisition of skill, the number of problems was made unusually large. The accumulations of several years have been carefully re-examined, re-arranged, and supplemented, and are now presented to the public for its candid consideration. Not the least valuable feature of this book is the careful gradation of the examples, securing thereby a natural and logical development of number work. No space is occupied with the presentation of theory, — that side of the subject being left to the succeeding book. The first thoughts are *what* and *how*, — these so presented that the processes shall be

easily comprehended and mastered. Subsequently, the *why* may be intelligently considered and readily understood.

The ADVANCED ARITHMETIC is the outgrowth of a somewhat similar experience in the class-room of a teachers' training-school. For many years an opportunity was afforded to study the effects upon large numbers of pupils of the current methods of instruction in arithmetic. The result of such observation was the conviction that the rational side of the subject is seriously neglected. An effort was made to supplement the ordinary text-book by a study of principles and by explanations of processes. The accumulations of fifteen years have been edited with all of the discrimination of which the authors were capable. Great care has been exercised in the presentation of principles and in the formulation of processes, to the end that the learner shall have every facility for the use of his reasoning powers, and at no point be relieved from the proper exercise of his mental activity and acumen. It is hoped that the book may contribute somewhat to the movement, now so happily going on, that looks toward the disestablishment of the method of pure authority, and the establishment of a method that makes its appeal to intelligence and reason.

The authors desire to express their appreciation of the excellent suggestions offered by many friends; but especial thanks are due Professor DAVID FELMLEY, of the Illinois State Normal School, for his discriminating criticisms and valuable assistance.

THE AUTHORS.

SUGGESTIONS.

METHOD is determined chiefly by aim. The answer which the teacher makes to the question, "Why should boys and girls study arithmetic?" will guide him in the details of instruction.

Arithmetic is one of the traditional "three R's." Some knowledge of its processes is necessary to any degree of intelligence. Its highly practical character is conceded by every one.

The arithmetical operations employed in ordinary business affairs are simple, but they must be performed with absolute accuracy and with great rapidity. They are based, primarily, upon the memory. The necessity for perfect familiarity with the fundamental facts of number becomes apparent. *Neither accuracy nor rapidity is possible without a thorough mastery of the primary work.* This mastery is acquired through constant repetition of the old in connection with the acquisition of the new. One of the teacher's maxims, constantly, must be "*Review!* REVIEW!! REVIEW!!!

But arithmetic has another and a higher function. It must cultivate that quick intelligence which is able to analyze given conditions and determine what should be done in the particular case.

A true problem in arithmetic is a statement or series of statements in which something is told and something

is asked; the answer to what is asked being implied in what is told. The chief activity of the child, in the solution of true problems, is, first, the analyzing of given conditions, and second, the performing of certain operations. The former of these activities should never become mechanical; the latter should pass into the mechanical stage as early as possible.

The study of advanced arithmetic differs in certain essential particulars from all other studies in the common-school course. The thought lacks the continuity of history or geography. The work consists of a series of efforts that are more or less distinct. Each problem stands alone in its statement; but the operations involved in it fall under certain general types. Arithmetic, consequently, requires a constant dealing with both particular and general notions. These general processes may be discovered in illustrative problems, and the possible number of them be absolutely exhausted. When this has been done, the "case" arrangement of problems should cease, and the pupils should be permitted to perform that analytic activity that discovers conditions in a problem, and that synthetic activity that unites it to its proper class. The problems in this book have been prepared with this thought in mind.

Another phase of arithmetic work should be clearly appreciated. It surpasses all other studies in the lower grades in the number of its generalizations and the ease with which they are made. The generally accepted principles that instruction should proceed from the concrete to the abstract, and from the individual to the general, have constant application. But the abstract should be the result of a conscious abstraction, and the general

should be a conscious generalization. As illustrating this thought, and showing how the general is reached from the individual, particular attention is called to the method of deriving rules from processes. The formulation and statement of a rule is a generalization.

It is a fundamental principle of true teaching, that whatever is done by the pupil shall be accomplished through his conscious, personal effort, and that whatever his acquisition, it shall be consciously his own, — not his in the memory alone, which is the same as his in the book, but his vitally and substantially, as the blood in his veins or the innate ideas of right and wrong. Such knowledge is of a rooted and growing order that gives satisfaction and power to its possessor.

To suggest and stimulate such teaching, and to secure such growth, many questions are asked and many directions are given, in this book, which are designed to throw the pupil upon his own resources. The questions cannot be answered by any statements found on the pages, — no need to tax the memory for words and phrases; yet all the questions and directions are simple and easily to be answered by the pupil, if he has thought his way clearly.

Too great emphasis cannot be given to the statement already made, that the work of analyzing should never become mechanical. While there is value in concise and definite formulas, there is infinitely greater value in *freedom*. The mind should be free to discover conditions, relations, and sequences; it should be as free in stating conclusions and results; but this freedom cannot exist if the reasoning is compelled, per force, to follow a memorized formula. The forms of analyses given in the fol-

lowing pages are presented as models to be studied and mastered, but not to be memorized. If the pupil, in his own way and in different words, shall clearly present the steps of reasoning and draw the correct conclusion, his work should be approved.

An additional suggestion must suffice. The teacher's knowledge of the subject should be organic. Arithmetic should be recognized as a science that is deduced from the idea of addition. When the so-called fundamental processes have been mastered, little remains but repetition. The fraction differs from the integer in that it introduces a double unity. The decimal fraction differs from the common fraction in the method of expressing its denominator. Percentage is "a case" in decimal fractions. Compound numbers differ from simple numbers because of the introduction of variable scales, etc., etc. As the power to generalize relieves the mind from the overwhelming burden of a countless multitude of individuals, so each new step in advance is easily held if correllated with the fundamental ideas.

J. W. C.

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THE NORMAL COURSE IN NUMBER.

THE NEW
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Part I.

SECTION I.

DEFINITIONS.

1. **Measuring** is the process of finding how many times a quantity contains a part of itself which is taken as a standard. Illustrate.

2. That portion of a measured quantity which is used as a standard is called a **Unit.**

When we count a basket of eggs, we measure the quantity of eggs by using one of them as a standard. Such a unit is a natural unit. When we measure a quantity of cloth, we use a portion of itself, called a yard, as a standard. Such a unit is an artificial unit.

3. From the repetitions of the unit in counting or measuring, the successive numbers, one, two, three, etc., arise; thus,

Number is that which answers the question "How many?"

4. **Arithmetic** is the science which treats of number and the methods of employing it in computation.

5. To simplify counting or measuring, units are gathered into equal groups, each of which forms a new unit. Thus, in measuring a basket of eggs, they are grouped into dozens. The quantity may be expressed as a number of single eggs or as a number of dozens.

A Decimal System of numbers is a system in which ones are grouped into tens; tens into tens of tens, or hundreds; hundreds into tens of hundreds, or thousands; thousands into ten-thousands, etc.

Ones are units of the first rank, or order; tens are units of the second order; hundreds, of the third order, etc.

Illustrate these groupings with bundles of splints.

The scale in any system of numbers is the number of units in each order required to form one of the next higher.

NOTATION.

6. The art of expressing numbers by means of characters is **Notation.**

7. A system of Notation which will express all numbers must include a set of characters to represent numbers and the laws for using them.

8. The Arabic System of Notation employs ten characters called figures. They are 1 (one), 2 (two), 3 (three), 4 (four), 5 (five), 6 (six), 7 (seven), 8 (eight), 9 (nine), 0 (cipher).

9. Any given figure always expresses the same number of units, but the order or kind of units is expressed by the *place* in which the figure is written.

Ones stand in the first place, tens in the next place to the left, hundreds in the third place, etc.; thus,

A figure standing in any place expresses units ten times as large as if standing one place to the right.

The cipher is used to fill vacant orders.

NUMERATION.

10. The art of reading numbers expressed by figures is **Numeration.**

11. Three orders form a **period.** The name of the lowest order in each period is ones; of the next higher is tens; of the third is hundreds.

The names of the first twelve periods in their order are as follows:

1. Units. 2. Thousands. 3. Millions. 4. Billions. 5. Trillions. 6. Quadrillions. 7. Quintillions. 8. Sextillions. 9. Septillions. 10. Octillions. 11. Nonillions. 12. Decillions.

NOTE.—The meaning of the prefix in the word *billion* is two; in the word *trillion* is three; in *quadrillion* is four, and so on. Observe that the number of any period above millions is two more than the meaning of the prefix in the name of that period.

12. Arrangement of orders and periods.

Trillions.			Billions.			Millions.			Thousands.			Units.		
Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Ones.
8	6	5,	4	0	6,	3	8	2,	1	0	4,	5	7	9

NOTE.—Learn the names of the periods in their order from *left to right*.

13. To read a number, group the figures into periods, beginning at the right, and separating the periods by commas. Beginning at the left, read the number in each period as if it stood alone; then add the name of the period.

NOTE.—The English system of Numeration is in use in England and upon the continent of Europe, except in France. It employs six orders for a period. In studying this system the meaning of the names of the periods is made plain. A million is a thousand thousand. A billion is the square of a million; a trillion, the third power of a million; a quadrillion, the fourth power of a million, etc.

14. Read the following numbers:

1. 2345.	**6.** 250849.	**11.** 683471.
2. 4638.	**7.** 381307.	**12.** 829406.
3. 7912.	**8.** 408391.	**13.** 200619.
4. 3105.	**9.** 716004.	**14.** 100054.
5. 26853.	**10.** 500836.	**15.** 973070.

16. 2530876.
17. 3890432.
18. 47063502.
19. 50780439.
20. 480983048.
21. 379068452016.
22. 690750142953.
23. 85968345620961.
24. 6390086133859016.
25. 7000492587291563295.
26. 8230075913748426950.
27. 2005300861943186627.
28. 10030006729062127390037.
29. 300750916400853269057040.

NOTE. — Frequent dictation exercises should be given with successively larger numbers until pupils have acquired proficiency in writing numbers.

15. Before writing the following numbers, tell how each will appear when written.

Illustration. Three thousand eight hundred seven is expressed by writing the following: three, comma, eight, cipher, seven.

1. Forty thousand six.
2. Ninety-seven thousand five hundred twelve.
3. Three hundred sixty-nine thousand twenty-four.
4. Four million eight thousand two.
5. Fifty-six million nineteen thousand thirty-three.
6. Eighty-one million five hundred thirteen thousand two hundred fifty-one.
7. Three hundred million ninety thousand four.
8. Five billion six million seven thousand eight.
9. Seventy-two billion six hundred thirty-five thousand two hundred fifty-one.
10. One hundred three billion two million seventeen thousand one hundred four.
11. Two trillion three billion four million five thousand six.
12. Ninety-one trillion two hundred seven billion sixty-nine million four thousand three
13. Eighty-six trillion one million twenty-three.

14. Two hundred sixteen trillion five hundred thousand.

15. Nineteen trillion four.

NOTE. 1. Teachers should supply dictation exercises in writing numbers until a good degree of proficiency is acquired.

2. Observe which of the number names are compound words.

3. Note that the word "and" is not used in these exercises.

16. THE ROMAN NOTATION.

This method expresses number by the use of certain print letters. They are I, V, X, L, C, D, M.

I = 1, V = 5, X = 10, L = 50, C = 100, D = 500, M = 1000. Their use is determined by the following

PRINCIPLES.

1. Repeating a letter repeats its value. II = 2, XX = 20.

2. When a letter is placed after one of greater value, the two express a number equal to the sum of their values. XV = 15, CI = 101.

3. When a letter is placed before one of greater value, the two express a number equal to the difference of their values. IX = 9, XC = 90. (Limited to IV, IX, XL, and XC.)

4. When a letter is placed between two, each of greater value, its value is taken from the sum of their values. XIX = 19, XIV = 14.

5. Placing a dash over a letter multiplies its value by a thousand. $\overline{\text{X}}$ = 10,000, $\overline{\text{M}}$ = 1,000,000.

EXERCISES.

1. Express by the Roman characters all numbers from one to one hundred.

2. 125.	**5** 419.	**8.** 752.	**11.** 1776.
3. 263.	**6.** 599.	**9.** 1066.	**12.** 1799.
4. 379.	**7.** 643.	**10.** 1492.	**13.** 1896.

NOTE. Use dictation exercises freely.

17. REDUCTION.

1. **Reduction is the process of changing the unit of a number without changing its value.**

2. Express 6 pints in quarts; 8 quarts in gallons; 6 feet in yards; 32 ounces in pounds; 20 mills in cents; 300 cents in dollars.

3. Have these numbers been changed to a larger or to a smaller unit?

The process of reducing a number to larger units is called **Reduction Ascending.**

4. How are pints reduced to quarts? quarts to gallons? feet to yards? ounces to pounds? mills to cents? ones to tens? tens to hundreds? tens to thousands? cents to dollars? Give many similar examples.

DIRECTION.

5. ***To reduce a number to a number of larger units, divide it by the number of the given units which makes one of the larger units.***

6. Express 3 quarts in pints; 3 gallons in quarts; 4 yards in feet; 3 dimes in cents; 5 tens in ones; 6 hundreds in tens; 7 thousands in hundreds, in tens.

7. Have these numbers been changed to a larger or to a smaller unit?

The process of reducing a number to smaller units is called **Reduction Descending.**

8. How are quarts reduced to pints? gallons to quarts? yards to feet? dimes to cents? tens to ones? hundreds to tens? thousands to hundreds? to tens?

DIRECTION.

9. ***To reduce a number to a number of smaller units, multiply it by the number of the smaller units to which one of the larger units is equal.***

NOTE.—A knowledge of the fundamental nature of the decimal system is of the utmost importance in arithmetical operations. This is obtained through practice in the two forms of Reduction. Multiply examples like the following·

18. 1. In 50,000 there are how many tens? hundreds? thousands? ten-thousands?

2. In 17,000,000 there are how many thousands? hundred-thousands? tens? hundreds? ones? ten-thousands?

3. In 38,000 mills there are how many cents? dimes? dollars?

4. In 65 dollars there are how many dimes? cents? mills?

19. 1 is what part of 10? One ten is what part of 100? One hundred is what part of 1000? In 1,111 each unit is what part of the unit standing in the first place at its left?

20. It is customary to fix the place of ones by placing a period at its right, thus: 1. When the period is thus used it is called the decimal point. From what has been observed what kind of unit will the right-hand figure in 1.1 express? What is one tenth of one tenth? What, then, is the kind of unit expressed by the right-hand figure in 1.11? What is one tenth of one hundredth? What kind of unit is expressed by the right-hand figure in 1.111? Similarly, show what kind of units is expressed by a figure in the fourth place at the right of the decimal point; in the fifth place; in the sixth place.

21. Table of six places at the right of the decimal point.

Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
.2	3	6	4	1	8

22. The name of any number is the same as the place in which its right-hand figure stands. Read the following numbers:

1. .6	11. .007	21. .00836
2. .8	12. .491	22. .04826
3. .2	13. .0005	23. .39627
4. .24	14. .0036	24. .30861
5. .37	15. .0683	25. .000003
6. .04	16. .2758	26. .000072
7. .127	17. .3085	27. .000739
8. .209	18. .4902	28. .004936
9. .842	19. .00006	29. .003972
10. .094	20. .00034	30. .386492

31. 16.07 (Read as 16 and 7 hundredths or as 1607 hundredths.)

32. 25.375	37. 6.00039
33. 239.004	38. 28643.907502
34. 508.0089	39. 10000.807501
35. 2851.3675	40. .197527
36. 3750.1049	

23. Perfect familiarity with the names and numbers of the places at the right of the decimal point is necessary for accurate and rapid writing. The name of a number is determined by the place of its right-hand figure. In writing numbers like the foregoing correctly the first time two things must be known: the number of figures required to express the number; the place in which the right-hand figure must stand to express the kind of units.

24. Write the following numbers:

1. Twenty-three hundredths.
2. Seven tenths.
3. Sixty-nine hundredths.

4. One hundred forty-eight thousandths.

5. Two hundred eleven thousandths.

6. Six hundred ninety-three thousandths.

7. Nine ten-thousandths.

8. Four ten-thousandths.

9. Thirty-two ten-thousandths.

10. One hundred eighty-three ten-thousandths.

11. Nine hundred seven ten-thousandths.

12. One thousand five hundred seventy-six ten thousandths.

13. Nine thousand four hundred three ten-thousandths.

14. Eight hundred-thousandths.

15. Five hundred-thousandths.

16. Forty-five hundred-thousandths.

17. Ninety-four hundred-thousandths.

18. Three hundred fifty-two hundred-thousandths.

19. Eight hundred ten hundred-thousandths.

20. Four thousand seven hundred nineteen hundred-thousandths.

21. Seven thousand twenty-three hundred-thousandths.

22. Two thousand six hundred-thousandths.

23. Thirty-four thousand six hundred twenty-one hundred-thousandths.

24. Fifty-nine thousand one hundred six hundred-thousandths.

25. Nineteen thousand three hundred-thousandths.

26. Fifty thousand one hundred-thousandths.

27. 25 millionths.

28. 8 millionths.

29. 63 millionths.

30. 478 millionths.

31. 2895 millionths.

32. 5008 millionths.

33. 37592 millionths.

34. 809066 millionths.

35. 26 and 68 hundredths.

36. 94 and 39 thousandths.

37. 290 and 463 hundred-thousandths.

38. 40073 and 50093 millionths.

39. 61 and 13 millionths.

40. Two thousand five and three thousand forty-six millionths.

25. Numbers like exercises 1 to 30, Art. **22**, are called Decimal Fractions. Numbers 31 to 39 are called Mixed Decimals. Name the Decimal Fractions in Art. **24**. Name the Mixed Decimals in the same section.

26. REDUCTIONS.

1. In .6 there are how many hundredths? How many thousandths? Write each of the numbers. How many ten-thousandths? Write the number. How many millionths? Write the number. Read 3.5 as hundredths, as thousandths, ten-thousandths, hundred-thousandths, millionths. Write each of the numbers. In .600000 there are how many tenths? ten-thousandths? hundredths? hundred-thousandths? millionths?

2. In 3.2 there are how many hundredths? How many ten-thousandths? How many millionths? Write each of the numbers. In 5.000000 how many hundred-thousandths? hundredths? thousandths? tenths? Write each of the numbers.

3. In 400000, there are how many tens? How many hundreds? How many ten-thousands? How many thousands?

SECTION II.

ADDITION.

27. Like numbers are numbers that are made by repeating the same unit.

28. The **Sum** of two or more like numbers is a number which contains all of their units.

29. The sign + (plus) between two numbers shows that their sum is to be found.

30. At first, to unite two numbers, we count the units of the second upon the first, thus: In uniting four blocks and three blocks, we think (or say) four, five, six, seven, as we place the three blocks one at a time with the four. In this way, *by counting objects*, we learn and commit to memory the following forty-five sums:

1	2	3	2	4	3	5	4	3
1	1	1	2	1	2	1	2	3
6	5	4	7	6	5	4	8	7
1	2	3	1	2	3	4	1	2
6	5	9	8	7	6	5	9	8
3	4	1	2	3	4	5	2	3
7	6	9	8	7	6	9	8	7
4	5	3	4	5	6	4	5	6
9	8	7	9	8	9	8	9	9
5	6	7	6	7	7	8	8	9

31. This series of forty-five sums of the nine primary numbers taken in twos is called the **Addition Table**.

The last twenty sums in the addition table may be learned without objects by uniting with the first number enough of the second to make ten; thus, $8 + 5 = 8 + 2 + 3 = 10 + 3 = 13$.

32. Addition is the process of finding the sum of two or more like numbers.

Since $7 + 5 = 12$, $17 + 5 = 22$, $27 + 5 = 32$, etc. Practice similarly with all of the "endings."

33. As a preparation for "column addition" give frequent exercises in adding by twos, threes, fours, etc., starting with one, two, three, etc., and carrying the work to 50.

34. EXERCISES.

The following problems are for seat, board, or home work, and for oral practice in recitation. Practice upon them until they can be performed with great rapidity. They contain frequent repetitions of the work in the addition table. Add from the bottom, naming results only, thus: (first problem) 7, 13, 21, 24, 26, etc. To test the accuracy of the work add downward.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
9	5	7	5	9	6	8	6	4	9
5	6	7	8	8	8	7	9	8	5
7	5	6	5	7	7	8	2	9	9
6	8	5	9	9	8	2	7	6	3
2	7	4	3	6	7	6	9	7	2
3	8	9	7	9	6	3	7	8	6
8	4	3	6	3	5	6	8	3	9
6	9	6	6	9	5	7	4	9	9
7	2	1	9	9	9	5	8	9	7

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
89	79	38	93	37	43	93	98
29	63	34	84	28	92	89	79
93	94	34	68	22	99	83	78
26	33	68	89	21	99	83	89
97	29	93	62	88	78	29	35
96	98	82	99	83	87	39	73

(19)	(20)	(21)	(22)
324	752	945	8685
561	433	887	4944
872	512	654	5636
324	311	472	3768
607	721	541	9483
830	869	635	7521
962	863	796	2754
	574	127	4856
		385	7039

(23)	(24)	(25)	(26)
3580	2846	77281	497529
6295	6389	94969	678315
3782	3528	88799	855274
9061	4893	68698	998865
3648	4678	89769	677345
7296	8253	84858	499586
6802	7594	48979	394759
3690	6031	87857	862979
3285	8539	36686	371789
9735	7426	76978	542963
	3869		752784

(27)	(28)	(29)	(30)
476583	9759869	898488	5492284
369868	8387698	764257	9785746
878725	4555966	701480	9637478
766958	1804795	925895	4476204
832777	8468872	699054	6544658
794489	6804487	854606	9669733
468985	7557688	988987	1368572
739849	5687899	504469	2943489
747494	8848786	566587	6948394
836366	9579665	677876	8164768
669118	3695467	757769	5819404
577669	6086464	515987	8578368
	8402326	492834	7533579

How were these numbers written for addition? Why? With what column was the addition begun in each case? Why? When the sum of the numbers expressed in any column exceeded nine, what was done?

35. FORMAL STATEMENT.

Write the numbers to be added so that units of the same order shall stand in the same column.

Beginning with the lowest order, find the sum of the numbers expressed in each column. If this sum exceeds nine, in any case, reduce it to the next higher denomination, placing the remainder, if any, under the column added, and adding the reduced number to the first term in the next higher order.

36. TEST OF ACCURACY.

1. Add the numbers expressed in each column, in the reverse order.

2. Separate the problem into two or more problems and then unite the several results.

What must be true of numbers in order that they may be united?

What is the denomination of the sum?

37. How many mills are there in one cent? cents in one dime? dimes in one dollar?

38. TABLE OF FEDERAL MONEY.

10 mills make 1 cent.
10 cents make 1 dime.
10 dimes make 1 dollar.

39. In writing Federal Money, separate the order of dimes from the order of dollars by a decimal point. 126 dollars, 4 dimes, 7 cents, and 6 mills is written, $126.476. It is usually read, 126 dollars, 47 cents, 6 mills.

40. Write the following:

1. 83 dollars, 27 cents, 4 mills.
2. 59 dollars, 20 cents, 1 mill.
3. 73 dollars, 5 cents, 2 mills.
4. 148 dollars, 2 dimes, 9 mills.
5. 300 dollars, 7 mills.

Read the above numbers,

(1) as dollars, dimes, cents, and mills.

(2) as dollars, cents, and mills.

41. Find the sum of the following numbers:

125 dollars, 26 cents, 8 mills.
64 dollars, 33 cents, 4 mills.
278 dollars, 5 cents.
471 dollars, 6 mills.
312 dollars, 59 cents, 7 mills.

42. Tell how the numbers are written for addition. Tell where the addition should begin. Describe the process, using a form similar to that on page 14.

43. Write, read, and add the following:

1. \$48.041, \$63.247, \$146.28, \$276.007, \$160.406.
2. \$361.79, \$483.062, \$583,802, \$1272.84, \$2169.176.
3. \$2678.145, \$5684.297, \$462.01, \$5000.36, \$790.46, \$579.614.
4. \$2638.95, \$5406.63, \$2384.25, \$76.52, \$857.35, \$96834.67, \$3790.48.
5. \$3762.05, \$67452.84, \$3568.90, \$35.70, \$49.32, \$5.83, \$23.71.
6. \$4.21, \$3.85, \$9.63, \$85.16, \$128.95, \$673.70, \$2895.30, \$853.60.
7. \$893.40, \$6.87, \$3708.90, \$7570.00, \$0.75, \$4603.55, \$3780.25.

8. $5.25, $60.70, $375.08, $95.80, $3617.50, $817.46, $3064.38, $0.85.

9. $3768.60, $479.00, $6328.50, $2.91, $325.75, $9.00, $754.08, $35.10.

10. $7000.00, $215.80, $725.60, $50.50, $1.87, $6384.50, $4536.40.

44. How many gills are there in one pint? pints in one quart? quarts in one gallon? These are used in measuring what?

Write the table of Liquid Measure.

The abbreviation for *gallon* is gal.; for *quart* is qt.; for *pint* is pt.; for *gill* is gi.

1. Add:

5	gal.	2	qt.	1	pt.	3	gi.
4	"	3	"	0	"	2	"
7	"	1	"	1	"	2	"
12	"	2	"	1	"	3	"
14	"	0	"	0	"	1	"

How are these numbers written for addition? Where does the addition begin? Describe the process as you did in Federal Money.

2. Add:

6	gal.	1	qt.	1	pt.	3	gi.
4	"	0	"	1	"	2	"
12	"	3	"	0	"	1	"
27	"	2	"	1	"	2	"
36	"	3	"	0	"	0	"

3. Add 8 gal. 3 qt. 1 pt. 3 gi.; 3 gal. 2 qt. 1 pt. 2 gi.; 4 gal. 1 qt. 1 pt.; 1 gal. 1 pt. 1 gi.; 6 gal. 3 qt. 3 gi.; 5 gal. 2 gi.

45. How many quarts are there in one peck? How many pecks in one bushel?

Write the table of Dry Measure.

1. **Add:**

5 bu.	3 pk.	6 qt.
6 "	2 "	4 "
9 "	1 "	6 "
28 "	0 "	7 "
16 "	2 "	5 "
4 "	3 "	2 "

2. Add 4 bu. 1 pk.; 7 bu. 3 pk. 2 qt.; 12 bu. 2 pk. 6 qt.; 19 bu. 1 pk.; 26 bu. 3 pk.; 14 bu.; 18 bu. 1 pk.; 2 pk.

3. Add 21 bu. 1 pk.; 36 bu. 2 pk.; 19 bu. 3 pk.; 2 pk.; 12 bu. 3 pk.; 6 bu. 4 qt.

46. Write the following problems very carefully on paper, slate, or blackboard, and solve:

1. $824 + 69 + 703 + 9208 + 29607 = ?$

2. $65 + 20007 + 893 + 566 + 15869 + 587 = ?$

3. $5607 + 20189 + 46827 + 463912 + 51872 + 56928 + 324501 + 873 = ?$

4. $70128 + 58694 + 79106 + 436912 + 586107 + 371009 + 400106 = ?$

5. $4007 + 93281 + 56185 + 47594 + 508069 + 724378 + 563128 = ?$

6. Paid $2480 for a farm, $268 for a span of horses, $65 for a wagon, $32 for a set of harness, $18 for a plough, $124 for a mowing-machine, and $384 for other utensils. What was the entire cost?

7. The area of Maine is 33,040 square miles; of New Hampshire, 9,305; of Vermont, 9,565; of Massachusetts, 8,315; of Rhode Island, 1,250; of Connecticut, 4,990. What is the entire area of the New England States?

8. The area of New York is 49,170 square miles; of New Jersey, 7,815; of Pennsylvania, 45,215; of Delaware, 2,050; of Maryland, 12,210; of District of Columbia, 70; of Virginia, 42,450; of West Virginia, 24,780. What is the area of the Middle States?

9. The area of North Carolina is 52,250 square miles; of South Carolina, 30,570; of Georgia, 59,475; of Florida, 58,680; of Tennessee, 42,050; of Alabama, 52,250; of Mississippi, 46,810; of Louisiana, 48,720; of Texas, 265,780; of Arkansas, 53,850; of Indian Territory, 64,690. What is the area of the Southern States?

10. What is the united area of the New England, Middle, and Southern States?

11. On Monday, a merchant sold 3 gal. 2 qt. 1 pt. of molasses; on Tuesday, 5 gal. 3 qt.; on Wednesday, 4 gal. 1 pt.; on Thursday, 6 gal. 1 qt. 1 pt.; on Friday, 2 gal. 2 qt. 1 pt.; on Saturday, 7 gal. 3 qt. 1 pt. How much did he sell in the entire week?

12. A dealer sold to A, 426 bu. 3 pk. 2 qt. of oats; to B, 329 bu. 3 pk. 5 qt.; to C, 189 bu. 1 pk. 7 qt.; to D, 426 bu. 2 pk. 5 qt.; to E, 562 bu. 3 pk. 6 qt. How much did he sell to all?

13. A farmer values his horses at $350, his cows at $275, his sheep at $411.75, his hogs at $129.25, and his poultry at $27.25. What is his value of all?

14. From New York to Albany is 143 miles; Albany to Suspension Bridge, 304 miles; Suspension Bridge to Detroit 230 miles; Detroit to Chicago, 284 miles. What distance from New York to Chicago?

15. A merchant bought at one time 224 barrels of flour for $1,344; at another, 217 barrels for $1,193.50; at another, 192 barrels for $1,056; at another, 486 barrels for $2,916. How many barrels did he buy? What was the cost of all?

16. A steamship sailed 239 miles each day for three days, and 227 miles each day for the next three days. How far did she sail in the six days?

17. Add .263, .187, 2.08, 4.019, 16.008, .563, .472, .008.

18. Add 26.0029, .0086, .0278, 43.0196, 217, .3059, .0062, .0487.

19. Add .00869, .0571, 36.426, 68.07956, .00964, .473, .8652, .17, .0086.

20. Add .000524, .00524, .0524, .068397, 6.080576, 12.008642, 99.0864, .000748, .429783.

21. A man gave each of his four sons $375; he gave his daughter as much as he gave to two sons; and to his wife as much as he gave to three sons. How much did he give to all?

22. A man left to his heirs a certain quantity of land. To the first he gave 320 acres; to the second as much as to the first, and 80 acres more; to the third as much as to the second, and 96 acres more; to the fourth as much as to all of the others, and to his wife as much as to the first three. How many acres did he bequeath?

23. In 1893, the enrollment in the public schools of Illinois was as follows: Number of male pupils enrolled in graded schools, 228,412; number of females, 235,885; number of male pupils enrolled in ungraded schools, 189,851; number of females, 171,937. What was the total enrollment for the year?

24. For the same year the number of male teachers in graded schools was 1,692; of female teachers, was 8,410; the number of male teachers in ungraded schools was 4,861; the number of female teachers was 9,277. What was the whole number of teachers employed?

25. For the same year the amount paid to male teachers in graded schools was $1,337,360.50; to female teachers, $1,168,903.82; to male teachers in ungraded schools, $4,272,782.46; to female teachers, $1,641,281.79. What was the whole amount paid to teachers?

26. The Permanent School Fund of Illinois is as follows: School Fund Proper, $613,362.96; the Surplus Revenue Fund, $335,592.32; the College Fund, $156,613.32; the Seminary Fund, $59,838.72; the County Funds, $158,616.63;

the Township Funds, $12,220,722.14; the University Fund, $606,207.64. What is the entire fund?

27. The following form is called a "Bill," or "Statement of account."

COLUMBUS, OHIO, May 15, 1896.

JAMES WATSON

To ROBERT S. WILLIAMS, Dr.

	1 Fifth Reader,	1	10		
	1 Advanced Arithmetic,		72		
	1 English Grammar,		60		
	1 Geography,	1	25		
	1 History United States,	1	10	4	77

Received payment,

ROBERT S. WILLIAMS.

28. John Jones, of Utica, N. Y., bought of Pixley & Co., September 12, 1895, the following articles. Prepare the bill.

1 suit of clothes, $16.50; 3 pair hose, 90¢; 1 light overcoat, $12; 6 pair cuffs, $1.35; 1 doz. collars, $2.30; 1 hat, $3.25; 1 umbrella, $1.85. Receipt the bill.

29. On January 24, 1896, Frank Walton bought of William Snow of Louisville, Ky., 1 barrel flour, $3.25; 3 hams, $3.83; 30 lbs. sugar, $1.45; 5 lbs. coffee, $1.85; 2 dozen oranges, $0.55; 6 cans tomatoes, $1.15. Bill not paid. Make the statement.

30. Bill of Robert Thompson, Bloomington, Ill., rendered to James Dixon, May 12, 1896. Items: Turkish lounge, $25.00; center table, $14.50; 6 chairs, $13.35; rocker, $4.75; chiffonier, $18.75; hall tree, $17.50.

31. Bill of Joseph Stoner, blacksmith, rendered to Charles Smith, December 10, 1895, Albany, N. Y. Items: sharpening 3 plows, $1.20; shoeing team, $2.40; setting tires, $1.65; repairing buggy, $1.50; babbitting harvester, $5.75. Receipt the bill.

47. The following device is often used by accountants:

86457	26
29063	21
840521	25
364307	35
428630	32
976538	27
2725516	

For the result read all of the last result and the right-hand figures of the previous results.

1. Find the value of the U. S. coinage of 1891 from the following statement: double eagles, $25,891,340; eagles, $1,956,000; half eagles, $1,347,065; quarter eagles, $27,600; silver dollars, $23,562,735; half dollars, $100,300; quarter dollars, $1,551,150; dimes, $2,304,671.60; nickels, $841,-715.50; cents, $470,723.50.

2. Money in circulation in United States, Dec. 1, 1894, was as follows: gold coin, $465,789,187; gold certificates, $58,925,899; silver dollars, $57,449,865; minor coins, $61,606,967; silver certificates, $332,317,084; "Sherman" notes, $124,574,906; United States notes, $276,910,489; currency certificates, $57,135,000; National Bank notes, $202,517,054. What was the total amount in circulation?

3. Distances along the Chicago and Alton Railroad—Chicago to Joliet, 37 miles; Joliet to Bloomington, 89 miles; Bloomington to Springfield, 59 miles; Springfield to Alton, 72 miles; Alton to St. Louis, 26 miles. What is the total length of the road?

4. Two ships meet in mid-ocean. One sails 416 miles eastward the first day, 386 miles the second day, and 369 miles the third day. The other sails 396 miles westward the first day, 278 the second day, and 339 the third day. How far apart are they at the end of the third day?

SECTION III.

SUBTRACTION.

48. As Addition is the process of uniting two or more like numbers, **Subtraction** is the process of separating a number into two smaller numbers. As such facts as $2 + 0 = 2$ are not counted in the addition table, so facts like $2 - 0 = 2$ are not counted in the subtraction table.

49. The following are the 81 primary problems in subtraction. Neither accuracy nor rapidity is possible until the difference between the numbers in each pair can be given with readiness.

2	3	3	4	4	4	5
1	2	1	3	2	1	4
5	5	5	6	6	6	6
3	2	1	5	4	3	2
6	7	7	7	7	7	7
1	6	5	4	3	2	1
8	8	8	8	8	8	8
7	6	5	4	3	2	1
9	9	9	9	9	9	9
8	7	6	5	4	3	2
9	10	10	10	10	10	10
1	9	8	7	6	5	4
10	10	10	11	11	11	11
3	2	1	9	8	7	6

11	11	11	11	12	12	12
5	4	3	2	9	8	7
12	12	12	12	13	13	13
6	5	4	3	9	8	7
13	13	13	14	14	14	14
6	5	4	9	8	7	6
14	15	15	15	15	16	16
5	9	8	7	6	9	8
16	17	17	18			
7	9	8	9			

50. John had 17 marbles and lost 8. How many had he left?

How many objects would be needed to illustrate this problem? Use the problem to test the accuracy of the following definitions:

51. **Subtraction is the process of separating a number into two parts, one of which is given for the purpose of finding the other.**

52. **The Minuend is a number that is to be separated into two parts, one of which is given for the purpose of finding the other.**

53. **The Subtrahend is the given part of the minuend.**

54. **The Remainder or Difference is the required part of the minuend.**

55. **The sign — (minus) when placed between two numbers shows that their difference is to be found.**

If the minuend is dollars, what will the subtrahend be? Why? What will the remainder be? Why?

56. **The minuend, subtrahend, and remainder are like numbers.**

John had 17 marbles and James had 12. How many more had John than James?

How many marbles would be needed to illustrate this problem? Will the definition given include this problem?

57. Problems in subtraction assume these two forms. When objects are employed they are reducible to the first form (see Art. **50**), since the 12 marbles that James had indicate the size of one of the two parts into which John's are to be separated.

58. A problem in subtraction is in its simplest form when each term in the minuend equals or exceeds the corresponding term in the subtrahend.

Illustration.

8462	Minuend.
5341	Subtrahend.

59. Problems are not generally in this form, but they must be made to assume it before the subtraction can be performed.

Problem.

$$\begin{array}{r} 721 \\ \underline{564} \end{array}$$

Is this problem in its simplest form? If not, show why it is not.

60. EXPLANATION.

1. Since the ones' term of the minuend is less than the corresponding term of the subtrahend, one of the tens in the minuend may be reduced to ones and added to the ones' term. 10 ones plus 1 equal 11 ones. 11 minus 4 equals 7.

2. Since the tens' term of the minuend is less than the corresponding term of the subtrahend, one of the hundreds in the minuend may be reduced to tens and added to the tens' term. 10 tens plus 1 ten equal 11 tens. 11 tens minus 6 tens equal 5 tens.

3. 6 hundreds minus 5 hundreds equal 1 hundred.

61. Since the minuend has been separated into the subtrahend and remainder their sum must equal the minuend. Hence,

To prove a problem in subtraction, find the sum of the subtrahend and remainder; if it equals the minuend, the work is correct.

62. Since the problem given is like all problems in subtraction, all may be solved as this has been. Hence, we may make the following

RULE FOR SUBTRACTION.

Write the subtrahend below the minuend, with units of the same order in the same column.

Beginning with the lowest denomination, subtract each term of the subtrahend from the corresponding term of the minuend. If any term of the subtrahend exceeds the corresponding term of the minuend, increase the smaller term by one of the next higher, and proceed as before.

63. EXAMPLES FOR PRACTICE.

1.	\$24.685 13.574	**6.**	5444454 4567956
2.	\$369.72 126.41	**7.**	4222332 2789679
3.	\$48.567 26.785	**8.**	26689888 16789899
4.	\$584.32 296.48	**9.**	92820 63574
5.	\$3557.888 1899.899	**10.**	102875 63986

11. 146238
87159

12. 196205
159438

13. 2666776
1789789

14. 611345566
235845897

15. 833344565
436789768

16. 746643455
366799687

17. 82345533
49455769

18. 923334454
583584889

19. 3445444554
1575678567

20. 5112556
2686897

21. 14455677
4857898

22. 84557666
59698979

23. 423445575
259576976

24. 21234566
17457988

25. 170308444
24154456

26. $800337.005
347942.462

27. 40080222
28848567

28. 847000221
512356345

29. $85200620.05
40065010.28

30. 403201058
250042006

Practice on these 30 problems until you can read the results with great rapidity.

	Minuend.	Subtrahend.	Remainder.
31.	824	63	?
32.	978	?	467
33.	?	826	279
34.	1276	?	638
35.	?	745	12864
36.	24671	?	18983

How find subtrahend when minuend and remainder are given? Why? Make a rule.

How find minuend when subtrahend and remainder are given? Why? Make a rule.

37. $60241 - 42374 + 26082 = ?$

38. $280621 + 460082 - 31892 - 42671 = ?$

39. $529706 - 31793 - 64802 + 5729 = ?$

40. A merchant bought 324 bu. 2 pk. of oats, and sold to A 59 bu. 3 pk. and to B 123 bu. 1 pk. How much did he have left?

41. A farmer had 861 acres of land, and gave 294 acres to his son. How many acres did he have left?

42. From a barrel containing 37 gal. 3 qt. 1 pt. and 3 gi. of vinegar, a merchant sold to A, 7 gal. 2 qt. 2 gi.; and to B, 4 gal. 3 qt. 1 pt. 2 gi. more than he did to A. How much was left in the barrel?

43. C and D were 350 miles apart. They traveled toward each other, C at the rate of 46 miles a day, and D at the rate of 37 miles a day. How far apart were they at the end of the second day? Which had traveled farther? How much?

44. Four men together owned 786 cattle. E owned 227, and F, 339. G owned as many less than E as F did more than E. How many did H own?

45. A merchant paid $2,500 for a quantity of silk. For other dry goods he paid $265 more than he did for the silk. For groceries he paid $683 less than what he had before expended. To one customer he sold goods amounting to $5,280; to another, $325 less than half as much; to another, $2,895 less than to the first, thus disposing of all of his stock. Did he gain or lose, and how much?

46. One cask contains 39 gal. 1 qt. 1 pt. 2 gi. of alcohol, and another 28 gal. 3 qt. 1 pt. 3 gi. How much more does the first contain than the second?

47. In one bin there are 873 bu. 2 pk. 5 qt. of corn; in another, 698 bu. 3 pk. 7 qt. How much less in the second than in the first?

48. A has \$650; B has \$25 less than half as much. C has \$125 less than both A and B. D has \$275 less than A, B, and C. How much have all of them?

49. A and B start from the same place at the same time and travel on the same road. A travels at the rate of 26 miles an hour, and B at the rate of 39 miles an hour. How far apart will they be at the end of 6 hours? If B should then stop, and A should travel at B's rate, how long would it take him to overtake B?

50. From A to B is 492 miles; from B to C is half as many miles, plus 42. How find the difference in the two distances? How many miles from A to C?

51. Three persons bought a mill valued at \$25642. The first paid \$6743.25; the second twice as much; and the third, the remainder. How much did the third pay?

52. At one time I spent $\frac{1}{8}$ of a dollar; at another, $\frac{3}{8}$ of a dollar; and at another, $\frac{2}{8}$ of a dollar. If I had but a dollar at the beginning, what part of it had I left? How many cents?

53. How many more days in the months of March, April, May, and June, counted together, than in the months of September and October?

54. When shopping, a lady bought ribbon for 36 cents, lace for \$1.48, gloves for \$1.75, and velvet for \$1.27. She gave in payment a five-dollar bill, and received her change in nickels and cents. How many nickels did she receive?

55. $8.27 - 6.52 = ?$

56. $264.008 - 79.169 = ?$

57. $.80641 - .27835 = ?$

58. $.01803 - .00657 = ?$

59. $2.061 - .8934 = ?$

60. 7.006521 — .009736 = ?

61. 4.069 + 72.0083 — 16.15328 = ?

62. .08631 + .2483 + .005687 — .0148 — 00693 = ?

63. 263.094 — 172.86 + 52.0048 = ?

64. \$821.054 + \$63.006 — \$279.838 — \$346.765 = ?

65. Bought several articles costing respectively 63¢, 89¢, 48¢, \$1.38, \$2.76, \$4.75. Gave merchant a \$20 bill. What change should I receive?

66. How many years after the discovery of America was Washington born? How old was Washington at the time of the Declaration of Independence? When he was first inaugurated as president? (Omit months and days.)

67. Lincoln was born how many years after Washington? How old was he at the time of his death? If still living, how old would he be? (Omit months and days.)

68. How many years from the battle of Bunker Hill to the attack on Fort Sumter? From the surrender at Yorktown to Lee's surrender? From the battle of Waterloo to the battle of Gettysburg?

69. The National Debt of U. S., on July 1, 1870, was \$2480672427.81. On July 1, 1894, it was \$1632253636.68. How much had it diminished in 24 years?

70. The area of France is 204092 square miles. That of Great Britain and Ireland is 120979 square miles. What is the difference of their areas?

Which of the States of the American Union is larger than either?

It is how much larger than France? Than Great Britain and Ireland?

Which is larger, France or California? How much?

Great Britain and Ireland or California? How much?

71. How many years ago was the Declaration of Independence signed? How many years from the signing of the Declaration of Independence to the close of the Civil War?

AREAS OF STATES AND TERRITORIES.

NEW ENGLAND.

Maine,	33,040.	Vt.,	9,565.	R. I.,	1,250.
N. H.,	9,305.	Mass.,	8,315.	Conn.,	4,990.

MIDDLE ATLANTIC GROUP.

N. Y.,	49,170.	Del.,	2,050.	Va.,	42,450.
N. J.,	7,815.	Md.,	12,210.	W. Va.,	24,780.
Penn.,	45,215.	D. C.,	70.		

COTTON STATES.

N. C.,	52,250.	Ala.,	52,250.	Texas,	265,780.
S. C.,	30,570.	Miss.,	46,810.	Ark.,	53,850.
Ga.	59,475.	La.,	48,720.	Tenn.,	42,050.
Fla.	58,680.				

CENTRAL STATES.

Ky.,	40,400.	Ill.,	56,650.	Wis.,	56,040.
Ohio,	41,060.	Mo,	69,415.	Minn.,	83,365.
Ind.,	36,350.	Mich.,	58,915.	Iowa,	56,025.

THE GREAT PLAIN.

N. D.,	70,795.	Neb.,	77,510.	Okla.,	39,030.
S. D.,	77,650.	Kan.,	82,080.	Ind. T.,	31,400.

MOUNTAIN STATES.

Mont.,	146,080.	Colo.,	103,925.	N. M.,	122,580.
Idaho,	84,800.	Utah,	84,970.	Ariz.,	113,020.
Wy.,	97,890.	Nev.,	110,700.		

PACIFIC STATES.

Cal.,	158,360.	Oreg.	96,030.	Wash.,	69,180.

Alaska, 531,409.

Find difference in area between —

71. Maine and the rest of New England.

72. Texas and the other Gulf States.

73. New England and Illinois.

74. The Central States and the Cotton States.

75. The Pacific States and the Middle Atlantic Group.

76. The Central States and the Great Plain.

77. The five States on the east bank of the Mississippi and the three States on the west bank.

78. How many States each equal to Illinois can be cut out of Texas?

79. Find the total area of all the States east of the Mississippi River, excluding Minnesota and Louisiana.

80. Find the total area of the original thirteen States.

81. A man expended $10,564 for 4 tracts of land. For the first he paid $1,968.50; for the second, $2,680; for the third, $3,127.50. What did he pay for the fourth?

82. The sum of the areas of Maine, Ky., Md., Penn., and a fifth State is 189,072 square miles. What is the area of the fifth State? Which is it?

83. A man bought three buildings. He paid for the first $7,846; for the second, $2,875 more; for the third, $3,182 less than for the second. He put in as part payment a farm for $2,125, and paid the remainder in cash. What was his cash payment?

Latitude and longitude are measured in degrees, minutes, and seconds. Sixty seconds (marked ″) make a minute. Sixty minutes (marked ′) make a degree (marked °).

84. New York is in 74° 3″ west longitude and Boston 71° 3′ 30″ west longitude. Boston is how far east of New York?

85. Chicago is in 87° 35′ west longitude. How far is it west of N. Y.? Of Boston?

86. Albany is 298 miles east of Buffalo and Chicago is 589 west of Buffalo. What is the distance from Albany to Chicago?

87. Berlin is 13° 23′ 43″ E. and New Orleans 90° 3′ 28″ W.; what is the difference of their longitudes?

88. Boston is in 42° 21′ 24″ N. latitude. The latitude of New York is 40° 42′ 43″ N. Boston is how much farther north than New York?

89. London is in latitude 51° 30′ 48″ N. It is how much farther north than Boston?

90. New Orleans is 29° 57′ N. and Rio Janeiro 22° 54′ S. What is their difference in latitude?

91. Find the time from July 6, 1888, to Sept. 10, 1896.

92. Find the time from March 12, 1889, to Oct. 18, 1895.

93. Find the time from June 15, 1887, to April 5, 1897.

NOTE. How many years from June 15, 1887, to June 15, 1896? How many months from June 15, 1896, to March 15, 1897? How many days in March after the 15th? To these add the 5 days in April.

94. Find the time from Aug. 21, 1890, to May 16, 1897.

95. Find the time from Sept. 12, 1891, to Dec. 25, 1897.

96. Find the time from Oct. 28, 1886, to June 19, 1895.

SECTION IV.

MULTIPLICATION.

64. What is the sum of 5 and 5? What is the sum of two 5's? What is the sum of 8 and 8 and 8? What is the sum of three 8's? What is the sum of five 6's? of four 9's? of seven 10's? This will suggest the manner in which the multiplication table is built up. As commonly used it consists of sums formed by repeating the nine primary numbers up to nine of each.

65. **Multiplication is a short method of finding the sum of two or more equal numbers.**

66. **The Multiplicand is one of the two or more equal numbers that are to be united.**

67. **The Multiplier is the number of equal numbers that are to be united.**

68. **The Product is the sum of two or more equal numbers that have been united by multiplication.**

69. **The sign of multiplication is an oblique cross. When the multiplier comes first, the sign is read *times*. When the multiplicand precedes, the sign is read *multiplied by*.**

70. Numbers are spoken of as abstract or concrete. This distinction is of little value except in Multiplication and Division. A number of named objects, as 6 books, is called **a concrete number.** A number whose unit is not named, as 7, or a number of numbers, as 5 nines, is called **abstract.** Accordingly,

1. **The Multiplicand may be abstract or concrete.**
2. **The Multiplier is abstract.**
3. **The Product is like the Multiplicand.**

71. Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

72. EXAMPLES FOR PRACTICE.

These problems and others like them should be used until the results can be spoken with great rapidity.

1. 134869725 × 2

2. 843512769 × 3

3. 4691058327 × 4

4. 9601475328 × 5

5. 7964031825 × 6

6. 6324507198 × 7

7. 31924065708 × 8

8. 2573960814 × 9

Go through these problems, giving the product of the multiplier and each term of the multiplicand without regard to the denomination of the product.

Repeat the operations, naming the denomination of each product.

Repeat the operations, naming the denomination of each product, reducing it to the next higher order when it exceeds nine, and giving the denomination of the remainder and of the reduced number in each case.

73. When the product in any case does not exceed nine, where should it be written? When the product exceeds nine, where should the remainder be written? What should be done with the reduced number?

74. Since the method employed in these cases is applicable in all similar cases, we may make the following rule for multiplication when the multiplier is less than 10.

RULE.

Write the multiplier under the lowest term of the multiplicand. Multiply this term of the multiplicand by the multiplier. If the product is less than 10, place it under the term multiplied. If the product is more than 10, reduce it to the next higher denomination, placing the remainder, if there should be one, under the term multiplied, and adding the reduced number to the product of the multiplier and the next higher term of the multiplicand.

75. Multiplication by tens, hundreds, etc.

Illustrative Example.

```
  8364
    22
 -----
 16728
16728
------
184008
```

76. In this problem the tens' term of the multiplier is how many times the units' term? The product of each term of the multiplicand and the tens' term of the multiplier is how

many times the product of the same terms and the units' term of the multiplier? Where, then, should these results be written?

77. Since a figure standing in tens' order expresses a number ten times as great as if standing in units' order, the products obtained by using the tens' term of the multiplier will belong one order higher than the terms multiplied. Hence,

To multiply by tens, proceed as before, placing the results one order to the left of the term multiplied.

78. Apply the same reasoning to the hundreds', thousands', etc., terms of the multiplier, and make a statement for each.

79. GENERAL RULE.

Write the multiplier under the multiplicand.

Beginning with the units' term of the multiplier, multiply the multiplicand by each term of the multiplier, successively, placing the right-hand figure of each partial product under the term by which it was obtained.

Unite the several partial products.

80. PROOFS.

1. ***Use the multiplicand as the multiplier.***

2. ***Divide the product by either of its factors. The quotient will be the other factor, if the work is correct.***

NOTE. Proof 2 cannot be used until after a study of division.

81. *Illustrative Analysis.* Multiply 826 by 352.

1. We are to unite 352 826's. We first unite 2 826's, then 50 826's, then 300 826's, and then unite these several products.

NOTE. 826's is read eight hundred twenty-sixes.

2. 2 times 826 ones are 1652 ones.

3. 50 times 826 are 5 times ten times 826. Ten times 826 are 826 tens. Since 5 times 826 tens are tens, the right-hand figure of this partial product will be written under the tens' figure of the multiplicand.

4. 300 times 826 are 3 times 100 times 826. 100 times 826 are 826 hundreds. Since 3 times 826 hundreds are hundreds, the right-hand figure of this partial product will be written under the hundreds' figure of the multiplicand.

5. Uniting the partial products gives the complete product.

82. EXAMPLES FOR PRACTICE.

1. Multiply 864 by 24; by 35, by 58; by 79.
2. Multiply 978 by 61; by 83; by 96; by 89.
3. Multiply 4625 by 189; by 265; by 374.
4. Multiply 3718 by 264; by 357; by 819.
5. Multiply 24689 by 345; by 678; by 921.
6. Multiply 13576 by 987; by 654; by 321.
7. Multiply 90876 by 234; by 567; by 892.
8. Multiply 34005 by 306; by 508; by 809.
9. 874 × 81326.
10. 596 × 493725.
11. 2086 × 321068.
12. 34708 × 97540016.
13. 65413 × 2496038.

83. To multiply by 10, 100, 1000, etc., annex as many zeros to the right of the multiplicand as there are in the multiplier.

NOTE. If the decimal point is used, it must be moved as many orders to the right as there are zeros in the multiplier, and the vacant orders must be filled with the zeros.

84. PROBLEMS.

1. Multiply 7642 by 10; by 100; by 10000; by 1000; by 100000.
2. Multiply 246092 by 1000; by 10; by 10000; by 100.
3. Multiply 9284 by 200; by 3000; by 60; by 70000.
4. Multiply 5681 by 2600; by 87000; by 24600; by 360.

85. GENERAL PROBLEMS.

1. What is the cost of 5 cords of wood at $6 a cord?

ANALYSIS. Since each cord cost $6, 5 cords cost 5 sixes of dollars (or 5 times $6), which are $30.

2. What is the cost of 236 acres of land at $72 an acre? Analyze as above.

NOTE. In such problems as number 2, where the multiplicand is smaller than the multiplier, the order of factors may be changed by the following analysis:

If the land cost $1 an acre, 236 acres would cost $236. Since the land cost $72 an acre, 236 acres cost 72 times $236. Use this form of analysis until it can be employed easily and rapidly.

3. If a man travel at the rate of 46 miles a day, how many miles will he travel in 27 days?

4. James earned $264; John, $432; William, twice as much as both James and John; Henry, three times as much as the difference between William's earnings and John's earnings. What did all earn?

5. Thomas bought 4 gal. 3 qt. 1 pt. 2 gi. of milk; Reuben bought 5 times as much. How much did both buy?

6. If 16 men can do a piece of work in 12 days, in how many days can one man do it?

7. If a railway train run 42 miles an hour, how many miles will it run in 678 hours?

8. A locomotive division is 126 miles long. How many miles does an engineer ride in July, if he makes a trip every day?

9. A merchant bought 7 loads of oats, each containing 63 bu. 2 pk. 5 qt. How many did he buy? What did they cost him, at 30 cents a bushel?

10. The distance from Bloomington to Chicago is 126 miles. How many feet of wire are there in 7 telegraph lines connecting the two cities?

11. A man had \$3,000. He bought 6 horses at \$125 each, 5 cows at \$57 each, 2 wagons at \$48 each, and 40 acres of land at \$40 an acre. How much money had he left?

12. Multiply the sum of 826 and 439 by twice their difference.

13. Multiply the sum of 4×729 and 8×563 by 9 times the difference between 23×48 and 65×76.

86. MULTIPLICATION BY FACTORS.

Since $6 = 2 \times 3$, 6 times any number $= 2 \times 5$ times, or 3×2 times, that number; hence, if I wish to multiply a number by 6, I may multiply it by 3 and that product by 2, or multiply it by 2 and that product by 3.

The same plan may be followed with any number that can be factored.

RULE.

Separate the multiplier into two or more factors, multiply by one of them, the resulting product by a second, and so continue until all of the factors have been used. The last product is the required result.

87. PROBLEMS.

1. Multiply 456 by 15, using factors of 15.

ANALYSIS. Since $15 = 3 \times 5$, 15 times 456 = 3 times 5 times 456. 5 times $456 = 2280$; 3 times $2280 = 6840$.

2. $5246 \times 18 = ?$

3. $6792 \times 25 = ?$

4. $24680 \times 48 = ?$

5. $56072 \times 64 = ?$

FORM.

$$\begin{array}{r} 5246 \\ 3 \\ \hline 15738 \\ 6 \\ \hline 94428 \end{array}$$

88. Multiplication by numbers slightly less than a power of 10.

1. $869 \times 99 = ?$

ANALYSIS. If the multiplier were 100, the product would be 86900. Since the multiplier is one less than 100, the product is once 869 less than 86900. $86900 - 869 = 86031$.

2. Multiply 7824 by 999; by 9999.

3. Multiply 862534 by 98; by 998; by 97; by 997; by 9997.

89. Learn the following table of aliquot parts;

	$16\frac{2}{3} = \frac{1}{6}$ of 100.	$166\frac{2}{3} = \frac{1}{6}$ of 1000.
$2\frac{1}{2} = \frac{1}{4}$ of 10.	$25 = \frac{1}{4}$ of 100.	$250 = \frac{1}{4}$ of 1000.
$5 = \frac{1}{2}$ of 10.	$50 = \frac{1}{2}$ of 100.	$500 = \frac{1}{2}$ of 1000.
$7\frac{1}{2} = \frac{3}{4}$ of 10.	$75 = \frac{3}{4}$ of 100.	$750 = \frac{3}{4}$ of 1000.
$3\frac{1}{3} = \frac{1}{3}$ of 10.	$33\frac{1}{3} = \frac{1}{3}$ of 100.	$333\frac{1}{3} = \frac{1}{3}$ of 1000.
$6\frac{2}{3} = \frac{2}{3}$ of 10.	$66\frac{2}{3} = \frac{2}{3}$ of 100.	$666\frac{2}{3} = \frac{2}{3}$ of 1000.
	$12\frac{1}{2} = \frac{1}{8}$ of 100.	$125 = \frac{1}{8}$ of 1000.
	$37\frac{1}{2} = \frac{3}{8}$ of 100.	$375 = \frac{3}{8}$ of 1000.
	$62\frac{1}{2} = \frac{5}{8}$ of 100.	$625 = \frac{5}{8}$ of 1000.
$8\frac{1}{3} = \frac{5}{6}$ of 10.	$83\frac{1}{3} = \frac{5}{6}$ of 100.	$833\frac{1}{3} = \frac{5}{6}$ of 1000.
$8\frac{3}{4} = \frac{7}{8}$ of 10.	$87\frac{1}{2} = \frac{7}{8}$ of 100.	$875 = \frac{7}{8}$ of 1000.

The twelfths and sixteenths of 100 and 1000 may be added for more extended work.

Illustrative Example.

$1284 \times 37\frac{1}{2} = ?$

FORM.

$$\begin{array}{r} 1284 \\ \hline 128400 \\ \hline 16050 \\ \hline 48150 \end{array}$$

ANALYSIS. $37\frac{1}{2} = \frac{3}{8}$ of 100. If the multiplier were 100, the product would be 128400. If the multiplier were $\frac{1}{8}$ of 100, the product would be $\frac{1}{8}$ of 128400, which equals 16050. Since the multiplier is $\frac{3}{8}$ of 100, the product is 3 times 16050, which equals 48150.

90. PROBLEMS.

1. Multiply 48464 by $62\frac{1}{2}$; by $12\frac{1}{2}$; by 625; by 375; by 75.

2. Multiply 58647 by $33\frac{1}{3}$; by $66\frac{2}{3}$; by $333\frac{1}{3}$; by $666\frac{2}{3}$.

3. Multiply 86484 by $16\frac{2}{3}$; by $166\frac{2}{3}$; by $83\frac{1}{3}$; by $833\frac{1}{3}$.

4. What is the cost of 24 dozens of eggs, at $12\frac{1}{2}$ cents a dozen?

ANALYSIS. At one dollar a dozen, 24 dozens cost $24. Since the price is $12\frac{1}{2}$ cents, or $\frac{1}{8}$ of a dollar a dozen, 24 dozens cost $\frac{1}{8}$ of $24, which equals $3.

5. What will $16\frac{2}{3}$ lbs. of sugar cost at 6 cents a pound? 25 lbs.? $33\frac{1}{3}$ lbs.? $66\frac{2}{3}$ lbs.? $83\frac{1}{3}$ lbs.?

ANALYSIS. 100 lbs. cost \$6. $\frac{1}{6}$ of 100 lbs. cost $\frac{1}{6}$ of \$6.

6. What is the cost of 324 objects at $8\frac{1}{3}$ cts. each? at $12\frac{1}{2}$ cts.? at $16\frac{2}{3}$ cts.? at 25 cts.? at $33\frac{1}{3}$ cts.? at $37\frac{1}{2}$ cts.? at 50 cts.? at 75 cts.?

7. Find the cost of 625 objects at $16\frac{2}{3}$ cts.; at 25 cts.; at 50 cts.; at $66\frac{2}{3}$ cts.; at 40 cts.

8. Find the cost of 824 objects at $33\frac{1}{3}$ cts.; at $62\frac{1}{2}$ cts.; at $37\frac{1}{2}$ cts.; at $83\frac{1}{3}$ cts.

9. Find the cost of 625 horses at \$$83\frac{1}{3}$ each; at \$$87\frac{1}{2}$ each; at \$$62\frac{1}{2}$ each.

10. Find the cost of 824 cattle at \$$37\frac{1}{2}$ each; at \$$33\frac{1}{3}$ each; at \$50 each.

91. MISCELLANEOUS PROBLEMS.

1. $8 \times .024 = ?$ $9 \times .0073 = ?$ $27 \times .1694 = ?$
2. $36 \times 3.056 = ?$ $48 \times 17.0835 = ?$ $59 \times 24.16947 = ?$
3. $83 \times .24356 = ?$ $92 \times .3607 = ?$ $98 \times 7.00869 = ?$
4. Multiply .086794 by 8; by 9; by 26; by 37.
5. Multiply 365.432 by 45; by 59; by 138; by 246.
6. $66\frac{2}{3} \times 7086957$.
7. 230000×569842.
8. 998×47952.
9. $(84625 + 53796) \times (63824 - 21706)$.
10. $(821 - 463 + 279) \times (425 + 872 - 328)$.
11. Multiply 3 gal. 2 qt. 1 pt. by 5; by 8; by 9; by 12.
12. Multiply 7 yd. 2 ft. 8 in. by 7; by 10; by 11.
13. Multiply 46 bu. 3 pk. by 15; by 24; by 38; by 49.
14. Bought 160 acres of land at \$65 an acre, and 80 acres at \$75 an acre. Sold 120 acres at \$80 an acre and the remainder at \$66.50 an acre. What was the gain or loss?

15. A dealer bought 3 horses at \$85 each; 5 at \$96 each; 7 at \$124.50 each. He shipped them to the city, the freight averaging \$12.50 each. He sold the cheapest at \$110.50 each; the second lot at \$128 each, and the third lot at \$164 each. If his personal expenses and the care of the horses amounted to \$32, what did he gain by the transaction?

16. Put the following problems in the form of bills with the teacher as the seller and yourself as purchaser:

1. 12 yds. calico at $6\frac{1}{4}$¢; 6 doz. eggs at $12\frac{1}{2}$¢; $3\frac{1}{2}$ lbs. coffee at 42¢; 15 yds. muslin at 11¢; 9 yds. summer silk at 86¢; 3 pairs shoes at \$1.45; 2 bu. potatoes at 65¢.

2. 4 yds. linen at 40¢; 2 pairs gloves at \$1.75; 6 yds. silk at \$1.25; 15 yds. sheeting at 25¢; 6 yds. pillow-casing at $12\frac{1}{2}$¢; 8 yds. towelling at 24¢; $\frac{1}{2}$ yd. velvet at \$1.25; 6 handkerchiefs at 25¢; 2 waists at \$1.25.

17. A stone falls 16 feet the first second, $(16 + 32)$ feet the second second, $(16 + 32 + 32)$ feet the third second, and so on. How many feet will it fall in 8 seconds?

18. Light travels 186000 miles a second. It takes 498 seconds for a light wave to pass from the sun to the earth. What is the distance?

19. A bushel of corn in the ear weighs 70 pounds; shelled, 56 pounds. How many pounds of cobs in a crib containing 1800 bushels of ears?

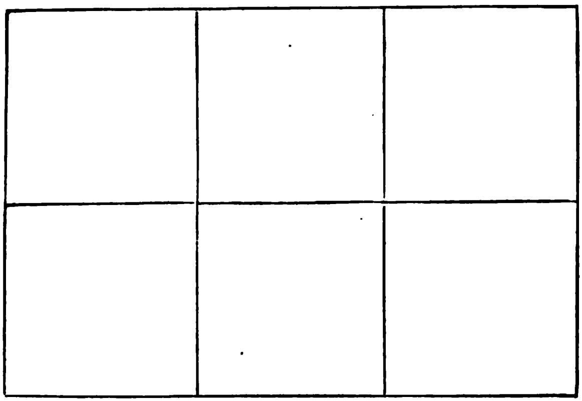
20. Make a pendulum by fastening a split bullet to a thread 30 inches long. Count the vibrations in a second. How many vibrations will it make in a minute? in an hour? in a day? in a week?

21. A car is loaded with 49 steel rails 32 feet long and weighing 78 pounds to the yard. The weight of the car and its load is 62592 pounds. What is the weight of the car?

22. If the above car be loaded with 512 bushels of shelled corn, what will the car and its load weigh?

23. What is the estimated number of words in a book containing 240 pages, each page averaging 350 words?

92. SURFACE MEASURE.



1. This figure is 3 inches long and 2 inches wide. How many square inches in each horizontal row? In the whole figure?

2. Draw a surface 5 inches long and 3 inches wide. How many square inches in each row? in the whole figure?

3. This page is 5 inches by 7 inches. How many square inches in a single row along the right-hand margin? How many such rows? How many square inches in the page?

NOTE. In the following problems the measurements should be made by the pupils, with tape-line, yard-stick, or foot-rule. The nearest integral number of units should be taken. Thus, if tho desk-top in the next problem is 20¾ inches long, it should be taken as 21 inches.

4. How many square inches in your desk-top? in all the desk-tops in the school-room?

5. How many square inches in a pane in your nearest window? in all the panes? in all the windows?

6. How many square feet in your school-room floor? square inches?

7. In a well-lighted school-room the floor-surface is not more than six times the window surface. Is your room "well-lighted"?

8. Would the leaves in this book cover all the glass in the school-room windows?

9. How many square feet of blackboard in your school-room? What did it cost? (Reckon natural slate @ 25¢ per sq. ft.; artificial slate at 11¢ per sq. ft.)

10. Draw on blackboard a diagram of the school-room floor, scale one inch to the foot.

11. Draw the same on paper, scale one quarter-inch to the foot.

12. Draw a diagram of the north wall using same scale.

13. Calculate separately the number of feet of wainscoting, blackboard, plaster, doors, windows.

14. How many square feet in a base-ball diamond?

NOTE. The teacher may increase this list indefinitely. The best problems are those drawn from the pupil's surroundings or from his studies.

15. How many square feet are there in a lot 99 feet wide and 208 feet deep?

16. What is the value of the above lot at $9\frac{1}{2}$ cents a square foot?

17. If flooring costs 3 cents a square foot, what did the lumber in the floor of your school-room cost?

18. If it costs 3 cents a square foot to lath and plaster a wall, what was the cost of plastering the ceiling of your school-room?

19. If matting sells for 4 cents a square foot, what will it cost to cover the floor of your school-room with it?

SECTION V.

DIVISION.

93. Separate 12 crayons into groups of 4 crayons each. Separate 12 crayons into 4 equal groups. In each of these processes the 12 has been separated into equal numbers; either of the processes is called **Division.** In the first process we are given the number to be separated and the *size* of the equal groups, — that is, the 12 are to be measured off into 4's. This kind of division is called **Measurement.** In the second process we are given the number to be separated and the *number* of equal groups to be made of it. This kind of division is called **Partition.**

94. DEFINITIONS.

1. **Division is the process of separating a number into equal numbers.**

2. **Measurement is the process of separating a number into equal numbers of a given size.**

3. The **Dividend** in **Measurement** is the number that is to be separated into equal numbers of a given size.

4. The **Divisor** in **Measurement** is one of the equal numbers into which the dividend is to be separated.

5. The **Quotient** in **Measurement** is the number of equal numbers into which the dividend has been separated.

6. **Partition is the process of separating a number into a given number of equal numbers.**

7. The **Dividend** in **Partition** is the number to be separated into a given number of equal numbers.

8. The **Divisor** in **Partition** is the number of equal numbers into which the dividend is to be separated.

9. The **Quotient in Partition** is one of the equal numbers into which the dividend has been separated.

10. The **Remainder** in each case is the undivided part of the dividend.

95 1. In measurement the divisor and dividend have the same unit. The quotient, being a number of numbers, is abstract. The remainder is like the dividend.

2. In partition the dividend and quotient are alike and the divisor is abstract. The remainder is like the dividend.

93. Which of the following problems illustrate measurement and which partition? Name divisor, dividend, and quotient in each case and show how the definitions apply.

PROBLEMS.

1. An 80-acre field was divided into 10-acre lots. How many did it make?

2. A stage coach went 6 miles an hour; how many hours were required to go 30 miles?

3. A school-room contains 54 seats arranged in 6 equal rows; how many seats are there in each row?

4. At 3 cents each, how many oranges can be bought for 30 cents?

5. If 5 barrels of flour cost $20, what is the price per barrel?

6. If a school of 42 pupils were divided into 6 equal classes, how many pupils would there be in each class?

7. With divisor and quotient in each of above problems, make a problem in multiplication; with dividend and quotient, a problem in division, and tell kind.

97. There are four signs of division. They are:), ÷, :, —

The divisor is placed at the left of the first sign, at the right of the second and third, and below the fourth.

Illustrations.

4) 12 (3. $12 \div 4 = 3.$ $12 : 4 = 3.$ $\frac{12}{4} = 3.$

98. By reversing the order of the multiplication table the measurement and partition tables are formed. Show how this is done with $6 \times 8 = 48$.

99. PROBLEMS IN MEASUREMENT.

1. Use 2 as a divisor and give the quotients for all numbers from 2 to 19.

Illustration. In 2 there is one 2. In 3 there is one 2 and half of another, etc.

2. Using 3 as a divisor, do the same with numbers from 3 to 29.

3. With 4 from 4 to 39.
4. With 5 from 5 to 49.
5. With 6 from 6 to 59.
6. With 7 from 7 to 69.
7. With 8 from 8 to 79.
8. With 9 from 9 to 89.
9. With 10 from 10 to 99.
10. With 11 from 11 to 109.
11. With 12 from 12 to 119.
12. With 16 from 16 to 144.

100. PROBLEMS IN PARTITION.

1. Use 2 as a divisor, and give quotients for all numbers from 2 to 19.

Illustration. One half of 2 is 1. One half of 3 is $1\frac{1}{2}$.

2. With 3 to 29.

Continue these exercises through the same numbers as in division.

101. EXAMPLES FOR PRACTICE IN MEASUREMENT.

1. Divide 8648 by 3.

FORM.

$$3\,\underline{)\,8648}$$
$$2882\tfrac{2}{3}$$

ANALYSIS. There are two 3's in 8 with a remainder of 2. Since the 8 is thousands, the quotient and remainder are thousands. 2 thousands = 20 hundreds. 20 hundreds and 6 hundreds are 26 hundreds. There are eight 3's in 26, with a remainder of 2. Since the 26 is hundreds, the quotient and remainder are hundreds. 2 hundreds = 20 tens. 20 tens and 4 tens are 24 tens. There are eight 3's in 24. Since the 24 is tens, the quotient is tens. There are two 3's in 8, with a remainder of 2. 2 is $\frac{2}{3}$ of 3. Hence in 8648 there are $2882\frac{2}{3}$ threes.

2. $7463 \div 4 = ?$	**7.** $560074 \div 9 = ?$
3. $9608 \div 5 = ?$	**8.** $730620 \div 10 = ?$
4. $21279 \div 6 = ?$	**9.** $582763 \div 11 = ?$
5. $39852 \div 7 = ?$	**10.** $9806436 \div 12 = ?$
6. $463024 \div 8 = ?$	**11.** $2668937 \div 16 = ?$

102. Explain the eleven preceding problems by partition.

FORM.

$$3\,\underline{)\,8648}$$
$$2882\tfrac{2}{3}$$

ANALYSIS. One third of 8 thousands is 2 thousands, with a remainder of 2 thousands 2 thousands = 20 hundreds. 20 hundreds and 6 hundreds are 26 hundreds. One third of 26 hundreds is 8 hundreds, with a remainder of 2 hundreds. 2 hundreds = 20 tens. 20 tens and 4 tens are 24 tens. One third of 24 tens is 8 tens. One third of 8 is 2, with a remainder of 2. $\frac{1}{3}$ of 2 is $\frac{2}{3}$. Hence one third of 8648 is $2882\frac{2}{3}$.

103. In problems like the preceding, in which the product of divisor and quotient, the remainder, and the new partial dividend are remembered and not written, the work is said to be by Short Division. The products are not written because they are included in the ordinary multiplication table.

Solve and analyze the following rapidly by both methods:

1. $289634 \div 9 = ?$	**6.** $24937068 \div 11 = ?$
2. $5879639 \div 11 = ?$	**7.** $569478370 \div 16 = ?$
3. $608579031 \div 12 = ?$	**8.** $439450682 \div 12 = ?$
4. $250769037 \div 16 = ?$	**9.** $713069458 \div 9 = ?$
5. $508763018 \div 12 = ?$	**10.** $864037926 \div 11 = ?$

104. LONG DIVISION.

Illustrative Problem. 98684 ÷ 42 = ?

FORM.

```
Divisor. Dividend. Quotient.
   42 ) 98684 ( 2349
        84
        146
        126
         208
         168
          404
          378
           26 Remainder.
```

105. EXPLANATION.

There are two 42's in 98. Multiplying the divisor by the first term of the quotient, we find their product to be 84, which we write beneath the partial dividend. The remainder is found to be 14. Since the 98 is thousands, the quotient and remainder are thousands. 14 thousands = 140 hundreds. To this number is added the 6 hundreds of the dividend. 140 hundreds and 6 hundreds = 146 hundreds.

In 146 there are three 42's. Performing the multiplication, to find how much 146 exceeds three 42's, the remainder is found to be 20. Since the 146 is hundreds, the quotient and remainder are hundreds. 20 hundreds = 200 tens. 200 tens + 8 tens = 208 tens. In 208 there are four 42's, with a remainder of 40. Since the 208 is tens, the quotient and remainder are tens. 40 tens = 400 ones. 400 ones + 4 ones = 404 ones. In 404 there are nine 42's, with a remainder of 26.

The 26 may be left as a remainder, or we may indicate the part of 42 that it is and join it to the quotient. It is $\frac{26}{42}$ of 42. Hence in 98684 there are 2349 42's and $\frac{26}{42}$ of another, or simply $2349\frac{26}{42}$ 42's.

To test the work find the product of the divisor and quotient, and to it add the remainder. The result should equal the dividend.

106. Analyze the same problem by partition, using the analysis previously given.

107. REMAINDERS.

Note that in many problems we have an integral quotient with a remainder, as, $25 \div 4 = 6$ with a remainder of 1; or we may complete the division obtaining a fractional quotient, $25 \div 4 = 6\frac{1}{4}$. Whether we shall say, 25 divided into 4's are six 4's with a remainder of 1, or 25 divided into 4's are six and one fourth 4's, depends upon the nature of the problem in which these numerical relations are found.

25 cents will buy how many tin cups at 4 cents each?

25 cents will buy how many pounds of sugar at 4 cents each?

In which of these problems is there an undivided remainder?

Make five concrete problems in measurement involving remainders; five in partition.

Make five concrete problems in measurement involving fractional quotients; five in partition.

108. RULE FOR LONG DIVISION.

At the left of the dividend take a partial dividend that will contain the divisor, and place the first term of the quotient at the right.

Multiply the divisor by the term of the quotient thus obtained, write the product beneath the partial dividend, and subtract.

To the remainder thus obtained annex the next term of the dividend for a second partial dividend, and proceed as before.

NOTE 1. If a new partial dividend will not contain the divisor, place a cipher in the quotient, annex the next term of the dividend, and proceed as before.

NOTE 2. In obtaining any term of the quotient, compare the first term of the divisor with the first part of the partial dividend.

NOTE 3. Prove the problems as in short division.

109. EXAMPLES FOR PRACTICE.

NOTE. Long Division usually begins with 13 as a divisor. In solving the following problems let the pupils make and refer to the multiplication tables for 13, 14, etc., to 19, until they can find readily any term of the quotient. Thus: $2 \times 13 = 26$; $3 \times 13 = 39$, etc.

1. Divide by 13: 31668; 56641; 49784; 73433; 987205; 1266238.

2. Divide by 14: 3430; 4536; 6490; 80132; 96039; 1219553; 1285310.

3. Divide by 15: 3705; 5777; 10269; 91185; 59947; 131549.

4. Divide by 16: 3952; 97264; 140318; 942448; 1376048; 892751; 394887; 1427759; 8895779; 1086331.

5. Divide by 17: 228225; 453151; 821993; 998746; 1201847; 15943688; 1482275; 6293332; 1172540.

6. Divide by 18: 244153; 438753; 644508; 988513; 877858; 1313982; 1743954; 1507995.

7. Divide by 19: 6574; 32908; 268669; 586915; 1290105; 1078000; 1798712; 1515876; 1311040; 1410003.

8. Divide 68324 by 20; 30; 40; 50; 60; 70; 80; 90.

9. Divide 47906 by 21; 31; 41; 51; 61; 71; 81; 91.

10. Divide 74583 by 22; 32; 42; 52; 62; 72; 82; 92.

11. Divide 194873 by 23; 33; 43; 53; 63; 73; 83; 93.

12. Divide 108460 by 24; 34; 44; 54; 64; 74; 84; 94.

13. Divide 298765 by 25; 35; 45; 55; 65; 75; 85; 95.

14. Divide 370625 by 26; 36; 46; 56; 66; 76; 86; 96.

15. Divide 872056 by 27; 37; 47; 57; 67; 77; 87; 97.

16. Divide 476921 by 28; 38; 48; 58; 68; 78; 88; 98.

17. Divide 572183 by 29; 39; 49; 59; 69; 79; 89; 99.

18. 109278 ÷ 234; 109278 ÷ 467.

19. 934605 ÷ 567; 934605 ÷ 1667.

20. 27732494 ÷ 4658; 27732494 ÷ 5943.

21. 794006387 ÷ 568946.

22. 74320876 ÷ 6958.

23. 14173345 ÷ 2005.

24. 14173345 ÷ 7069.

25. 602305812 ÷ 70003.

26. 602305812 ÷ 8604.

27. A bought 37 acres of land, for which he paid $2,664. What was the price per acre?

Measurement or partition? Why?

ANALYSIS. Since 37 acres cost $2,664, each acre cost one thirty-seventh of $2,664.

28. At $72 an acre, how many acres of land can be bought for $2,664?

Measurement or partition? Why?

ANALYSIS. Since each acre cost $72, as many acres can be bought for $2,664 as there are 72's in 2,664. There are thirty-seven 72's in 2,664. Hence, for $2,664, 37 acres can be bought, at $72 an acre.

Analyze each of the following:

29. At 43 miles an hour, how long will it take a train to run 1,677 miles?

30. If a train run 1,677 miles in 39 hours, what is the rate per hour?

31. 38 pieces of cloth cost $1,786. What was the average price?

32 At $47 a piece, how many pieces of cloth can be bought for $1,786?

33. A farmer sold his wheat at 97 cents a bushel, receiving $353.08. How many bushels did he sell?

34. A farmer sold 364 bushels of wheat for $353.08. What was the price per bushel?

35. 15 bu. 3 pk. 4 qt. of oats were divided into 4 equal piles. What amount was there in each pile?

36. Divide 24 gal. 3 qt. 1 pt. 2 gi. of vinegar into 7 equal parts. What is the amount in each part?

37. The divisor is 328, the quotient 407, and the remainder 279. What is the dividend?

38. Make a rule for finding the dividend when the divisor, quotient, and remainder are given.

39. The dividend is 364,280, the quotient 877, and the remainder 325. What is the divisor?

40. Make a rule for finding the divisor when the dividend, remainder, and quotient are given.

110. Division by 10, 100, etc.

1. $8640 \div 10$. 2. $4900 \div 100$. 3. $596000 \div 1000$.

4. Make a rule based on the three preceding problems.

5. $58764 \div 100$.

What is the quotient in Problem 5? the remainder? Make a rule for such cases. Does this include every case?

RULE FOR DIVIDING BY A POWER OF 10.

Cut off from the right of the dividend as many figures as there are ciphers in the divisor. The part thus cut off is the remainder, and the rest of the dividend is the quotient.

6. Divide 79640 by 10; 100; 1000; 10000.

7. Make and solve six problems like the above.

111. Changing problems from measurement to partition, or from partition to measurement.

1. Paid 75 cents for oranges at 5 cents each. How many did I buy?

Give analysis like that of Problem 28, Art. **109**.

Change to partition.

ANALYSIS. If the oranges had cost one cent each, 75 cents would have bought 75 oranges. Since they cost 5 cents each, 75 cents bought one fifth of 75 oranges. One fifth of 75 oranges is 15 oranges.

2. 25 acres of land cost $650. What was the average price per acre?

Give analysis like that of Problem 27, Art. **109**.

Change to measurement.

ANALYSIS. If each acre had cost $1, 25 acres would have cost $25. Since 25 acres cost $650, each acre cost as many times $1 as there are 25's in 650. There are 26 25's in 650. Hence, each acre cost $26.

3. Review Problems 29–34, inclusive, Art. **109**, making the changes as indicated above.

112. Division by Aliquot Parts.

Review the aliquot parts of 100 and 1000 given in multiplication.

1. Divide 73800 by $37\frac{1}{2}$.

FORM.

$$\begin{array}{r} 73800 \\ \hline 738 \\ \hline 5904 \\ \hline 1968 \end{array}$$

ANALYSIS. $37\frac{1}{2}$ is $\frac{3}{8}$ of 100.

If the divisor were 100, the quotient would be 738.

If the divisor were $\frac{1}{8}$ of 100, the quotient would be eight 738's, which are 5904.

Since the divisor is $\frac{3}{8}$ of 100, the quotient is $\frac{1}{3}$ of 5904, which is 1968.

2. Divide 465500 by 25; $37\frac{1}{2}$; $62\frac{1}{2}$; $87\frac{1}{2}$.
3. Divide 39683000 by 125; 375; 625; 875.
4. Divide 269500 by $8\frac{1}{3}$; $16\frac{2}{3}$; $33\frac{1}{3}$; $41\frac{2}{3}$; $58\frac{1}{3}$; $91\frac{2}{3}$.
5. Divide 5005000 by $16\frac{2}{3}$; $33\frac{1}{3}$; $41\frac{2}{3}$; $58\frac{1}{3}$; $91\frac{2}{3}$.

6. How many dozens of eggs, at $12\frac{1}{2}$ cents a dozen, can be bought for \$2? \$5? \$3.75? \$6.50?

ANALYSES. (*a*) If the eggs were \$1 a dozen, \$2 would buy 2 dozens. Since the eggs are $\frac{1}{8}$ of \$1 a dozen, \$2 will buy 8×2 dozens, which = 16 dozens.

Or (*b*) At $12\frac{1}{2}$ cents a dozen, \$1 will buy 8 dozens, and \$2 will buy twice 8 dozens, which equals 16 dozens.

7. How many pounds of butter can be bought for \$7.50 at 25 cents a pound? at $33\frac{1}{3}$ cents? at $16\frac{2}{3}$ cents? at 50 cents?

8. How many yards of cloth can be bought for \$10.50 at $37\frac{1}{2}$ cents a yard? at $62\frac{1}{2}$ cents? at $87\frac{1}{2}$ cents?

113. Division by Factors.

Division may sometimes be simplified by dividing by the factors of the divisor, thus performing the problem by "short division" instead of by "long division." The only difficulty is in understanding the remainders.

1. Divide 6936 by 12.

Explanation. $\frac{1}{12}$ of $6936 = \frac{1}{3}$ of $\frac{1}{4}$ of 6936. $\frac{1}{4}$ of $6936 = 1734$. $\frac{1}{3}$ of $1734 = 578$.

2. $11745 \div 15 = ?$

3. $135072 \div 24 = ?$

4. $150010 \div 35 = ?$

5. $5377792 \div 56 = ?$

114. To Find the True Remainder.

$8661 \div 42 = ?$

Explanation. $42 = 2 \times 3 \times 7$. This problem, and all similar problems, may be read in this way: How many 42's are there in 8661?

There are 4330 2's in 8661, and a remainder of 1. If 4330 2's be separated into groups, each of which shall contain three 2's, there will be 1443 such groups, with a remainder of one 2. Each of these groups contains three 2's, which equals one 6.

Separating 1443 6's into groups each of which shall contain seven 6's, we find that there are 206 such groups, and a remainder of one 6. 8661 contains 206 42's, with a remainder of one 1, one 2, and one 6, whose sum is 9.

The first remainder in such cases is ones; each of the units in the second remainder equals the first divisor; each of the units in the third remainder is obviously equal to the product of the first and second divisor, and so on.

Explain also as a problem in partition.

115. RULE FOR FINDING THE TRUE REMAINDER.

Multiply the remainder obtained by each division by all of the preceding divisors. To the sum of these products add the first remainder.

1. $43259 \div 110$. $110 = 2 \times 5 \times 11$.
2. $64727 \div 385$. $385 = 5 \times 7 \times 11$.
3. $583077 \div 308$. $308 = 4 \times 7 \times 11$.
4. $9386457 \div 343$.

116. 1. $\$10.72 \div 4 = ?$ (Explain by partition.) $\$8.76 \div 6 = ?$ $\$78.52 \div 8 = ?$

2. $10.72 \div 4 = ?$ $8.76 \div 6 = ?$ $78.52 \div 8 = ?$
3. $87.437 \div 7$; $9.6864 \div 16$; $.624 \div 25$; $.0834 \div 75$.
4. $.00384 \div 4$; by 48; by 64.
5. $.16842 \div 24$; by 225; by 192.

117. OUTLINE REVIEW.

1. Numbers may be:
   (1) Expressed.
   (2) United.
   (3) Separated.
2. Numbers are expressed by:
   (1) The Arabic method.
   (2) The Roman method.

3. Numbers are united by:
   (1) Addition.
   (2) Multiplication.

4. In addition the numbers are alike, and may be equal or unequal.

5. In multiplication the numbers are alike and equal.

6. The product is of the same unit as the multiplicand. The multiplier is abstract.

7. Numbers may be separated by:
   (1) Subtraction.
   (2) Measurement.
   (3) Partition.

8. In subtraction a number is separated into two parts that may be equal or unequal.

9. The subtrahend and remainder are of the same unit as the minuend.

10. In measurement and partition numbers are separated into two or more equal parts. In measurement the size of the parts is given, and the number is required.

11. The dividend and divisor are of the same denomination, and the quotient is abstract

12. In partition the number of parts is given, and the size is required.

13. The dividend and quotient are of the same unit, and the divisor is abstract.

118. ORAL EXERCISES.

1. 4 times 6 are how many times 8?
2. 6 times 8 are how many times 12?
3. 4 times 14 are how many times 8?
4. 5 times 12 are how many times 15?
5. 8 times 9 are how many times 4?
6. 9 times 12 are how many times 6 times 3?
7. 12 times 8 are how many times 4 times 6?

8. 5 times 6 plus 4 times 8 are how many times 2?

9. 3 times 16 plus 4 times 6 are how many times 2 times 9?

10. 4 times 16 less 3 times 4 are how many times 13?

11. 5 times 13 plus 2 times 5 are how many times 25?

12. 4 times 17 plus 3 times 9 are how many times 19?

13. 4 times 19 plus 2 times 7 less 3 times 17 are how many times 13?

14. 5 times 16 are how many times 10?

15. 4 times 16 plus 6 are how many times 7?

16. If 2 oranges cost 10 cents, what will 7 oranges cost?

17. If 9 yards of muslin cost 108 cents, what will 7 yards cost?

18. If 11 books cost $4.40, what will 6 books cost?

19. If a man travels 72 miles in 4 hours, how many miles will he travel in 5 hours going at the same rate?

20. If 12 acres of oats yield 480 bushels, how many bushels will 17 acres yield at the same average?

21. If $24 buy 12 yards of cloth, how many yards will $36 buy at the same price?

22. If 11 cords of wood cost $44, how many cords can be bought for $72 at the same price? for $84? for $92? for $68?

23. If 7 men can do a piece of work in 10 days, in how many days can 2 men working at the same rate do the work? 5 men? 14 men?

24. If 8 men can do a piece of work in 12 days, how many men would be required to do the same work in 6 days? in 8 days? in 16 days? in 24 days?

25. Sold 3 dozen eggs at 12 cents, and 2 pounds of butter at 24 cents. What is the change out of a dollar?

26. Sold 5 articles at 10 cents, 2 articles at 12 cents, and 2 at 10 cents. Find change out of a dollar.

27. Sold 3 articles at 15 cents, 3 at 10 cents, and 1 at 8 cents. Find change for a dollar.

28. Sold 4 articles at 15 cents, 2 at 8 cents, and 1 at 14 cents. Find change for a dollar.

29. In how many days can 5 men earn as much as 10 men can earn in 4 days?

30. Five 9's are how many 15's?

31. How many days will be required for 8 men to do the work performed by 12 men in 4 days?

32. Twelve 7's are how many 21's?

33. If 4 oranges are worth 12 apples, how many apples are 23 oranges worth?

34. How many 9's in three 21's + 18?

35. What is the cost of 24 bushels of apples if 16 bushels cost $9.60?

36. 7 times 8, — 5 is how many 17's?

37. In 12 days 8 men will earn as much as how many in 16 days?

38. 16 times $37\frac{1}{2}$ is how many times $12\frac{1}{2}$?

39. How many bushels of oats at 25 cents can be bought for 125 bushels of wheat at 75 cents?

40. 25 times $16\frac{2}{3}$ is how many times $83\frac{1}{3}$?

41. Bought 13 barrels of flour for $91; gave 9 of them for apples at $3 a barrel. How many were received?

42. 8 times 12 — 9 are how many 29's?

43. Bought 13 barrels of flour for $78; gave 6 of them for 12 barrels of apples. What were the apples a barrel?

119. THE LAW OF SIGNS.

The brackets [ ], parentheses ( ), braces { }, and vinculum $\overline{\quad}$ are called symbols of aggregation; enclosed expressions are called bracketed expressions; operations within

the bracketed expressions must be performed first, thus: $12 - [5 \times 2] + [30 \div 6] = 12 - 10 + 5 = 7$.

If no brackets occur, multiplications and divisions are performed in order before any additions or subtractions, thus: $12 - 5 \times 2 + 30 \div 5 \times 3 = 12 - 10 + 18 = 20$.

EXERCISES.

1. $(8 - \overline{2 + 3}) \times (6 + 7 - 9) = ?$
2. $4 \times [6 - \{11 - (5 + 3)\} + 2] = ?$
3. $19 + 3 \times 3 - 64 \div 8 + 5 \times 4 = ?$
4. $[30 \div 5 \times 2 + 9 \times 2] \div [10 + 60 \div 12] = ?$
5. $[60 \div 6 + 4 \times 3] \div [15 - 4] \times [90 \times 2 \div 3 + 40] = ?$

120. 1. $(86429 - 4786) - (7512 - 482)$.

2. $86429 - 4786 - 7512 - 482$.

3. The divisor is 879, the quotient 46, and the remainder 23; what is the dividend?

4. The remainder is 279 and the subtrahend 673; what is the minuend?

5. The product is 4212, and the multiplier 78; what is the multiplicand?

6. The minuend is 964, and the remainder 278; what is the subtrahend?

7. The product is 195 and the multiplicand 39; what is the multiplier?

8. Find the sum of

3 bu.	2 pk.	4 qt.
5 "	1 "	6 "
7 "	3 "	2 "
4 "	0 "	7 "
12 "	3 "	5 "

9. A merchant, having 23 gal. 2 qt. 1 pt. 2 gi. of vinegar, sold 17 gal. 3 qt. 1 pt. 3 gi. What did he have left?

10. A had 125 bushels of corn; B had twice as much and 25 bushels more; C had as much as both A and B; D had as much as the difference between A's and B's. The corn was sold at $37\frac{1}{2}$ cents a bushel. What did it bring?

11. How many seconds are there in a minute? minutes in an hour? hours in a day? days in a week?

12. A man worked 6 days of 8 h. 23 m. 46 s. each. How many hours, minutes, and seconds did he work?

13. In 6 days a man put in 50 h. 22 m. 36 s. What was his daily average?

14. $(24 - 6) \div 3 = ?$ $24 - 6 \div 3 = ?$

15. $(3 + 4) \times 2 = ?$ $3 + 4 \times 2 = ?$

16. $8 \times 2 - (6 - 2) \div 2 \times 4 + 2 \times 13 \div 5 = ?$

17. I bought 125 acres of land at \$60 an acre. I spent \$1,250 to fence it. At what price an acre must I sell it to gain \$1,000?

18. If 12 men can do a piece of work in 15 days, in how many days can 20 men do it?

19. 16 men undertook a piece of work that would take them 24 days; when it was half done, 10 of them left. In how many days could those remaining finish it?

20. A farmer bought 20 pounds of sugar at 6 cents a pound; 10 yards of cloth at 75 cents a yard; 2 pounds of tea at 50 cents a pound; 15 pounds of coffee at 20 cents a pound. He sold the merchant an equal number of bushels of potatoes and apples, getting 40 cents a bushel for the former, and 60 cents a bushel for the latter, and received 70 cents in change. How many bushels of each did he sell?

21. A merchant bought 48 yards of cloth at 62 cents a yard, and 81 yards at 75 cents a yard. He sold the former at 81 cents a yard, and the latter at such a price as to gain \$20.37 on the whole transaction. At what price did he sell it per yard?

22. How many States equal in area to New England can be made from Texas? What is the remainder? It is nearest the area of what State? Is about what part of it?

23. The total area of the cotton States is how many times that of New England?

24. Compare that portion of the United States lying west of the Mississippi River with the portion lying east of it.

25. The populations of the New England States in 1890 were as follows: Me., 666,086; N. H., 376,530; Vt., 349,290; Mass., 2,238,943; R. I., 345,506; Conn., 746,258. Find the population per square mile for each. Arrange them in the order of the density of population.

26. If your State were as densely populated as Massachusetts, how many people would it contain?

27. Draw a row of 6 square inches. How many such rows are needed to make 24 square inches? 36 square inches? 60? 144?

28. Your tablet page is 9 inches long. If it were one inch wide, what would be its area? How many inches wide must it be to contain 45 square inches? 63 square inches? 81? 108? 144?

29. The sheet upon which I write contains 80 square inches; it is 10 inches long; how wide is it?

ANALYSIS. Were the sheet one inch wide, it would contain 10 square inches. To contain 80 square inches it must be as many inches wide as there are 10's in 80. There are 8 10's in 80. Hence the sheet is 8 inches wide.

30. In the window at my elbow, each pane is 14 inches wide and contains 392 square inches. How long is each?

31. What is the width of the desk-top upon which I write? Its area is 999 square inches, and its length 37 inches.

32. The floor of this room is 26 feet wide and contains 819 square feet. How long is it? How many square yards of linoleum must I buy to cover it?

33. A 40-acre field is 1,320 feet square; how wide a strip along one side contains an acre (43,560 square feet)? How many corn-rows, 3 feet 8 inches apart, can be planted in this strip the long way?

34. A man bought a piece of land 40 rods long and 24 rods wide. He bought a second piece containing as many square rods but 30 rods wide; what was its length? He paid $72.50 an acre (160 square rods) for the two pieces; what did they cost?

35. A town lot having a depth of 161 feet and containing 19,642 square feet sold for $2,013; what was the price per front foot?

36. Lumber is sold by the thousand feet, the unit being a board 12 inches long, 12 inches wide, and one inch thick. If flooring is worth $25 a thousand, what did the lumber cost to floor your school-room?

37. If at the above price the flooring for a room cost $32, how many square feet were there in the floor? If the room was 40 feet long, what was its width?

38. A man's crop of oats brought him $493.90, the selling price being 22 cents a bushel. The field in which he raised them was 80 rods wide and 90 rods long. What was the average yield per acre?

39. Bought wheat at 63 cents a bushel, paying for it $2,844.12; how many bushels were bought?

40. At 21 cents a bushel, how many bushels will $188.74 buy?

41. The President of the United States receives $50,000 a year; how much is that for each day of 1896?

42. An Illinois farmer sold his farm, consisting of 320 acres, at $82.50 an acre, and invested the proceeds in western land at $30 an acre; how many acres did he buy?

43. Sold a piece of property for $3,825; another for $4,682; a third for $5,620. After paying a debt of $6,327

he invested the remainder in land at $65 an acre. How many acres did he get?

44. How much will it cost to cover your school-room floor with matting costing 32 cents a yard?

45. If it cost $33.60 to cover the floor of a room 36 feet long with matting a yard wide, worth 28 cents a yard, what is the width of the room?

46. Place 4 inch-cubes in line. Place 3 such rows side by side. Place another cube upon each of the 12 cubes. How long is the solid you have formed? How wide? How high? How many cubic inches in a solid 4 in. × 3 in. × 2 in.?

47. How many inch-cubes are needed to build a solid 6 in. × 5 in. × 4 in.?

48. How many inch-cubes will fill a chalk box 6 in. × 3 in. × 3 in.?

NOTE. The number of cubic units that a box or vessel holds is called its capacity.

49. What is the capacity of an upper drawer of my desk which is 21 in. × 10 in. × 3 in.?

ANALYSIS. A drawer 1 inch long, 1 inch wide, and 1 inch deep holds 1 cubic inch. A drawer 21 inches long, 1 inch wide, and 1 inch deep holds 21 times 1 cubic inch, which is 21 cubic inches. A drawer 21 inches long, 10 inches wide, and 1 inch deep holds 10 times 21 cubic inches, which is 210 cubic inches. A drawer 21 inches long, 10 inches wide, and 3 inches deep holds 3 times 210 cubic inches, which is 630 cubic inches.

50. The next drawer is 21 in. × 10 in. × 4 in. What is its capacity?

51. The lowest drawer contains 1,680 cubic inches. Its bottom is 21 in. × 10 in. How many cubic inches are needed to cover the bottom 1 inch deep? How many such layers to fill the drawer? What is the depth of the drawer?

52. 231 cubic inches equal 1 gallon. A wagon tank is 10 feet long and 33 inches wide, inside measure. 360 gallons fill it to what depth?

53. What is the capacity of my school-room, 30 ft. × 24 ft. × 15 ft.?

54. A cubic foot of air, at 70 degrees Fahrenheit, weighs 525 grains. How many grains of air in the school-room?

55. 7000 grains equal 1 pound. How many pounds of air in your school-room?

56. The coined gold of the world is estimated at 10,800 cubic feet. To what height will it reach if piled uniformly so as to cover your school-room floor?

NOTE. The number of cubic units in a solid is called its volume.

57. What is the volume of a brick 8 in. × 4 in. × 2 in.?

58. What is the volume of a pine sill 8 in. × 12 in. × 24 ft.?

NOTE. In describing timbers the dimensions are given in this order: thickness, width, length.

59. Dry pine weighs 29 pounds to the cubic foot. What is the weight of the sill?

60. How many men are needed to carry it, no man lifting more than 200 pounds?

61. Joliet limestone weighs 160 pounds to the cubic foot. What is the weight of a stone step 6 in. × 3 ft. × 12 ft.?

121. PROPERTIES OF NUMBERS.

1. Numbers are **Integral, Fractional,** or **Mixed.**

2. **An Integral Number** is a number of whole units.

3. Integral numbers are classified with respect to their divisibility into **Composite, Prime, Even,** and **Odd.**

4. **A Factor** of a number is one of the two or more integral numbers which being multiplied together will produce that number. It is consequently a divisor of that number.

5. **A Composite Number** is a number that has factors beside itself and 1.

4, 10, 35, are composite numbers.

6. A Prime Number is a number that has no factors except itself and 1.

1, 2, 3, 5, etc., are prime numbers.

7. Two numbers are **prime to each other** when they have no common factor.

8. An Even Number is a number that contains 2 as a factor.

4, 6, 8, 10, etc., are even numbers.

9. An Odd Number is a number that does not contain 2 as a factor.

3, 5, 7, 9, etc., are odd numbers.

NOTE. Readiness in factoring depends upon knowing certain properties of numbers, hence they should be carefully noted.

122. PRINCIPLES.

1. A factor of a number is a factor of any number of times that number.

This is obvious, since every time the number is repeated the factor is repeated.

Illustration. 5 is a factor of 10. It must then be a factor of any number of 10's.

2. A common factor of two numbers is a factor of their sum.

Illustration. 3 is a factor of 6 and of 9. It must then be a factor of 15. For 6 is two 3's and 9 is three 3's, hence the sum of 6 and 9 is five 3's.

Demonstration. Since each number is some number of times the common factor, their sum must be some number of times the common factor.

3. A divisor of two numbers is a divisor of their difference.

Illustration. If 5 is a common factor of two numbers, as 15 and 25, each must be composed of 5's. If one exceeds

the other it must be because it has more 5's, hence the difference must be 5's.

Demonstration. Since each is some number of times the common divisor, if they differ it is because one of them contains the common divisor more times than the other, hence their difference is divisible by the common divisor.

123. TESTS OF DIVISIBILITY.

1. Any number is divisible by 2 if its right-hand figure is 2, 4, 6, 8, or 0.

2. Any number is divisible by 3 if the sum of the ones expressed by its digits is divisible by 3.

Thus, 263457 is divisible by 3 because the sum of 2, 6, 3, 4, 5, and 7 is divisible by 3.

3. A number is divisible by 4 if the number expressed by the two right-hand digits is divisible by 4.

Thus, 39484 is divisible by 4 because 84 is divisible by 4.

4. A number is divisible by 5 if the right-hand digit is 0 or 5.

5. Any number is divisible by 6 if divisible by 2 and 3.

Thus, 27684 is divisible by 6 because it is even, and the sum of the ones expressed by its digits is divisible by 3.

6. A number is divisible by 8 if the number expressed by the three right-hand digits is divisible by 8.

Thus, 6252144 is divisible by 8 because 144 is divisible by 8.

7. A number is divisible by 9 if the sum of the ones expressed by its digits is divisible by 9.

Thus, 8645373 is divisible by 9 because $8 + 6 + 4 + 5 + 3 + 7 + 3 = 36$, a multiple of 9.

8. A number is divisible by 10 if the right-hand figure is 0.

9. **A number is divisible by 11 if the sum of the ones expressed by the digits in the odd orders equals the sum of the ones expressed by the digits in the even orders, or if the difference of these sums is a multiple of 11.**

Examine 8730645.

The odd orders are occupied by 5, 6, 3, and 8. The sum of these numbers is 22. The even orders are occupied by 4, 0, 7. The sum of these numbers is 11. $22 - 11 = 11$; hence, this number is divisible by 11.

10. The tests given are those most commonly used. Others might be added, but they are of little practical value.

Make a test for 12, for 15, and for 18.

11. When shall we conclude that a number is prime?

Examine the number 293. Is it divisible by 2? Why? By 3? Why? Should 4 be tried? Why? Is it divisible by 5? by 7? by 11? by 13? by 17? Why not try 6, 8, 9, 10, 12, 14, and 16? What numbers have really been tried as divisors? Examine the quotients obtained by these successive divisions. How large are they? Shall 19 be tried? What would be true of its quotient? What would be true if any number larger than 19 should be tried? Is 293 prime? How do you know?

12. We may conclude that a number is prime if the successive prime numbers have been tried as divisors until the quotient is less than the divisor, provided there has been a remainder after each division.

If the divisions should be continued, the quotients would be found to be numbers already tried as divisors. No larger number than the prime last used can be an exact divisor, for, if it were, the quotient would be an exact divisor also; but this we have seen to be impossible.

124. Exercises in testing Divisibility.

Tell whether or not the following numbers are divisible by three, and give the reason.

Illustrative Example. 45687 is divisible by three because the sum of the 1's expressed by its digits, 30, is divisible by three.

1. 6087546.
2. 790439.
3. 802563.
4. 587642.
5. 7300872.
6. 14568396.

Apply tests for 4, 5, 6, on the following numbers, and give reasons:

7. 24640.
8. 73860.
9. 915465.
10. 145380.
11. 13060.
12. 785630364.

Apply tests for 8 and 9 on the following:

13. 45144.
14. 846396.
15. 4591872.
16. 1046832.
17. 5685309.
18. 43549488.

Apply test for 11 on the following:

Illustrative Example. 604759969.

The sum of the 1's expressed by the digits in the odd orders is 33, and by the digits in the even orders is 22. The difference of these sums is 11, hence the number is divisible by 11.

19. 497882605.
20. 65834078.
21. 138071406.
22. 24086937.
23. 356543847.
24. 958263547.

Are the following numbers prime? Give the reason for your answer.

25. 887, 941, 767, 1187, 1201, 899.

125. Demonstration of Tests.

TEST FOR FOUR.

1. Any number of more than two orders may be regarded as some number of hundreds plus the number expressed by the two right-hand figures.

Since one hundred is divisible by four, any number of hundreds must be, by Principle 1. If the number expressed by the two right-hand figures is divisible by four, the whole number is, by Principle 2.

Illustrative Number. 79284 = 79200 + 84.

79200 is divisible by four, by Principle 1. 84 is divisible by four; hence, 79284 is divisible by four.

TEST FOR FIVE.

2. Any number of more than one order may be regarded as some number of tens plus the number expressed by the right-hand figure. If the right-hand figure is zero, the number is tens; and since one ten is divisible by five, the number must be divisible by five, by Principle 2.

Illustrate with a number.

TEST FOR EIGHT.

3. Any number of more than three orders may be regarded as some number of thousands plus the number expressed by three right-hand figures. Since one thousand is divisible by eight, any number of thousands is so divisible, by Principle 1. If the number expressed by the three right-hand digits is divisible by eight, the whole number must be, by Principle 2.

Illustrate with a number.

TEST FOR NINE.

4. To understand this test examine the nature of the decimal system.

$10 = 9 + 1.$	$20 = (2 \times 9) + 2.$	$30 = 27 + 3.$
$100 = 99 + 1.$	$200 = 2 \times 99 + 2.$	$300 = 297 + 3.$
$1000 = 999 + 1.$	$2000 = 2 \times 999 + 2.$	$3000 = 2997 + 3.$

From this partial table it is evident that 1 of any order exceeds some multiple of 9 by 1; 2 of any order exceeds some multiple of 9 by 2; hence, we may say:

A digit in any place expresses a number that exceeds some multiple of 9 by as many ones as the digit expresses when standing alone.

Since a given number is the sum of the numbers expressed by its several digits ($349 = 300 + 40 + 9$), it follows that any number exceeds some multiple of 9 by the sum of the ones expressed by its separate digits.

If this sum is a multiple of 9, the given number is the sum of two multiples of 9 and is therefore divisible by 9. (Principle 2.)

TEST FOR ELEVEN.

5. The following statements will make this test clear:

(1) One ten is one less than a multiple of eleven; two tens are two less; and any number of tens are as many less as there are tens.

A similar statement may be made for thousands, hundred-thousands, ten-millions, etc. But tens, thousands, hundred-thousands, ten-millions, etc., occupy orders whose numbers, counting from the right, are even. Hence, a digit standing in an order whose number is even, expresses a number which is as many less than a multiple of eleven as the number of ones expressed by the digit.

(2) In a similar manner it may be shown that a digit standing in an order whose number is odd expresses a number which is as many more than a multiple of eleven as there are ones expressed by the digit.

(3) If these remainders are equal, they balance each other, and the number is a multiple of eleven. If one set of remainders exceeds the other by a multiple of eleven, it must follow that the whole number is a multiple of eleven.

126. FACTORING.

1. Learn to apply the test readily. Try the successive primes, beginning, usually, with 2.

2. If the right-hand figure is 0, the factors 2 and 5 are readily recognized. In such numbers as 25000, each 0 implies 2 and 5, hence the factors may be read off at once:

2, 5, 2, 5, 2, 5, 5, 5.

3. Numbers ending in 25, 50, or 75 may be factored by inspection, by remembering that each hundred contains 4 25's.

Illustration. $1575 = 1500 + 75$. $1500 = 60 \times 25$. $75 = 3 \times 25$. $60 \times 25 + 3 \times 25 = 63 \times 25 = 3 \times 3 \times 7 \times 5 \times 5$.

Write the prime factors of numbers to 100, in the following form:

$$4 = 2 \times 2.$$
$$6 = 2 \times 3.$$
$$8 = 2 \times 2 \times 2. \quad \text{Etc.}$$

This expression may be read, "$4 = 2$ times 2," or "the prime factors of 4 are 2 and 2."

Learn the prime numbers to 100 so that they can be repeated easily in ten seconds.

Prime factor the following:

(1)	(2)	(3)	(4)
102	201	301	400
105	203	304	403
108	217	310	407
120	221	319	427
125	247	323	437
150	250	343	451
152	259	361	469
164	287	371	473
175	289	380	481
186	299	391	497

(5)	(6)	(7)	(8)
500	650	825	10560
525	675	833	15824
529	700	851	24860
539	703	869	55625
583	731	899	73812
595	749	900	100000
600	767	917	121212
611	799	940	255850
629	800	950	640000
637	804	975	1000000

127. CANCELLATION.

Cancellation is a method of shortening the work in problems involving only multiplication and division.

PRINCIPLES.

(1) **Dividing any one of a series of factors by any number divides their product by that number.**

(2) **Dividing dividend and divisor by the same number does not change the quotient.**

1. Divide $8 \times 9 \times 6$ by 4; by 9; by 12; by 72.

Illustration. $\frac{1}{12}$ of $8 \times 9 \times 6 = \frac{1}{4}$ of $\frac{1}{3}$ of $8 \times 9 \times 6$. $\frac{1}{4}$ of $8 \times 9 \times 6 = 2 \times 9 \times 6$. $\frac{1}{3}$ of $2 \times 9 \times 6 = 2 \times 3 \times 6$.

2. $5 \times 8 \times 0 \times 6 = ?$

Illustrative Problem. $\dfrac{24 \times 45 \times 60}{18 \times 20} = ?$

EXPLANATION.

This is a problem in division in which divisor and dividend are partially factored. I shorten the operation by dividing divisor and dividend by 9. $\frac{1}{9}$ of the divisor is 2×20. $\frac{1}{9}$ of the dividend is $24 \times 5 \times 60$. I further shorten the operation by dividing divisor and dividend by 4. $\frac{1}{4}$ of the dividend is $6 \times 5 \times 60$, etc.

3. $\dfrac{12 \times 36 \times 51}{24 \times 18 \times 34} = ?$

4. Divide $27 \times 35 \times 52$ by $18 \times 7 \times 13$.

5. Divide $92 \times 87 \times 57 \times 69$ by $23 \times 23 \times 19 \times 29$.

6. Divide $140 \times 169 \times 510$ by 39×68.

7. How many baskets of eggs, each containing 12 dozens, at 15 cents a dozen, will pay for 8 bolts of cloth, each containing 24 yards, at 30 cents a yard?

128. ANALYSIS.

1. If 63 books cost $126, what will 125 books cost?

FORM.

$$\frac{\$126 \times 125}{63}.$$

ANALYSIS. Since the question asks for the cost of certain articles, I begin with $126, writing it above a short horizontal line. If 63 books cost $126, each book will cost one sixty-third of $126, which is expressed by writing 63 below the line as a divisor. 125 books will cost 125 times this number of dollars, which is expressed by writing 125 above the line as a factor of the dividend. Cancelling the common factors and completing the work, the result is $250.

2. If 15 men can do a piece of work in 7 days, in how many days can 21 men do the same work?

3. If 24 men dig a ditch in 18 days, how many would be required to dig the same ditch in 27 days?

4. If 11 tons of hay can be made from 5 acres, at the same rate, how many tons can be made from 65 acres?

5. If 12 acres of land raise 720 bushels of corn, how many acres would be needed to raise 1,800 bushels at the same rate?

6. If 26 horses eat a certain quantity of grain in 39 days, how many days would it last 338 horses?

7. If a certain quantity of grain last 46 horses 34 days, how many horses would eat the same amount in 391 days?

8. 64 men can do a piece of work in 57 days, working 9 hours a day; in how many days can 38 do the same work, working 8 hours a day?

9. If 42 men do a piece of work in 18 days, working 10 hours a day, how many men can do the same work in 90 days, working 7 hours a day?

10. If 91 men can do a certain amount of work in 54 days, working 9 hours a day, how many hours a day must 162 men work to perform the same labor in 39 days?

11. The interest on what amount of money at 8 per cent, for 87 days, equals the interest on $2,500 at 7 per cent for 261 days?

12. At what rate will the interest on $3,200 for 92 days equal the interest on $4,800 for 46 days, at 6 per cent?

13. For how many days must $965 be loaned in order that the interest on it at 5 per cent shall equal the interest on $2,123 for 125 days, at 11 per cent?

14. How many men working 12 hours a day will be needed to dig a ditch 1,500 ft. long, 8 ft. wide, and 6 ft. deep, in 250 days, if 36 men in 180 days of 9 hours each can dig a ditch 1,080 ft. long, 9 ft. wide, and 12 ft. deep, the work to be uniformly difficult?

For further problems see Simple and Compound Proportion.

SECTION VI.

FRACTIONS.

129. DEFINITIONS.

1. A fractional number, or, more briefly, **a fraction**, is a number that is composed of one or more **fractional units**. *Illustration.* $\frac{2}{3}$, $\frac{5}{12}$.

2. A fractional unit is one of the equal parts into which a whole has been separated. *Illustration.* $\frac{1}{3}$, $\frac{1}{12}$.

3. The **numerator** of a fraction is the number of fractional units in the fractional number. In the fractions $\frac{2}{3}$, $\frac{5}{8}$, the numerators are 2 and 5.

4. The **denominator** of a fraction is the number that shows the size of the fractional unit. This it does by showing the number of equal parts into which the whole has been separated. *Illustration.* In the fractions $\frac{2}{5}$, $\frac{7}{12}$, 5 and 12 are the denominators. The numerator and denominator are the terms of the fraction.

5. Fractions are classified, with respect to their value, into proper and improper.

6. A **proper fraction** is a fraction whose value is less than one. Its numerator is less than its denominator. *Illustration.* $\frac{5}{8}$, $\frac{7}{13}$.

7. An **improper fraction** is a fraction whose value is equal to or greater than one. Its numerator is equal to or greater than its denominator. *Illustration.* $\frac{8}{8}$, $\frac{9}{7}$.

8. Fractions are classified, with respect to their form, into simple, complex, and compound.

9. A **simple fraction** is a fraction whose terms are integers. *Illustration.* $\frac{5}{8}$, $\frac{4}{5}$.

10. **A complex fraction is a fraction which contains a fraction in one or both of its terms.**

Illustration. $\frac{4}{\frac{2}{3}}$, $\frac{\frac{2}{5}}{7}$, $\frac{\frac{3}{5}}{\frac{4}{7}}$, $\frac{4\frac{1}{2}}{8}$, etc.

11. **A compound fraction** consists of two or more simple fractions joined by *of*. *Illustration.* $\frac{2}{3}$ of $\frac{4}{5}$.

12. **A mixed number** is composed of an integer and a fraction. *Illustration.* $4\frac{5}{8}$.

13. Tell whether each of the following is proper or improper, and give the definition in each case.

$$\tfrac{4}{5},\ \tfrac{5}{4},\ \frac{2\frac{1}{2}}{3},\ \frac{3\frac{1}{2}}{2\frac{1}{4}},\ \tfrac{3}{4} \text{ of } \tfrac{5}{6},\ \frac{\frac{2}{3}}{\frac{5}{6}}.$$

14. Tell whether each of the following is simple, complex, or compound, and give the definition in each case.

$$\tfrac{5}{8},\ \tfrac{8}{7},\ \frac{3\frac{1}{2}}{7},\ \tfrac{2}{5} \text{ of } \tfrac{5}{6},\ \frac{\frac{2}{3}}{\frac{3}{4}},\ \frac{8}{\frac{2}{5}}.$$

15. A fraction is in its lowest terms when the numerator and denominator are prime to each other.

130. PRINCIPLES.

I. Multiplying the numerator of a fraction by an integer multiplies the fraction by the integer.

Since the number of fractional units in the fractional number is multiplied by the integer, while their size is unchanged, the fraction is multiplied by the integer. Illustrate.

II. Dividing the numerator of a fraction by an integer divides the fraction by the integer.

Since the number of fractional units in the fractional number is divided by the integer, while their size is unchanged, the fraction is divided by the integer. Illustrate.

III. Multiplying the denominator of a fraction by an integer divides the fraction by the integer.

If the number of equal parts into which a unit has been separated be doubled, each part will be one half as large as before. If the denominator of a fraction be multiplied by any integer, the unit will be divided into as many times the number of parts that it was before as the integer is times one. The resulting fractional units will be the same part of the former fractional units that one is of the integer. Since the number of fractional units is unchanged, the fraction must be divided by the integer. Illustrate.

IV. Dividing the denominator of a fraction by an integer multiplies the fraction by the integer.

If the number of equal parts into which a unit has been separated be divided by an integer, the resulting fractional units will be as many times the former fractional units as the integer is times one. Since the numerator is unchanged, the fraction is multiplied by the integer. Illustrate.

V. Multiplying both terms of a fraction by the same number does not change its value.

Illustration. $\frac{1}{2} = \frac{4}{8}$. There are 4 times as many fractional units in $\frac{4}{8}$ as in $\frac{1}{2}$, but each is only $\frac{1}{4}$ as large.

VI. Dividing both terms of a fraction by the same number does not change its value.

Illustration. $\frac{4}{8} = \frac{1}{2}$. There are only $\frac{1}{4}$ as many fractional units in $\frac{1}{2}$ as in $\frac{4}{8}$, but each is 4 times as large.

131. REDUCTION.

Reduction of fractions is the process of changing their denomination without changing their value.

Review Reduction, page 6.

132. *Illustrative Example.* In \$5 there are how many quarters?

ANALYSIS. Since in \$1 there are 4 quarters, in \$5 there are 5 fours of quarters, which are 20 quarters.

1. In \$7 there are how many tenths of \$1? in \$8? in \$9? in \$16? in \$86?

2. How many quarter-yards in 5 yards? in 12 yards? in 15 yards? in 25 yards? in 63 yards?

3. Change 7 to fifths; 8 to thirds; 12 to eighths; 15 to sixteenths.

4. Tell how to reduce any integer to an equivalent fraction having any denominator.

133. *Illustrative Example.* In \$$7\frac{3}{4}$ there are how many quarters?

ANALYSIS. Since in \$1 there are 4 quarters, in \$7 there are 7 fours of quarters, which are 28 quarters. 28 quarters and 3 quarters are 31 quarters.

PROBLEMS.

Change the following to improper fractions and analyze the process.

NOTE. Illustrate Problems 1–3 with paper circles.

1. $5\frac{1}{2}$.	**6.** $8\frac{2}{3}$.	**11.** $64\frac{3}{7}$.	**16.** $6\frac{1}{4}$.	**21.** $33\frac{1}{3}$.
2. $2\frac{3}{4}$.	**7.** $12\frac{9}{10}$.	**12.** $85\frac{19}{20}$.	**17.** $18\frac{3}{4}$.	**22.** $56\frac{1}{4}$.
3. $3\frac{3}{8}$.	**8.** $15\frac{3}{4}$.	**13.** $91\frac{6}{17}$.	**18.** $41\frac{2}{3}$.	**23.** $66\frac{2}{3}$.
4. $3\frac{2}{5}$.	**9.** $20\frac{14}{15}$.	**14.** $34\frac{7}{27}$.	**19.** $26\frac{2}{3}$.	**24.** $81\frac{1}{4}$.
5. $5\frac{7}{12}$.	**10.** $25\frac{3}{4}$.	**15.** $512\frac{5}{11}$.	**20.** $62\frac{1}{2}$.	**25.** $83\frac{3}{4}$.

134. Solve the above problems, giving the following

ANALYSIS. $5\frac{1}{2} = ?$ Since in one there are 2 halves, in any integer there are twice as many halves as ones; hence in 5 there are two times 5 halves, which are $\frac{10}{2}$, $\frac{10}{2} + \frac{1}{2} = \frac{11}{2}$.

Show how the following rule may be made from the above analysis.

RULE.

For reducing a mixed number to an improper fraction.

Multiply the integer by the denominator of the fraction. To this result add the numerator, and write the sum over the denominator.

135. Change:

1. 8 to 11ths.	**7.** 9 to 15ths.	**13.** $18\frac{3}{7}$ to 7ths.
2. $6\frac{2}{5}$ to 5ths.	**8.** 10 to 18ths.	**14.** $34\frac{3}{17}$ to 17ths.
3. 12 to 10ths.	**9.** $11\frac{5}{6}$ to 6ths.	**15.** $46\frac{9}{25}$ to 25ths.
4. $12\frac{1}{3}$ to 3ds.	**10.** 15 to 21sts.	**16.** $50\frac{7}{30}$ to 30ths.
5. 7 to 8ths.	**11.** $14\frac{3}{10}$ to 10ths.	**17.** $65\frac{11}{12}$ to 12ths.
6. $7\frac{3}{8}$ to 8ths.	**12.** $17\frac{9}{16}$ to 16ths.	**18.** $72\frac{7}{18}$ to 18ths.

136. *Illustrative Example.*

Change 24 quarters of a dollar to dollars.

ANALYSIS. Since in $1 there are 4 quarters, in 24 quarters of a dollar there are as many dollars as there are fours in 24. There are 6 fours in 24; hence in 24 quarters of a dollar there are $6.

Similarly reduce the following fractions to whole numbers:

1. $\frac{8}{2}$, $\frac{6}{3}$, $\frac{10}{5}$, $\frac{12}{4}$.
2. $\frac{12}{3}$, $\frac{14}{7}$, $\frac{12}{6}$, $\frac{16}{4}$.
3. $\frac{16}{8}$, $\frac{20}{5}$, $\frac{21}{7}$, $\frac{25}{5}$.
4. $\frac{30}{10}$, $\frac{36}{12}$, $\frac{40}{20}$, $\frac{42}{14}$.
5. $\frac{65}{13}$, $\frac{72}{18}$, $\frac{72}{24}$, $\frac{72}{36}$.
6. $\frac{75}{15}$, $\frac{75}{25}$, $\frac{76}{4}$, $\frac{76}{19}$.
7. $\frac{78}{3}$, $\frac{78}{6}$, $\frac{78}{13}$, $\frac{81}{9}$.
8. $\frac{81}{27}$, $\frac{87}{3}$, $\frac{87}{29}$, $\frac{91}{13}$.
9. $\frac{128}{8}$, $\frac{128}{16}$, $\frac{135}{5}$, $\frac{144}{16}$.
10. $\frac{225}{9}$, $\frac{225}{25}$, $\frac{250}{50}$, $\frac{288}{36}$.

137. *Illustrative Example.* Reduce $\frac{27}{5}$ to a mixed number.

ANALYSIS. Since in one there are $\frac{5}{5}$, in $\frac{27}{5}$ there are as many ones as there are fives in 27. There are $5\frac{2}{5}$ fives in 27; hence there are $5\frac{2}{5}$ ones in $\frac{27}{5}$.

Change the following fractions to whole or mixed numbers:

1. $\frac{14}{3}$, $\frac{21}{5}$, $\frac{32}{7}$.
2. $\frac{37}{8}$, $\frac{38}{6}$, $\frac{45}{4}$.
3. $\frac{48}{3}$, $\frac{48}{5}$, $\frac{48}{12}$.
4. $\frac{63}{10}$, $\frac{65}{11}$, $\frac{68}{7}$.
5. $\frac{78}{12}$, $\frac{83}{13}$, $\frac{85}{17}$.
6. $\frac{91}{14}$, $\frac{91}{13}$, $\frac{92}{17}$.
7. $\frac{127}{15}$, $\frac{234}{19}$, $\frac{318}{23}$.
8. $\frac{479}{27}$, $\frac{529}{23}$, $\frac{576}{26}$.
9. $\frac{234}{31}$, $\frac{261}{38}$, $\frac{279}{43}$.
10. $\frac{321}{47}$, $\frac{355}{54}$, $\frac{441}{63}$.
11. $\frac{433}{72}$, $\frac{516}{86}$, $\frac{873}{97}$.
12. $\frac{625}{125}$, $\frac{526}{134}$, $\frac{810}{162}$.
13. $\frac{569}{218}$, $\frac{834}{312}$, $\frac{1256}{420}$.
14. $\frac{92}{23}$, $\frac{96}{24}$, $\frac{119}{17}$.
15. $\frac{153}{17}$, $\frac{113}{19}$, $\frac{152}{19}$.
16. $\frac{2826}{33}$, $\frac{3824}{35}$.

RULE.

To reduce an improper fraction to a whole or mixed number, divide the numerator by the denominator.

17. $7\frac{2}{3}$ feet are how many thirds of a foot? $\frac{28}{3}$ of a foot equal how many feet?

18. $12\frac{2}{5}$ yards equal how many fifths of a yard? $\frac{87}{5}$ of a yard equal how many yards?

19. $\$17\frac{3}{10}$ equal how many dimes? $\$\frac{187}{10}$ equal how many dollars?

20. $15\frac{3}{8}$ pounds are how many eighths of a pound? $\frac{99}{8}$ of a pound are how many pounds?

NOTE. Give many dictation exercises until facility in solution and analysis is acquired.

138. *Illustrative Example.* Reduce $\frac{25}{30}$ to its lowest terms.

ANALYSIS. Dividing numerator and denominator by 5, the resulting terms are prime to each other, hence the fraction is in its lowest terms. $\frac{5}{6}$ equals $\frac{25}{30}$, by Principle 6; or, $\frac{5}{6}$ equals $\frac{25}{30}$ because the resulting fractional units are 5 times as large as the former, and there are but $\frac{1}{5}$ as many of them.

Reduce the following to their lowest terms:

1. $\frac{6}{8}$, $\frac{10}{15}$, $\frac{12}{18}$, $\frac{25}{30}$.
2. $\frac{24}{32}$, $\frac{27}{36}$, $\frac{22}{77}$, $\frac{18}{54}$.
3. $\frac{63}{84}$, $\frac{44}{76}$, $\frac{52}{60}$, $\frac{27}{81}$.
4. $\frac{36}{72}$, $\frac{39}{78}$, $\frac{65}{95}$, $\frac{78}{104}$.
5. $\frac{42}{56}$, $\frac{48}{60}$, $\frac{52}{65}$, $\frac{63}{81}$.
6. $\frac{72}{84}$, $\frac{78}{91}$, $\frac{81}{90}$, $\frac{84}{98}$.
7. $\frac{66}{99}$, $\frac{82}{123}$, $\frac{86}{129}$, $\frac{90}{99}$.
8. $\frac{88}{99}$, $\frac{94}{141}$, $\frac{96}{144}$, $\frac{100}{125}$.
9. $\frac{144}{180}$, $\frac{288}{306}$, $\frac{775}{950}$.
10. $\frac{836}{924}$, $\frac{875}{975}$.

RULE.

To reduce a fraction to its lowest terms, continue the division of the terms of the fraction by the same number until they are prime to each other.

Express the following in the simplest form:

11. $\frac{25}{75}$ of \$1, $\$\frac{45}{60}$, $\$\frac{36}{108}$, $\$\frac{24}{60}$.
12. $\frac{17}{51}$ of a yard, $\frac{24}{96}$ yard, $\frac{125}{300}$ yard, $\frac{108}{240}$ yard.
13. $\frac{15}{35}$ of a bushel, $\frac{18}{21}$ bushel, $\frac{9}{24}$ bushel, $\frac{25}{45}$ bushel.
14. $\frac{7}{42}$ of a mile, $\frac{21}{49}$ mile, $\frac{48}{75}$ mile, $\frac{51}{85}$ mile.

139. *Illustrative Example.* Change $\frac{7}{12}$ to 60ths.

1. Analysis. To change $\frac{7}{12}$ to 60ths, each 12th must be separated into 5 equal parts. $\frac{7}{12}$ contain 7 times $\frac{5}{60}$, which equals $\frac{35}{60}$. Hence, to change $\frac{7}{12}$ to 60ths I multiply both terms by 5.

2. Since in one there are $\frac{60}{60}$, in $\frac{7}{12}$ there are $\frac{7}{12}$ of $\frac{60}{60}$. $\frac{1}{12}$ of $\frac{60}{60}$ is $\frac{5}{60}$. $\frac{7}{12}$ of $\frac{60}{60}$ is $7 \times \frac{5}{60} = \frac{35}{60}$.

Is this reduction ascending or descending?

PROBLEMS.

Change:

1. $\frac{5}{8}$ to 40ths.
2. $\frac{7}{15}$ to 45ths.
3. $\frac{7}{8}$ to 56ths.
4. $\frac{4}{9}$ to 36ths.
5. $\frac{1}{20}$ to 80ths.
6. $\frac{7}{25}$ to 100ths.
7. $\frac{9}{11}$ to 55ths.
8. $\frac{3}{4}$ to 60ths.
9. $\frac{7}{12}$ to 96ths.

RULE.

To change a fraction to an equivalent fraction having any denominator, multiply both terms of the fraction by the quotient arising from dividing the desired denominator by the given denominator.

Note. Give many dictation exercises.

Change:

10. $\frac{5}{6}$ to 18ths.
11. $\frac{11}{12}$ to 48ths.
12. $\frac{9}{16}$ to 96ths.
13. $\frac{7}{10}$ to 120ths.
14. $\frac{17}{21}$ to 105ths.
15. $\frac{19}{24}$ to 144ths.
16. $\frac{31}{32}$ to 128ths.
17. $\frac{5}{36}$ to 108ths.
18. $\frac{19}{37}$ to 333ds.
19. $\frac{23}{40}$ to 360ths.
20. $\frac{37}{42}$ to 630ths.
21. $\frac{41}{44}$ to 924ths.
22. $\frac{44}{49}$ to 343ds.
23. $\frac{5}{54}$ to 378ths.
24. $\frac{17}{86}$ to 860ths.
25. $\frac{97}{125}$ to 1000ths.
26. $\frac{83}{150}$ to 1350ths.
27. $\frac{127}{256}$ to 2816ths.
28. $\frac{324}{353}$ to 2118ths.
29. $\frac{469}{521}$ to 1563ds.
30. $\frac{599}{631}$ to 2524ths.
31. $\frac{72}{650}$ to 2600ths.

140. ADDITION OF FRACTIONS.

1. Define addition, sum. Only what numbers can be united? Add $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$. Add $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

2. What is true of the fractions in each of these problems? How is their addition performed?

3. In the problem $\frac{2}{3} + \frac{3}{4} = ?$ the fractional units in the first fraction are unlike those in the second. Before these two fractions can be united they must be made alike; that is, they must be changed to equivalent fractions having the same fractional unit. The denominators will then be alike; that is, the new fractions will have a common denominator.

4. If each of the thirds be divided into two equal parts, what are the resulting fractional units called? into four? Name three other fractional units that may be made from thirds. Which is the largest of all these fractional units? In the same way, name successive fractional units that may be made from fourths. Have you found any that can be made from either thirds or fourths? Which were they? $\frac{2}{3}$ equals how many twelfths? $\frac{3}{4}$ equals how many twelfths? Then $\frac{2}{3} + \frac{3}{4} = ?$

PROBLEMS.

1. $\frac{1}{2} + \frac{1}{4}$.	5. $\frac{2}{3} + \frac{5}{6}$.	9. $\frac{2}{3} + \frac{1}{2} + \frac{5}{6}$.
2. $\frac{2}{3} + \frac{3}{5}$.	6. $\frac{3}{4} + \frac{5}{6}$.	10. $\frac{3}{4} + \frac{1}{6} + \frac{2}{3}$.
3. $\frac{1}{4} + \frac{2}{5}$.	7. $\frac{4}{5} + \frac{1}{6}$.	11. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
4. $\frac{1}{2} + \frac{4}{5}$.	8. $\frac{1}{3} + \frac{5}{8}$.	12. $\frac{2}{3} + \frac{3}{5} + \frac{5}{6}$.

It is thus seen that to unite unlike fractions they must be changed to equivalent fractions having a common denominator, hence we must learn how to find this common denominator.

141. LEAST COMMON MULTIPLE.

1. Let us consider Problem 2 in Art. 140.

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$, etc. These denominators contain what common factor? Every number containing 3 as a

factor is a multiple of 3. Name multiples of 4, 5, 6, 8, 10.

$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20}$, etc. These denominators are multiples of 5.

15, found in both sets of denominators, is a multiple of both 3 and 5, hence is called a **common multiple.** Name two other common multiples of 3 and 5.

Name two common multiples of 2, 3, and 4; of 3, 6, and 9; of 4, 5, and 8; of 3, 4, 5, and 6. Name three common multiples of 2, 4, and 5; of 3, 8, and 6; of 4, 5, and 8.

2. A **multiple** of a number is an integral number of times that number. It is consequently exactly divisible by that number.

3. A **common multiple** of two or more numbers is a number that is a multiple of each of them. It is consequently divisible by each of them.

4. A multiple of a number contains all of the prime factors of that number.

5. A common multiple of two or more numbers contains all of the prime factors of each of them.

6. How many common multiples may two or more numbers have?

7. Name three common multiples of 2 and 3; of 4 and 5; of 3, 4, and 6. Which is the least common multiple in each case?

8. The **least common multiple** of two or more numbers is the least number that is a multiple of each of them.

9. It is the product of such prime factors, and such only, as are necessary to produce the several numbers.

142. *Illustrative Example.* 18, 24, 40.

FORM.

$$\left.\begin{array}{l} 18 = 2 \times 3 \times 3 \\ 24 = 2 \times 2 \times 2 \times 3 \\ 40 = 2 \times 2 \times 2 \times 5 \end{array}\right\} 2 \times 2 \times 2 \times 5 \times 3 \times 3 = 360.$$

Analysis. The prime factors of 18 are 2 and two 3's; of 24 are three 2's and 3; of 40 are three 2's and 5. Since the l. c. m. of these numbers must contain 40, it must contain its prime factors, which I use as factors of the l. c. m. Since the l. c. m. must contain 24, it must contain also the prime factor of 24 not found in 40; hence, I use 3 as a prime factor of the l. c. m. Since the l. c. m. must contain 18, it must contain also the prime factor of 18 not found in 40 or 24; hence, I use 3 as a factor of the l. c. m. The product of 2, 2, 2, 5, 3, and 3 is the l. c. m. of these numbers, because it contains such prime factors, and only such, as are necessary to produce the several numbers.

EXAMPLES.

Find the l. c. m. of the following:

1. 2, 3, 4.
2. 3, 4, 6.
3. 4, 6, 9.
4. 5, 6, 8.
5. 6, 8, 9, 10.
6. 4, 5, 6, 8.
7. 3, 5, 6, 8.
8. 5, 6, 8, 12.
9. 4, 5, 6, 7.
10. 5, 6, 7, 8, 9.
11. 6, 8, 9, 10.
12. 5, 7, 10, 11, 12.
13. 8, 9, 10, 12, 14.
14. 7, 8, 10, 12.
15. 8, 9, 12, 15.
16. 6, 12, 15, 16.
17. 3, 5, 8, 10, 12.
18. 2, 5, 6, 7, 8.
19. 6, 7, 8, 9, 10.
20. 7, 8, 10, 12, 14.
21. 8, 9, 10, 11, 12.
22. 8, 12, 20, 30.
23. 6, 15, 18, 36.
24. 15, 25, 30, 40.
25. 18, 24, 36, 50, 60.
26. 24, 36, 60, 75.
27. 80, 120, 160, 18.
28. 75, 130, 145, 190.
29. 125, 160, 225, 260.
30. 336, 345, 425.

143. The Inspection Method.

Illustrative Example. Find l. c. m. of 10, 12, 18, 30, 36, 45.

FORM.

10
12
18
30
36
$45 \times 2 \times 2 = 180$, l. c. m.

Analysis. Since 10 is a divisor of 30, any multiple of 30 contains 10. I therefore strike out 10. Similarly I strike out 12 and 18, divisors of 36. The largest factor of 30 found in 45 is 15; the other factor is 2; so the l. c. m. must contain 45×2. The largest factor of 36 in 45 is 9, the other factor is 4, or 2×2; hence the l. c. m. is $45 \times 2 \times 2$, or 180.

EXAMPLES.

Find the l. c. m. of the following:

1. 8, 16, 30, 48, 60, 75.
2. 7, 15, 28, 42, 75, 90.
3. 21, 45, 63, 72, 84.
4. 28, 44, 56, 70, 88.
5. 39, 52, 64, 78, 91.
6. 42, 58, 84, 91, 98.
7. 36, 52, 65, 72, 84.
8. 4, 5, 6, 8, 12, 16, 25, 30.
9. 27, 38, 45, 54, 60, 76, 90.
10. 28, 30, 40, 56, 60.
11. 51, 68, 78, 88, 91.
12. 23, 46, 69, 92.
13. 15, 30, 45, 50, 60, 90.
14. 3, 7, 13, 25, 49, 56.
15. 5, 11, 13, 29.
16. 6, 8, 12, 16, 24, 36, 144.

144. EXAMPLES.

1. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$.
2. $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$.
3. $\frac{3}{4} + \frac{1}{6} + \frac{5}{9}$.
4. $\frac{4}{5} + \frac{5}{6} + \frac{3}{8}$.
5. $\frac{1}{6} + \frac{5}{8} + \frac{2}{9} + \frac{7}{10}$.
6. $\frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{7}{8}$.
7. $\frac{1}{3} + \frac{2}{5} + \frac{1}{6} + \frac{3}{8}$.
8. $\frac{4}{5} + \frac{5}{6} + \frac{7}{8} + \frac{5}{12}$.
9. $\frac{1}{4} + \frac{3}{5} + \frac{4}{6} + \frac{2}{7}$.
10. $\frac{3}{5} + \frac{1}{6} + \frac{3}{7} + \frac{7}{8} + \frac{5}{9}$.
11. $\frac{2}{5} + \frac{1}{8} + \frac{7}{9} + \frac{9}{10}$.
12. $\frac{4}{5} + \frac{6}{7} + \frac{3}{10} + \frac{2}{11} + \frac{7}{12}$.
13. $\frac{3}{8} + \frac{8}{9} + \frac{1}{10} + \frac{11}{12} + \frac{1}{14}$.
14. $\frac{5}{8} + \frac{4}{9} + \frac{11}{12} + \frac{7}{15}$.
15. $\frac{5}{6} + \frac{5}{12} + \frac{8}{15} + \frac{3}{16}$.
16. $\frac{2}{3} + \frac{4}{5} + \frac{7}{8} + \frac{9}{10} + \frac{11}{12}$.
17. $\frac{1}{2} + \frac{2}{5} + \frac{1}{6} + \frac{6}{7} + \frac{7}{8}$.
18. $\frac{5}{6} + \frac{4}{7} + \frac{3}{8} + \frac{8}{9} + \frac{1}{10}$.
19. $\frac{6}{7} + \frac{5}{8} + \frac{9}{10} + \frac{11}{12} + \frac{3}{14}$.
20. $\frac{7}{8} + \frac{4}{9} + \frac{7}{10} + \frac{1}{12} + \frac{7}{12}$.
21. $\frac{2}{3} + \frac{3}{4} + \frac{5}{3} + \frac{7}{12}$.
22. $\frac{2}{3} + \frac{4}{5} + \frac{7}{12}$.

23. $\frac{3}{4} + \frac{5}{8} + \frac{9}{10} + \frac{11}{12}$.

24. $\frac{2}{5} + \frac{5}{7} + \frac{8}{9} + \frac{11}{12}$.

25. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$.

26. $\frac{6}{7} + \frac{7}{8} + \frac{8}{9} + \frac{9}{10}$.

27. $\frac{6}{11} + \frac{8}{13} + \frac{15}{22} + \frac{19}{26}$.

28. $\frac{11}{12} + \frac{8}{15} + \frac{13}{20} + \frac{18}{25} + \frac{19}{30}$.

29. $\frac{2}{5} + \frac{5}{6} + \frac{3}{8} + \frac{4}{9} + \frac{7}{10} + \frac{5}{12}$.

30. $\frac{4}{7} + \frac{8}{15} + \frac{19}{20} + \frac{23}{24} + \frac{11}{30}$.

145. *Short Method.* $\frac{1}{3} + \frac{1}{4} = ?$ Since the denominators are prime to each other, their product is their l. c. m. Since the numerator is one in each case, the denominator of the second becomes the numerator of the first reduced fraction, and the denominator of the first, the numerator of the second.

RULE.

To find the sum of two fractions whose numerators are one, place the sum of their denominators over their product.

EXAMPLES.

1. $\frac{1}{2} + \frac{1}{3} = ?$
2. $\frac{1}{3} + \frac{1}{4} = ?$
3. $\frac{1}{3} + \frac{1}{7} = ?$
4. $\frac{1}{5} + \frac{1}{9} = ?$
5. $\frac{1}{12} + \frac{1}{13} = ?$
6. $\frac{1}{11} + \frac{1}{15} = ?$
7. $\frac{1}{12} + \frac{1}{25} = ?$
8. $\frac{1}{4} + \frac{1}{6} = ?$
9. $\frac{1}{8} + \frac{1}{20} = ?$

Is the rule applicable to fractions whose denominators are not prime to each other? In such cases is the result in its lowest terms?

Modify the preceding rule for such cases as $\frac{2}{3} + \frac{2}{5}$.

Give the results rapidly for the following problems:

1. $\frac{2}{3} + \frac{2}{5} = ?$
2. $\frac{3}{4} + \frac{3}{7} = ?$
3. $\frac{4}{5} + \frac{4}{9} = ?$
4. $\frac{5}{8} + \frac{5}{9} = ?$
5. $\frac{7}{10} + \frac{7}{15} = ?$
6. $\frac{6}{7} + \frac{6}{11} = ?$
7. $\frac{8}{9} + \frac{8}{15} = ?$
8. $\frac{9}{10} + \frac{9}{14} = ?$
9. $\frac{3}{17} + \frac{3}{20} = ?$

NOTE. Dictate many similar problems.

146. Practice adding small fractions mentally until facility is acquired.

Illustrative Example. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{12} + \frac{5}{6} = ?$

FORM.

$\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$. $\quad \frac{1}{4} + \frac{5}{8} = \frac{7}{8}$. $\quad \frac{7}{8} + \frac{7}{12} = \frac{21}{24} + \frac{14}{24} = 1\frac{11}{24}$.
$\frac{11}{24} + \frac{5}{6} = \frac{11}{24} + \frac{20}{24} = 1\frac{7}{24}$. $\quad 1 + 1 + 1\frac{7}{24} = 3\frac{7}{24}$.

1. $\frac{1}{4} + \frac{2}{3} + \frac{5}{6} + \frac{5}{8} = ?$
2. $\frac{4}{5} + \frac{1}{2} + \frac{7}{10} + \frac{3}{4} + \frac{5}{6} = ?$
3. $\frac{2}{3} + \frac{3}{4} + \frac{1}{6} + \frac{5}{8} + \frac{7}{12} = ?$
4. $\frac{1}{7} + \frac{5}{14} + \frac{1}{2} + \frac{7}{10} + \frac{4}{5} + \frac{1}{12} = ?$
5. $\frac{5}{8} + \frac{5}{6} + \frac{5}{12} + \frac{3}{4} + \frac{1}{2} + \frac{2}{3} = ?$

147. In adding mixed numbers do not reduce them to improper fractions.

Illustrative Example. Add $7\frac{8}{9}$, $9\frac{5}{6}$, $3\frac{11}{12}$.

FORM.

$7\frac{8}{9}$	32	
$9\frac{5}{6}$	30	36
$3\frac{11}{12}$	33	
	95	
2	72	
	23	
$21\frac{23}{36}$		

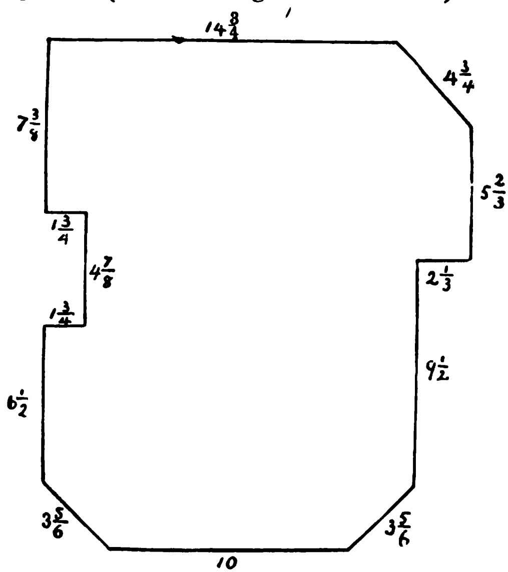
NOTE. The common denominator is written at the right of a vertical line, the new numerators in column for ease in adding. The quotient, 2, is written in ones' column.

1. $2\frac{1}{2} + 3\frac{1}{3} + 1\frac{5}{8} + 7\frac{2}{5}$.
2. $5\frac{1}{6} + 12\frac{2}{3} + 4\frac{7}{12} + 8\frac{11}{15}$.
3. $12\frac{18}{25} + 26\frac{11}{15} + 33\frac{2}{5} + 86\frac{5}{6}$.
4. $61\frac{7}{8} + 124\frac{11}{12} + 96\frac{17}{32} + 216\frac{1}{4}$.
5. $77\frac{5}{24} + 66\frac{7}{12} + 118\frac{2}{3} + 45\frac{1}{2}$.
6. $125\frac{18}{35} + 99\frac{5}{7} + 231\frac{3}{5} + 184\frac{2}{3}$.
7. $286\frac{5}{17} + 324\frac{1}{2} + 789\frac{13}{51} + 612\frac{2}{3}$.
8. $177\frac{5}{13} + 268\frac{11}{26} + 317\frac{1}{2} + 439\frac{3}{4}$.
9. $48\frac{5}{19} + 64\frac{4}{5} + 92\frac{1}{2} + 354\frac{9}{10}$.
10. $291\frac{7}{11} + 37\frac{3}{10} + 63\frac{19}{55}$.

11. Four rolls of carpet contain respectively $98\frac{3}{8}$ yd., $97\frac{4}{5}$ yd., $112\frac{3}{4}$ yd., $96\frac{7}{18}$ yd. What is the total amount?

12. What is the length of border required in papering this room?

Floor plan. (Dimensions given are in feet.)



13. In a box weighing $12\frac{1}{2}$ pounds, a grocer packed for shipment $15\frac{7}{8}$ pounds of ham, $\frac{13}{16}$ of a pound of tea, $3\frac{3}{4}$ pounds of coffee, $6\frac{1}{2}$ pounds of sugar. What was the total weight of the package?

14. The United States coins weigh,— cent 48 grains, 5-cent piece $73\frac{1}{6}$ grains, dime $38\frac{7}{12}$ grains, quarter-dollar $96\frac{9}{20}$ grains, half-dollar $192\frac{9}{10}$ grains, dollar $412\frac{1}{2}$ grains, quarter-eagle $64\frac{1}{2}$ grains, half-eagle 129 grains, eagle 258 grains, double-eagle 516 grains. What is the entire weight of the series?

15. A man has in his purse 2 silver dollars, 3 half-dollars, 6 dimes, and 7 5-cent pieces. What is the weight of the whole? (Addition.)

16. How many acres are there in 4 tracts of land, the first containing $88\frac{2}{3}$, the second, $112\frac{2}{3}$ acres; the third, $146\frac{1}{2}$ acres; and the fourth, $39\frac{5}{12}$ acres?

17. Find the sum of $124\frac{1}{2}$ pounds, $316\frac{3}{4}$ pounds, $518\frac{7}{8}$ pounds, $209\frac{11}{16}$ pounds, and $77\frac{1}{4}$ pounds.

18. Find the amount of coal in 5 car-loads weighing as follows: $28\frac{2}{5}$ tons, $29\frac{7}{8}$ tons, $30\frac{3}{10}$ tons, $27\frac{19}{20}$ tons, and $31\frac{3}{4}$ tons.

19. A mail carrier traveled $12\frac{5}{16}$ miles on Monday, $11\frac{2}{3}$ miles on Tuesday, $13\frac{7}{8}$ miles on Wednesday, $10\frac{3}{4}$ miles on Thursday, $9\frac{7}{12}$ miles on Friday, and $14\frac{15}{16}$ miles on Saturday. Find the whole distance traveled in the 6 days.

148. SUBTRACTION OF FRACTIONS.

1. Define subtraction, minuend, subtrahend, remainder.

2. *Illustrative Example.* $\frac{3}{4} - \frac{1}{4} = ?$

ANALYSIS. If $\frac{3}{4}$ be separated into 2 parts, one of which is $\frac{1}{4}$, the other will be $\frac{2}{4}$ or $\frac{1}{2}$.

$\frac{7}{8} - \frac{3}{8} = ?$ $\quad \frac{11}{12} - \frac{7}{12} = ?$ $\quad \frac{13}{15} - \frac{8}{15} = ?$ $\quad \frac{17}{18} - \frac{5}{18} = ?$

When the denominators are alike, how is the subtraction performed?

3. $\frac{7}{8} - \frac{3}{4} = ?$ How many eighths are needed to make $\frac{1}{4}$? to make $\frac{3}{4}$? How many eighths remain?

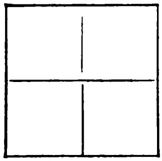
4. $\frac{11}{15} - \frac{2}{5}$; $\frac{17}{20} - \frac{3}{4}$; $\frac{19}{20} - \frac{3}{5}$; $\frac{3}{5} - \frac{7}{12}$.

5. *Illustrative Problem.* Separate $\frac{3}{4}$ of a sheet of paper into two parts, one of which shall be $\frac{2}{3}$ of a sheet.

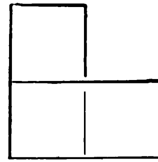
This $\frac{3}{4}$ of a sheet of paper is to be separated into two such parts that one of them shall be $\frac{2}{3}$ of a sheet. Thirds are not readily formed from fourths; hence, I change the fourths into something from which thirds may be made.

Fold the fourths together. Fold what you now have into three equal parts. Open the sheet. It is now folded into

Fold the sheet into four equal parts; thus,



Fold down one of the fourths; thus,



how many equal parts? Show $\frac{1}{3}$ of the sheet. How many twelfths does it contain? Show $\frac{2}{3}$ of the sheet. Now tear off the fourth first folded down. What part of the sheet is left? How many twelfths are there in it? How many are needed to make $\frac{2}{3}$ of a sheet? How many, then, should you tear off? What is left? What, then, does $\frac{3}{4} - \frac{2}{3}$ equal?

6. Application to an abstract problem.

$\frac{4}{5} - \frac{1}{4} = ?$ I am to separate $\frac{4}{5}$ into two such parts that one of them shall be $\frac{1}{4}$. Since $\frac{1}{4}$ is not easily formed from fifths, I change $\frac{4}{5}$ to twentieths, from which $\frac{1}{4}$ may be made. $\frac{4}{5} = \frac{16}{20}$. $\frac{5}{20}$ are needed to make $\frac{1}{4}$. If $\frac{16}{20}$ be separated into two parts, one of which is $\frac{5}{20}$, the other is $\frac{11}{20}$; hence, $\frac{4}{5} - \frac{1}{4} = \frac{11}{20}$.

Explain the following in the same manner:

1. $\frac{2}{3} - \frac{2}{5} = ?$	5. $\frac{5}{7} - \frac{1}{4} = ?$	9. $\frac{9}{10} - \frac{5}{6} = ?$
2. $\frac{3}{5} - \frac{1}{6} = ?$	6. $\frac{6}{7} - \frac{2}{5} = ?$	10. $\frac{7}{10} - \frac{4}{7} - ?$
3. $\frac{3}{4} - \frac{2}{7} = ?$	7. $\frac{7}{8} - \frac{2}{3} = ?$	11. $\frac{11}{12} - \frac{3}{5} = ?$
4. $\frac{5}{6} - \frac{3}{8} = ?$	8. $\frac{5}{8} - \frac{1}{6} = ?$	12. $\frac{9}{14} - \frac{3}{8} = ?$

7. From the foregoing the following analysis may be derived:

$\frac{5}{6} - \frac{2}{7} = ?$ Since these fractions are unlike, I change them to equivalent fractions having the l. c. d.

$$\frac{5}{6} = \frac{35}{42}. \qquad \frac{2}{7} = \frac{12}{42}. \qquad \frac{35}{42} - \frac{12}{42} = \frac{23}{42}.$$

Explain the following problems as above:

1. $\frac{4}{7} - \frac{2}{5} = ?$	4. $\frac{7}{12} - \frac{2}{13} = ?$
2. $\frac{6}{11} - \frac{1}{6} = ?$	5. $\frac{11}{18} - \frac{4}{11} = ?$
3. $\frac{5}{9} - \frac{2}{7} = ?$	6. $\frac{12}{13} - \frac{6}{7} = ?$

7. $\frac{19}{21} - \frac{7}{12} = ?$

8. $\frac{18}{19} - \frac{3}{5} = ?$

9. $\frac{5}{21} - \frac{1}{6} = ?$

10. $\frac{18}{25} - \frac{7}{12} = ?$

11. $\frac{29}{35} - \frac{11}{15} = ?$

12. $\frac{31}{33} - \frac{17}{22} = ?$

13. $\frac{22}{39} - \frac{4}{23} = ?$

14. $\frac{37}{45} - \frac{3}{20} = ?$

15. $\frac{43}{50} - \frac{7}{40} = ?$

16. $\frac{28}{52} - \frac{12}{65} = ?$

17. $\frac{23}{33} - \frac{34}{55} = ?$

18. $\frac{59}{60} - \frac{13}{40} = ?$

Form a rule from the analysis just given.

149. Review Art. **145.** Make a rule for problems like the following. Give results rapidly.

1. $\frac{1}{2} - \frac{1}{3}$.
2. $\frac{1}{3} - \frac{1}{4}$.
3. $\frac{1}{4} - \frac{1}{5}$.
4. $\frac{1}{2} - \frac{1}{5}$.
5. $\frac{1}{3} - \frac{1}{5}$.
6. $\frac{1}{5} - \frac{1}{8}$.
7. $\frac{1}{4} - \frac{1}{10}$.
8. $\frac{1}{6} - \frac{1}{7}$.
9. $\frac{1}{7} - \frac{1}{12}$.
10. $\frac{1}{7} - \frac{1}{11}$.
11. $\frac{1}{8} - \frac{1}{9}$.
12. $\frac{1}{8} - \frac{1}{25}$.
13. $\frac{1}{11} - \frac{1}{12}$.
14. $\frac{1}{15} - \frac{1}{20}$.
15. $\frac{1}{12} - \frac{1}{13}$.
16. $\frac{1}{7} - \frac{1}{15}$.
17. $\frac{1}{10} - \frac{1}{19}$.
18. $\frac{2}{3} - \frac{2}{5}$.
19. $\frac{2}{3} - \frac{2}{4}$.
20. $\frac{2}{3} - \frac{2}{7}$.
21. $\frac{2}{5} - \frac{2}{7}$.
22. $\frac{2}{5} - \frac{2}{9}$.
23. $\frac{2}{5} - \frac{2}{11}$.
24. $\frac{2}{6} - \frac{2}{7}$.
25. $\frac{2}{6} - \frac{2}{8}$.
26. $\frac{2}{6} - \frac{2}{9}$.
27. $\frac{3}{5} - \frac{3}{7}$.
28. $\frac{3}{5} - \frac{3}{8}$.
29. $\frac{3}{5} - \frac{3}{10}$.
30. $\frac{3}{5} - \frac{3}{11}$.
31. $\frac{3}{7} - \frac{3}{10}$.
32. $\frac{4}{5} - \frac{4}{7}$.
33. $\frac{4}{7} - \frac{4}{11}$.
34. $\frac{4}{7} - \frac{4}{13}$.
35. $\frac{5}{6} - \frac{5}{7}$.
36. $\frac{5}{8} - \frac{5}{11}$.
37. $\frac{5}{9} - \frac{5}{10}$.
38. $\frac{8}{9} - \frac{8}{11}$.
39. $\frac{1}{12} - \frac{1}{13}$.

150. *Illustrative Problem.* $7\frac{1}{3} - 4\frac{3}{4} =$ what?

FORM.

6	16	
~~7~~$\frac{1}{3}$	~~4~~	12
$4\frac{3}{4}$	9	
$2\frac{7}{12}$	7	

ANALYSIS. $\frac{1}{3} = \frac{4}{12}$; $\frac{3}{4} = \frac{9}{12}$. Since $\frac{4}{12}$ is less than $\frac{9}{12}$, I take one of the 7 ones, leaving 6 ones, and reduce it to twelfths. $1 = \frac{12}{12}$; $\frac{12}{12} + \frac{4}{12} = \frac{16}{12}$; $\frac{16}{12} - \frac{9}{12} = \frac{7}{12}$; $6 - 4 = 2$. Hence, $7\frac{1}{3} - 4\frac{3}{4} = 2\frac{7}{12}$.

NOTE. That instead of adding $\frac{12}{12}$ to $\frac{4}{12}$ I may subtract the $\frac{9}{12}$ from the $\frac{12}{12}$ and add the remainder, $\frac{3}{12}$, to the $\frac{4}{12}$, thus obtaining $\frac{7}{12}$, as before.

Observe that the process is identical with that of subtraction of whole numbers.

Explain the following in the same way:

1. $17\frac{1}{5} - 8\frac{2}{5} = ?$
2. $26\frac{3}{7} - 19\frac{5}{8} = ?$
3. $124\frac{3}{10} - 98\frac{2}{5} = ?$
4. $317\frac{7}{12} - 268\frac{4}{5} = ?$
5. $91\frac{7}{15} - 48\frac{5}{9} = ?$
6. $461\frac{19}{21} - 178\frac{41}{42} = ?$
7. $83\frac{5}{12} - 59\frac{4}{5} = ?$
8. $92\frac{11}{17} - 46\frac{42}{51} = ?$
9. $126\frac{17}{24} - 97\frac{15}{16} = ?$
10. $532\frac{18}{25} - 483\frac{23}{30} = ?$
11. $624\frac{18}{31} - 279\frac{37}{40} = ?$
12. $1217\frac{19}{63} - 968\frac{18}{25} = ?$

151. ADDITION AND SUBTRACTION.

1. From the sum of $\frac{5}{8}$ and $\frac{7}{12}$ take their difference.

2. $7\frac{8}{15} + 5\frac{2}{5} - 6\frac{7}{8} - 3\frac{1}{3} + 7\frac{11}{12} = ?$

3. The remainder is $\frac{7}{15}$ and the subtrahend $\frac{5}{8}$. What is the minuend?

4. What must be added to $6\frac{19}{21}$ to produce $11\frac{13}{14}$?

5. A man received $\$4\frac{3}{4}$ for butter, $\$5\frac{2}{5}$ for cheese, and an amount equal to their sum for vegetables. He paid $\$3\frac{1}{3}$ for sugar, $\$4\frac{4}{5}$ for coffee, and an amount equal to their difference for tea. What amount was left?

6. $8\frac{2}{5} - (4\frac{1}{11} - 1\frac{9}{10}) = ?$

7. $4\frac{7}{8} + (6\frac{11}{12} - 2\frac{6}{7}) = ?$

8. $5\frac{13}{14} - (2\frac{1}{5} + 1\frac{11}{12}) = ?$

9. A man bought 160 acres of land. To A, he sold $24\frac{2}{3}$ acres; to B, $41\frac{5}{12}$ acres; to C, as much as to A and B; to D, as much as the difference between A's and B's. How many acres were left?

10. $(8\frac{13}{14} + 4\frac{7}{8}) - (2\frac{5}{12} + 3\frac{4}{5}) = ?$

11. 3 barrels contain $156\frac{3}{4}$ gallons of oil. In the first are $51\frac{5}{8}$ gallons; in the second $49\frac{5}{8}$ gallons. How many gallons in the third?

12. Monday night the Mississippi River at St. Louis stood at 15 feet above low-water mark. Tuesday it rose $1\frac{7}{8}$ feet,

Wednesday $1\frac{1}{3}$ feet; Thursday $\frac{5}{8}$ feet; Friday it fell $1\frac{1}{2}$ feet; Saturday $2\frac{1}{3}$ feet. What was its height Saturday night?

13. John is $5\frac{3}{8}$ years older than Thomas. Thomas is $2\frac{5}{8}$ years younger than Harry and $1\frac{3}{4}$ years older than Richard. John is how much older than Harry?

14. A pole $22\frac{5}{8}$ feet long is broken in two. One piece is $2\frac{3}{4}$ feet longer than the other. What is the length of each piece?

QUERY. How much added to the shorter would make it equal to the longer? Adding this to the pole, would give what total length?

15. From Bloomington to Decatur on the Illinois Central railroad the distance is $43\frac{5}{8}$ miles. Clinton is $1\frac{4}{15}$ miles nearer to Decatur than to Bloomington. How far is Clinton from each?

16. A owns $12\frac{7}{18}$ acres of land; B owns $7\frac{8}{9}$ acres more than A. C owns as much as both A and B, and D owns as much as the difference between A's and B's; find B's, C's, and D's. Find the whole amount owned.

17. $\frac{2}{3}$ of A's money increased by $\frac{4}{5}$ of his money lacks $\frac{1}{8}$ of his money of being \$955. How much money had he?

18. The distance from Albany to Syracuse is 148 miles. A starts from Albany for Syracuse and B from Syracuse for Albany at the same time. A walks $23\frac{3}{4}$ miles the first day, $18\frac{5}{6}$ miles the second day, $24\frac{7}{12}$ miles the third, and $29\frac{11}{16}$ miles the fourth. B travels in the same time $15\frac{2}{3}$ miles, $19\frac{9}{24}$ miles, $26\frac{1}{2}$ miles, and $31\frac{7}{8}$ miles. Make diagrams and show:

a. How far apart they were at the end of each day.

b. How far each is from the starting-point at the end of each day.

c. How far each is from his destination at the end of each day.

19. A piece of cloth contains $7\frac{5}{12}$ yards. What will be left after using $\frac{7}{8}$ of a yard for a vest, $2\frac{3}{4}$ yards for a coat, and $2\frac{1}{6}$ yards for a pair of pantaloons?

20. A farmer having 7 apple-trees gathered from them as follows: $2\frac{1}{5}$ barrels, $3\frac{7}{8}$ barrels, $4\frac{5}{6}$ barrels, $5\frac{7}{12}$ barrels, $2\frac{3}{4}$ barrels, $3\frac{2}{3}$ barrels, $5\frac{1}{2}$ barrels. He sold to one man $12\frac{3}{4}$ barrels; to a second, $2\frac{1}{6}$ barrels; to a third $4\frac{3}{8}$ barrels. How many barrels were left?

21. In 1834 the amount of gold in the eagle was reduced from $247\frac{1}{2}$ grains to $232\frac{1}{3}$ grains. How much was taken out?

22. From an ounce (480 grains) of standard gold were minted an eagle, a half-eagle, and a quarter-eagle. How many grains remained? (See Art. **147**, Problem 14.)

23. In 1853 the weight of the dime was reduced from $41\frac{1}{4}$ grains to $38\frac{7}{12}$ grains. How many grains were taken out?

24. From a standard silver dollar, $412\frac{1}{2}$ grains, 10 dimes are coined? How many grains of silver remain?

152. MULTIPLICATION OF FRACTIONS.

1. Define multiplication, multiplicand, multiplier, product.

What does the numerator show? What is the effect produced by multiplying the numerator? Why?

Unite five 2's of thirds; six 3's of fifths; seven 5's of eighths. What did you do in each of these cases? How, then, may you multiply a fraction by an integer?

2. Since the numerator of a fraction expresses the number of fractional units in the fractional number, it is evident that a fraction is multiplied by multiplying its numerator.

NOTE. The sign × was first introduced by William Oughtred in 1631. At first the multiplier was uniformly placed after the sign. Now the multiplier frequently precedes it.

The sign × is read "multiplied by" when the multiplier follows; as 7 lbs. × 5, 9A. × $\frac{2}{3}$, $\frac{3}{4}$ oz. × 8 ("three fourths of an ounce multiplied by eight").

The sign is read "times" when the multiplier preceding it is an integer or a mixed number, as 5×7 lbs., $6\frac{3}{4} \times 8$ ft. ("six and three fourths times eight feet").

The sign is read "of" when the multiplier before it is a simple fraction; as $\frac{3}{4} \times \$20$ ("three fourths of twenty dollars").

The sign is read "by" when the factors are dimensions; as, a pane $14'' \times 32''$ ("a pane fourteen inches by thirty-two inches"). A door $3' - 8'' \times 7' - 6''$ ("a door three feet eight by seven feet six").

153. THE MULTIPLIER AN INTEGER.

PROBLEMS.

1. Multiply $\frac{4}{5}$ by 7.

ANALYSIS. Seven 4's of fifths are $\frac{28}{5}$. In $\frac{28}{5}$ there are as many ones as there are 5's in 28. There are $5\frac{3}{5}$ fives in 28; hence $\frac{28}{5} = 5\frac{3}{5}$.

2. $\frac{2}{3} \times 4.$	9. $\frac{7}{10} \times 9.$	16. $8\frac{10}{11} \times 21.$
3. $\frac{3}{4} \times 5.$	10. $\frac{7}{12} \times 11.$	17. $15\frac{8}{9} \times 32.$
4. $\frac{5}{6} \times 5.$	11. $\frac{8}{9} \times 10.$	18. $26\frac{18}{19} \times 13.$
5. $\frac{7}{8} \times 7.$	12. $\frac{7}{11} \times 6.$	19. $9\frac{2}{5} \times 14.$
6. $\frac{3}{5} \times 9.$	13. $\frac{8}{17} \times 9.$	20. $35\frac{11}{12} \times 43.$
7. $\frac{7}{9} \times 4.$	14. $5\frac{2}{5} \times 8.$	21. $52\frac{7}{8} \times 49.$
8. $\frac{6}{7} \times 8.$	15. $7\frac{3}{5} \times 12.$	22. $69\frac{8}{9} \times 56.$

NOTE. Observe that in these problems the *number* of fractional units is multiplied in each case.

154. PROBLEMS.

1. Multiply $\frac{5}{8} \times 4$.

What fractional unit is four times as large as an eighth? What, then, is 4 times $\frac{5}{8}$? Here we multiply the *size* of the fractional units, by dividing the denominator by the integer, hence we say "4 times $\frac{5}{8}$ is $\frac{5}{2}$;" $\frac{5}{2}$ is clearly 4 times $\frac{5}{8}$, since it has the same number of fractional units and they are 4 times as large.

2. $\frac{8}{21} \times 7$.	14. $\frac{17}{42} \times 14$.	26. $\frac{113}{144} \times 72$.
3. $\frac{7}{16} \times 8$.	15. $\frac{14}{57} \times 19$.	27. $\frac{127}{256} \times 64$.
4. $\frac{11}{24} \times 12$.	16. $\frac{49}{68} \times 34$.	28. $\frac{11}{384} \times 48$.
5. $\frac{19}{26} \times 13$.	17. $\frac{11}{70} \times 10$.	29. $\frac{119}{266} \times 38$.
6. $\frac{18}{49} \times 7$.	18. $\frac{13}{72} \times 18$.	30. $\frac{17}{225} \times 75$.
7. $\frac{28}{57} \times 19$.	19. $\frac{9}{74} \times 37$.	31. $\frac{29}{525} \times 75$.
8. $\frac{17}{63} \times 21$.	20. $\frac{8}{75} \times 25$.	32. $\frac{211}{512} \times 64$.
9. $\frac{38}{51} \times 17$.	21. $\frac{21}{76} \times 19$.	33. $\frac{13}{343} \times 49$.
10. $\frac{49}{52} \times 26$.	22. $\frac{17}{78} \times 13$.	34. $\frac{581}{625} \times 25$.
11. $\frac{13}{55} \times 11$.	23. $\frac{10}{81} \times 27$.	35. $\frac{97}{576} \times 24$.
12. $\frac{3}{40} \times 8$.	24. $\frac{19}{87} \times 29$.	36. $\frac{85}{784} \times 28$.
13. $\frac{3}{40} \times 20$.	25. $\frac{11}{84} \times 28$.	37. $\frac{863}{1089} \times 33$.

155. PROBLEMS.

It has been shown in Art. **86** that the continued product of a multiplicand and the factors of a multiplier is the same as the product of the multiplicand and the multiplier itself; thus: $25 \times 3 \times 2 = 25 \times 6$.

In such problems as $\frac{8}{15} \times 10$, we multiply the size of the fractional units by 5, obtaining $\frac{8}{3}$, then the number of fractional units by 2, obtaining $\frac{16}{3}$.

1. $\frac{19}{27} \times 12 = ?$

ANALYSIS. 12 times $\frac{19}{27} = 4$ times 3 times $\frac{19}{27}$. 3 times $\frac{19}{27} = \frac{19}{9}$. 4 times $\frac{19}{9} = \frac{76}{9} = 8\frac{4}{9}$.

2. $\frac{5}{8} \times 12$.	12. $\frac{25}{28} \times 56$.	22. $\frac{91}{144} \times 180$.
3. $\frac{7}{12} \times 9$.	13. $\frac{7}{27} \times 63$.	23. $\frac{131}{160} \times 48$.
4. $\frac{11}{14} \times 28$.	14. $\frac{29}{30} \times 75$.	24. $\frac{253}{324} \times 72$.
5. $\frac{15}{16} \times 24$.	15. $\frac{13}{35} \times 65$.	25. $\frac{19}{325} \times 125$.
6. $\frac{13}{18} \times 20$.	16. $\frac{31}{42} \times 28$.	26. $\frac{217}{400} \times 120$.
7. $\frac{19}{20} \times 35$.	17. $\frac{45}{52} \times 65$.	27. $\frac{93}{436} \times 218$.
8. $\frac{16}{21} \times 24$.	18. $\frac{31}{58} \times 87$.	28. $\frac{329}{576} \times 240$.
9 $\frac{17}{22} \times 33$.	19. $\frac{16}{75} \times 100$.	29. $\frac{283}{625} \times 250$.
10. $\frac{23}{24} \times 40$.	20. $\frac{73}{108} \times 144$.	30. $\frac{419}{630} \times 144$.
11. $\frac{18}{25} \times 45$.	21. $\frac{83}{136} \times 51$.	31. $\frac{479}{1225} \times 245$.

Which of these three methods may always be employed? Which is the most convenient?

RULE.

To multiply a fraction by an integer, divide the denominator by the integer, if possible; if not possible, divide the denominator by the largest possible factor of the integer; if the denominator is not so divisible, multiply the numerator by the integer.

Since dividing the denominator by a factor of the integer is the same as omitting that factor from the denominator and the integer, the rule may be shortened:

Omit all factors common to the integer and the denominator, and multiply the numerator by the remaining factor of the integer.

156. THE MULTIPLIER A FRACTION.

PROBLEMS.

1. Multiply 8 by $\frac{3}{4}$.

This means find $\frac{3}{4}$ of 8. Whenever the multiplier is a fraction, the problem may be read in the same way. There are two processes involved: finding $\frac{1}{4}$ of 8 (partition), and uniting three such parts (multiplication).

Analysis. $\frac{1}{4}$ of 8 is 2. $\frac{3}{4}$ of 8 are 3 twos, which are 6

Find:

2. $\frac{5}{6}$ of 12; $\frac{4}{5}$ of 15; $\frac{6}{7}$ of 21; $\frac{13}{14}$ of 28.

3. $\frac{2}{3}$ of 6 bushels; $\frac{2}{3}$ of $\frac{6}{7}$; $\frac{3}{5}$ of 10 cents; $\frac{3}{5}$ of $\frac{10}{13}$.

4. $\frac{5}{8}$ of \$24; $\frac{5}{8}$ of $\frac{24}{7}$; $\frac{3}{10}$ of 40 acres; $\frac{3}{10}$ of $\frac{40}{3}$.

In the preceding examples the multiplicand is first divided by the denominator of the multiplier, and the quotient is multiplied by the numerator. Since the order of the operations is immaterial, we may first multiply the multiplicand by the numerator and divide this product by the denominator. Generally this will be more convenient; hence, the

RULE.

To multiply by a fraction, multiply by the numerator and divide the product by the denominator.

5. Multiply 8 by $\frac{4}{5}$; 9 by $\frac{2}{3}$; 12 by $\frac{5}{7}$; 15 by $\frac{4}{5}$.

6. Multiply 21 by $\frac{3}{7}$; 24 by $\frac{2}{5}$; 30 by $\frac{5}{6}$; 32 by $\frac{2}{3}$.

7. Multiply $\frac{5}{6}$ by $\frac{2}{5}$; $\frac{8}{9}$ by $\frac{3}{4}$; $\frac{9}{10}$ by $\frac{2}{3}$; $\frac{11}{25}$ by $\frac{5}{6}$.

8. Multiply $\frac{15}{28}$ by $\frac{4}{5}$; $\frac{32}{39}$ by $\frac{13}{16}$; $\frac{18}{25}$ by $\frac{7}{9}$; $\frac{27}{28}$ by $\frac{8}{9}$.

9. Multiply $\frac{81}{82}$ by $\frac{26}{27}$; $\frac{81}{95}$ by $\frac{12}{13}$; $\frac{99}{100}$ by $\frac{17}{33}$; $\frac{125}{126}$ by $\frac{42}{25}$.

10. Find $\frac{7}{12}$ of $\frac{5}{8}$. How find $\frac{1}{12}$ of $\frac{5}{8}$? How $\frac{7}{12}$ of $\frac{5}{8}$?

11. Find $\frac{5}{6}$ of $\frac{7}{11}$; $\frac{4}{7}$ of $\frac{8}{13}$; $\frac{2}{3}$ of $\frac{4}{5}$; $\frac{2}{5}$ of $\frac{3}{7}$.

12. Find $\frac{3}{7}$ of $\frac{4}{11}$; $\frac{4}{5}$ of $\frac{8}{9}$; $\frac{6}{7}$ of $\frac{5}{11}$; $\frac{7}{8}$ of $\frac{3}{4}$.

13. Find $\frac{7}{9}$ of $\frac{5}{6}$; $\frac{6}{7}$ of $\frac{12}{13}$; $\frac{7}{10}$ of $\frac{13}{17}$; $\frac{2}{3}$ of $\frac{26}{27}$.

In these problems, how is the numerator of the product formed? the denominator? Where cancellation is possible, what should first be done?

RULE.

To multiply a fraction by a fraction, cancel all factors common to a numerator and a denominator, and multiply together the remaining factors of the numerators for the numerator of the product, and the remaining factors of the denominators for the denominator of the product.

157. PROBLEMS.

Multiply:

1. $\frac{28}{45}$ by 15. **4.** 129 by $\frac{11}{15}$. **7.** $\frac{18}{49}$ by $\frac{8}{9}$.

2. $\frac{17}{24}$ by 48. **5.** $\frac{18}{25}$ by 32. **8.** $\frac{39}{41}$ by $\frac{29}{31}$.

3. 54 by $\frac{17}{27}$. **6.** $\frac{9}{25}$ by $\frac{5}{9}$. **9.** $\frac{5}{8}$ of $\frac{7}{12}$ by $\frac{5}{6}$ of $\frac{9}{10}$.

10. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ by $\frac{11}{12}$ of $\frac{24}{33}$ of $\frac{9}{10}$.

Find:

11. $\frac{17}{25}$ of 560. **12.** $\frac{29}{35}$ of 784. **13.** $\frac{83}{90}$ of 1872.

Multiply:

14. $\frac{5}{6}$ of $\frac{5}{8}$ by $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{8}{15}$. **15.** $8\frac{2}{3}$ by 6.

ANALYSIS. 6 times $\frac{2}{3}$ is 4. 6 times 8 is 48. $48 + 4 = 52$.

Multiply:

16. $12\frac{4}{5}$ by 10. **18.** $125\frac{7}{12}$ by 48. **20.** $624\frac{7}{32}$ by 86.

17. $15\frac{5}{8}$ by 24. **19.** $584\frac{19}{25}$ by 50. **21.** $11\frac{2}{3}$ by $8\frac{1}{4}$.

FORM.

$$\begin{array}{r} 11\frac{2}{3} \\ 8\frac{1}{4} \\ \hline 2\frac{11}{12} \\ 5\frac{1}{3} \\ 88 \\ \hline 96\frac{1}{4} \end{array}$$

Use this method when the fractions are small.

ANALYSIS. $\frac{1}{4}$ of $11 = 2$ and a remainder of 3. $3 = \frac{9}{3}$. $\frac{9}{3} + \frac{2}{3} = \frac{11}{3}$. $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$. $8 \times \frac{2}{3} = 5\frac{1}{3}$. $8 \times 11 = 88$, etc.

Multiply:

22. $15\frac{2}{5}$ by $12\frac{1}{3}$. **23.** $86\frac{3}{4}$ by $27\frac{1}{5}$. **24.** $1248\frac{7}{12}$ by $492\frac{7}{8}$.

Reduce the mixed numbers to improper fractions.

25. What is the cost of $\frac{7}{12}$ of an acre of land at $88 an acre?

26. What is the cost of $\frac{7}{15}$ of a quire of paper at 23 cents a quire?

27. What is the cost of $4\frac{2}{3}$ cords of wood at $4.75 a cord?

28. What is the cost of $23\frac{7}{8}$ yards of broadcloth at $\$2\frac{4}{5}$ a yard?

29. What is the cost of $34\frac{7}{15}$ tons of coal at $2.57 a ton?

30. What is $\frac{2}{3}$ of $\frac{5}{8}$? $\frac{3}{5}$ of $\frac{7}{12}$ of $\frac{11}{14}$?

31. What is $\frac{4}{7}$ of $\frac{5}{12}$ of $\frac{11}{14}$ of 50?

32. What is $\frac{7}{15}$ of $2\frac{1}{8}$ of $\frac{11}{12}$ of $5\frac{1}{6}$ of 12?

33. Find the cost of $3\frac{1}{3}$ yards of cloth at $3.75; of $4\frac{5}{8}$ yards at $2.40; of $15\frac{3}{4}$ yards at $3.20.

34. Bought $4\frac{1}{4}$ cords of wood at $3.50; $5\frac{3}{8}$ cords at $4.40; $6\frac{2}{3}$ cords at $4.56. Sold the wood at an average price of $5.00 a cord. What was the gain?

35. A farmer gathered $42\frac{1}{3}$ loads of corn averaging $33\frac{3}{8}$ bushels from his 25-acre field. What was the total yield?

36. Multiply $17\frac{2}{5}$ by $14\frac{1}{3}$; $26\frac{3}{8}$ by $29\frac{3}{4}$; $132\frac{5}{12}$ by $68\frac{3}{8}$; $694\frac{2}{3}$ by $87\frac{1}{2}$.

37. A steamer ran at an average rate of $386\frac{5}{8}$ miles for $5\frac{2}{3}$ successive days. What distance was covered?

38. Find the cost of $469\frac{2}{5}$ bushels of wheat at $60\frac{3}{4}$ cents.

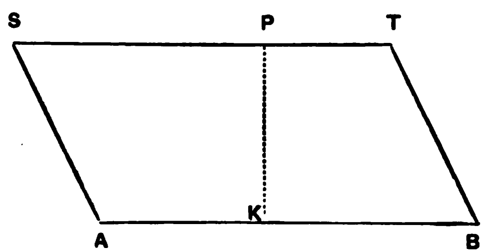
39. Find the cost of $35\frac{3}{4}$ pounds of coffee at $28\frac{5}{8}$ cents.

40. A father worked 6 days at $\$2\frac{3}{4}$ per day, his son 5 days at $\$1\frac{3}{4}$, his daughter 4 days at $\$\frac{4}{5}$; what were their total earnings for the week?

41. What is the area of a door $3\frac{1}{2}$ feet $\times$ $7\frac{1}{4}$ feet?

42. Draw a square $5\frac{1}{2}$ inches by $5\frac{1}{2}$ inches. Draw lines dividing it into square inches. Multiply $5\frac{1}{2}$ by $5\frac{1}{2}$ and point out each partial product in the diagram.

43. The sheet on which I now write is $7\frac{3}{4}'' \times 10\frac{3}{8}''$. How many square inches?

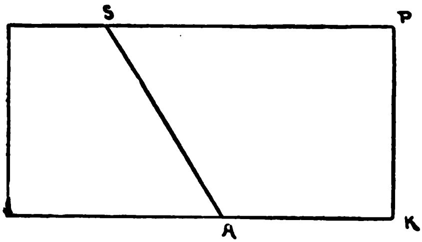


RHOMBOID.

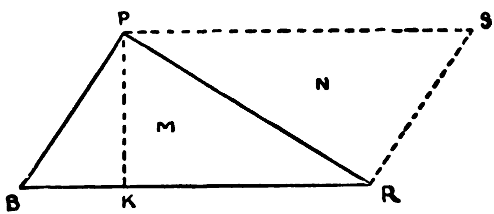
44. A rhomboid is a four-sided figure with parallel sides and oblique angles. It was shown in Art. **92** that the area of a rectangle is the number of square units in a row the length of the rectangle multiplied by the number of such rows; or, briefly, area = base $\times$ altitude.

In the rhomboid the perpendicular distance from the base to the side opposite is the altitude.

45. Cut a rhomboid out of paper. Fold the base AB upon itself, creasing it along the altitude PK. Cut along PK; arrange the parts as in the second figure. The base, altitude, and area of this rectangle are the same as in the rhomboid; hence area of rhomboid = base × altitude.



46. Cut out two equal paper triangles, M and N. Place a pair of equal sides together thus, making the rhomboid BS.



Since the base and altitude of triangle M and of the rhomboid are equal, area of triangle $= \frac{\text{base} \times \text{altitude}}{2}$

47. Cut out five paper triangles, measure accurately their bases and altitudes, and determine their areas.

48. Determine the area of any triangular spaces in the school-yard. Draw each on some convenient scale.

49. Draw and calculate the area of the following triangles. Use the scale of one inch to the foot in the first set, one quarter-inch to the foot in the second set.

	Base.	Altitude.		Base.	Altitude.
50.	4 ft.	$2\frac{1}{2}$ ft.	**55.**	20 ft.	$17\frac{1}{2}$ ft.
51.	2 yd.	$1\frac{1}{4}$ ft.	**56.**	$18\frac{3}{4}$ ft.	$6\frac{1}{2}$ ft.
52.	$3\frac{5}{8}$ ft.	$4\frac{3}{4}$ ft.	**57.**	$9\frac{3}{5}$ ft.	$14\frac{2}{3}$ ft.
53.	$4\frac{7}{16}$ ft.	$2\frac{3}{8}$ ft.	**58.**	6 ft.	$14\frac{3}{7}$ ft.
54.	$3\frac{3}{5}$ ft.	$1\frac{1}{4}$ yd.	**59.**	$4\frac{3}{5}$ yd.	$3\frac{3}{4}$ yd.

60. A cubic foot of water weighs $62\frac{1}{2}$ pounds. Ice is $\frac{23}{25}$ as heavy as water. What is the weight of a cubic foot of ice?

61. What is the weight of a cubic foot of Joliet limestone which is $2\frac{3}{5}$ times as heavy as water? of dry pine $\frac{12}{25}$ as heavy?

62. A gallon of water weighs $8\frac{1}{3}$ pounds. What is the weight of a gallon of mercury $13\frac{3}{5}$ times as heavy? of a gallon of milk which is $\frac{103}{100}$ as heavy as water?

63. Standard silver is $\frac{9}{10}$ pure. How many grains of silver in the standard dollar of $412\frac{1}{2}$ grains?

64. The rear wheel of a bicycle is $7\frac{1}{3}$ feet in circumference, and revolves $2\frac{1}{2}$ times as often as the pedals. How many miles are traveled in 1,000 revolutions of the pedals? (1 mile = 5,280 feet.)

65. From what number can $5\frac{19}{36}$ be taken nine times with no remainder?

158. DIVISION OF FRACTIONS.

1. Define Measurement, Divisor, Dividend, Quotient.

2. Define Partition, Divisor, Dividend, Quotient.

3. Illustrate a problem in measurement and one in partition by using objects.

159. Divisor an Integer.

Illustrative Problem. **1.** Divide $\frac{10}{11}$ into 5 equal parts.

Is this a problem in measurement or partition? Why? How can it be performed with objects?

According to the definitions heretofore given, this is a problem in partition. It may be read: Find $\frac{1}{5}$ of $\frac{10}{11}$; $\frac{1}{5}$ of $\frac{10}{11} = \frac{2}{11}$.

2. Divide $\frac{18}{25}$ by 6; $\frac{15}{16}$ by 3; $\frac{21}{22}$ by 7; $\frac{24}{25}$ by 12; $\frac{27}{31}$ by 9; $\frac{39}{40}$ by 13.

3. Divide $\frac{48}{49}$ by 16; $\frac{51}{58}$ by 17; $\frac{57}{61}$ by 19; $\frac{68}{69}$ by 17; $\frac{87}{101}$ by 29.

4. Divide $\frac{90}{91}$ by 15; $\frac{91}{15}$ by 13; $\frac{95}{18}$ by 19; $\frac{144}{21}$ by 24; $4\frac{2}{3}$ by 7; $8\frac{2}{5}$ by 14; $5\frac{2}{11}$ by 19.

How may all of these divisions be performed? Make a rule based upon the solution of these problems.

Illustrative Problem. **5.** Divide $\frac{7}{8}$ by 5.

How does this problem differ from the preceding? In what other way may a fraction be divided by an integer? $\frac{1}{5}$ of $\frac{7}{8} = \frac{7}{40}$, obtained by multiplying the denominator by 5, which divides each fractional unit by 5.

Explain fully the effect of multiplying the denominator by an integer.

6. Divide $\frac{5}{12}$ by 4; $\frac{6}{7}$ by 7; $\frac{11}{12}$ by 9; $2\frac{1}{3}$ by 10; $4\frac{2}{5}$ by 13.

7. Divide $\frac{3}{4}$ by 8; $\frac{5}{9}$ by 11; $\frac{7}{15}$ by 12; $\frac{9}{10}$ by 16.

8. Divide $3\frac{1}{3}$ by 7; $5\frac{2}{5}$ by 8; $7\frac{3}{4}$ by 12; $8\frac{5}{6}$ by 9.

9. Divide $\frac{8}{15}$ by 12.

ANALYSIS. $\frac{1}{12} = \frac{1}{3}$ of $\frac{1}{4}$. Hence $\frac{1}{12}$ of $\frac{8}{15} = \frac{1}{3}$ of $\frac{1}{4}$ of $\frac{8}{15}$. $\frac{1}{4}$ of $\frac{8}{15} = \frac{2}{15}$. $\frac{1}{3}$ of $\frac{2}{15} = \frac{2}{45}$.

10. Divide $\frac{12}{13}$ by 15; $\frac{10}{11}$ by 20; $\frac{20}{21}$ by 25; $\frac{21}{25}$ by 14; $\frac{27}{28}$ by 18; $\frac{35}{36}$ by 14; $\frac{39}{40}$ by 26.

11. Divide $\frac{51}{40}$ by 34; $\frac{52}{63}$ by 39; $\frac{54}{65}$ by 36; $\frac{57}{20}$ by 38; $\frac{68}{15}$ by 85.

How was the division performed in the first set ᴗ. problems? How in the second set? How in the third? Make a rule based on these solutions. What cancellations should be performed?

160. Divisor a Fraction.

Illustrative Problem. **1.** Divide 4 by $\frac{2}{3}$.

ANALYSIS. To divide 4 by $\frac{2}{3}$ is to separate 4 into equal parts, each of which is $\frac{2}{3}$. $4 = \frac{12}{3}$. In $\frac{12}{3}$ there are six 2's of thirds (or 6 times $\frac{2}{3}$).

Illustrate this problem by folding 4 paper squares into thirds, and then separating the thirds into groups of two each.

2. Divide 8 by $\frac{2}{5}$.

$8 = \frac{40}{5}$. In $\frac{40}{5}$ there are 20 times $\frac{2}{5}$.

What is done with the integer? How is the division performed? Make a rule.

Divide:

3. 7 by $\frac{3}{4}$. **4.** 12 by $\frac{5}{8}$. **5.** 10 by $\frac{6}{7}$.

RULE.

To divide a whole number by a fraction, reduce the whole number to the same denomination as the fraction, and divide the numerator of the dividend by the numerator of the divisor.

Divide:

6. 18 by $\frac{9}{10}$. **8.** 32 by $\frac{14}{15}$. **10.** 63 by $\frac{21}{22}$.

7. 24 by $\frac{10}{11}$. **9.** 45 by $\frac{19}{20}$. **11.** 36 by $\frac{18}{19}$.

12. How many boxes each holding $\frac{2}{3}$ of a quart can be filled from 8 quarts of berries?

13. How many yards of cloth at $\frac{5}{6}$ of a dollar a yard can be bought for $12?

14. If a man can dig a ditch $\frac{7}{12}$ of a rod in length in an hour, how many rods can he dig in $3\frac{2}{3}$ days of 9 hours each?

15. At $2\frac{3}{5}$ cents each, how many apples can be bought for 52 cents?

Reduce the mixed number to an improper fraction $2\frac{3}{5} = \frac{13}{5}$.

16. At $\$2\frac{1}{4}$ a yard, how many yards of cloth can be bought for \$63?

17. At $\$3\frac{3}{7}$ a day, how many days must a man work to earn \$144?

18. At $\$24\frac{3}{8}$ an acre, how many acres can be bought for \$724?

19. If one horse cost $\$124\frac{11}{16}$, how many horses can be bought for \$3,990?

161. *Illustrative Problem.* **1.** Divide 1 by $\frac{2}{5}$.

ANALYSIS. $1 = \frac{5}{5}$. In $\frac{5}{5}$ there are as many $\frac{2}{5}$ as there are 2's in 5. The quotient of 5 by 2 may be expressed thus: $\frac{5}{2}$. Hence $1 \div \frac{2}{5} = \frac{5}{2}$.

Explain the following in the same way:

Divide 1 by $\frac{2}{3}$; by $\frac{4}{5}$; by $\frac{6}{7}$; by $\frac{7}{8}$; by $\frac{10}{11}$; by $\frac{11}{10}$; by $\frac{14}{15}$; by $\frac{19}{20}$; by $\frac{20}{17}$; by $\frac{21}{16}$.

NOTE. The denominator is the number of fractional units in 1. The numerator is the number of fractional units in the fraction; hence the rule.

RULE.

To divide 1 by a fraction, divide the denominator of the fraction by its numerator.

NOTE. This is usually called "inverting the divisor."

2. Divide 15 by $\frac{2}{7}$.

ANALYSIS. If the dividend were 1, the quotient would be $\frac{7}{2}$. Since the dividend is 15, the quotient is 15 times $\frac{7}{2}$, which equals, etc.

Analyze Problems 2 to 11 (Art. **160**) by this method.

162. Division of a Fraction by a Fraction.

Illustrative Problem. Divide $\frac{3}{4}$ by $\frac{1}{3}$.

Fold a paper square into fourths. Tear out one of the fourths. We are now to see how many pieces, each of which is one third of the paper square, can be made from the three fourths. Since thirds are not easily made from fourths, we change the three fourths to nine twelfths. Four twelfths

make one third. Nine twelfths make two thirds with one twelfth left, which is one fourth of another third. Hence, in $\frac{3}{4}$ of a sheet of paper there are $2\frac{1}{4}$ thirds of a sheet.

Explain the following problems by the same method.

Divide:

1. $\frac{3}{4}$ by $\frac{1}{5}$.
3. $\frac{5}{6}$ by $\frac{2}{5}$.
5. $\frac{7}{8}$ by $\frac{2}{11}$.
2. $\frac{2}{3}$ by $\frac{1}{4}$.
4. $\frac{5}{7}$ by $\frac{3}{5}$.
6. $\frac{6}{7}$ by $\frac{2}{3}$.

Now omit the reference to objects. What do you do with divisor and dividend?

RULE.

To divide a fraction by a fraction, change them to equivalent fractions having the l. c. d., and divide the numerator of the dividend by the numerator of the divisor.

7. Divide $\frac{9}{10}$ by $\frac{1}{7}$.

ANALYSIS. $\frac{9}{10} = \frac{63}{70}$. $\frac{1}{7} = \frac{10}{70}$. $\frac{63}{70} \div \frac{10}{70} = 63 \div 10 = \frac{63}{10} = 6\frac{3}{10}$.

8. Divide $\frac{1}{5}$ by $\frac{5}{6}$.

ANALYSIS. $\frac{1}{5} = \frac{6}{30}$. $\frac{5}{6} = \frac{25}{30}$. $\frac{6}{30} \div \frac{25}{30} = 6 \div 25 = \frac{6}{25}$.

9. Divide $\frac{7}{11}$ by $\frac{2}{7}$; $\frac{8}{15}$ by $\frac{4}{9}$; $\frac{4}{9}$ by $\frac{8}{15}$; $\frac{17}{21}$ by $\frac{3}{8}$.

10. Divide $2\frac{1}{2}$ by $1\frac{2}{3}$; $3\frac{1}{7}$ by $2\frac{1}{5}$; $1\frac{5}{11}$ by $3\frac{2}{3}$.

163. Shorter Analysis.

1. Divide $\frac{5}{6}$ by $\frac{4}{5}$.

ANALYSIS. $1 \div \frac{4}{5} = \frac{5}{4}$. $\frac{5}{6} \div \frac{4}{5} = \frac{5}{6}$ of $\frac{5}{4} = \frac{25}{24}$.

2. Divide $\frac{2}{3}$ by $\frac{2}{5}$; $\frac{3}{4}$ by $\frac{3}{5}$; $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{5}{6}$ by $\frac{3}{7}$; $\frac{6}{7}$ by $\frac{12}{7}$; $\frac{7}{8}$ by $\frac{21}{40}$; $\frac{11}{12}$ by $\frac{24}{33}$.

Cancel common factors.

RULE.

To divide a fraction by a fraction, invert the divisor and proceed as in multiplication.

Use the following form, $\frac{19}{20} \div \frac{7}{10} = \frac{19}{20} \times \frac{10}{7} = \frac{19}{14}$.

3. Divide $\frac{19}{20}$ by $\frac{7}{10}$; $\frac{42}{51}$ by $\frac{7}{34}$; $\frac{17}{27}$ by $\frac{85}{81}$.

164. PROBLEMS.

1. At $\$\frac{2}{7}$ a pound, how many pounds of coffee can be bought for $\$3\frac{1}{5}$?

2. If a man travel $4\frac{3}{5}$ miles an hour, in how many hours will he travel $23\frac{7}{8}$ miles?

3. At $\$2\frac{1}{8}$ a yard, how many yards of cloth can be bought for $\$6\frac{4}{5}$?

4. If each bag hold $1\frac{5}{8}$ bushels, how many bags will be needed to hold $40\frac{1}{3}$ bushels of oats?

5. At $\$\frac{10}{11}$ a yard, how much cloth will $\$\frac{2}{7}$ buy?

6. If an acre of land will yield $23\frac{5}{12}$ bushels of wheat, how many acres are necessary to yield $1{,}874\frac{4}{5}$ bushels?

7. $(\frac{2}{3} \div \frac{7}{8}) \div (\frac{8}{7} \times \frac{2}{5}) = ?$

8. $(\frac{2}{3} + \frac{4}{7}) \div (\frac{5}{6} - \frac{2}{7}) = ?$

9. $\frac{5}{8}$ of $\frac{2}{5}$ of $\frac{7}{12} \div \frac{3}{8}$ of $\frac{4}{7}$ of $\frac{8}{15} = ?$

10. $(\frac{5}{9} \times \frac{18}{25}) \div (\frac{18}{25} \div \frac{5}{9}) = ?$

11. Divide $\frac{18}{25}$ by 9; $\frac{15}{16}$ by 25; $\frac{7}{12}$ by 10.

12. Divide 6 by $\frac{4}{5}$; 12 by $\frac{5}{6}$; 15 by $\frac{5}{8}$.

165. COMPLEX FRACTIONS.

Problems in division of fractions are sometimes written in the form of a fraction; thus, $\frac{2}{3} \div \frac{3}{4}$ may be written $\dfrac{\frac{2}{3}}{\frac{3}{4}}$. Such expressions are called **Complex Fractions.**

1. The following method of reading complex fractions is recommended. The complex fraction whose numerator is $\frac{3}{4}$ and denominator $\frac{5}{8}$.

Read:

$$\dfrac{\frac{3}{4}}{\frac{5}{8}};\quad \dfrac{2}{\frac{4}{7}};\quad \dfrac{\frac{3}{4}}{7};\quad \dfrac{\frac{5}{6}}{\frac{7}{8}};\quad \dfrac{\dfrac{2\frac{1}{3}}{5}}{8};\quad \dfrac{\frac{5}{8}}{4\frac{1}{5}},\quad \dfrac{\dfrac{\frac{2}{3}}{4}}{6}.$$

2. The longest straight line used separates the numerator from the denominator. This line may be regarded as a sign

of division. The expression above it is the dividend, and that below it the divisor. Solve the following as problems in division.

Reduce:

1. $\dfrac{\frac{3}{4}}{\frac{5}{8}}$.	5. $\dfrac{\frac{5}{6}}{\frac{10}{11}}$.	9. $\dfrac{\frac{5}{12} \div \frac{15}{28}}{\frac{15}{28} \div \frac{5}{12}}$.
2. $\dfrac{2\frac{1}{3}}{4\frac{1}{5}}$.	6. $\dfrac{8 \div \frac{2}{5}}{\frac{2}{5} \div 8}$.	10. $\dfrac{\frac{5}{7} \text{ of } 2\frac{1}{3}}{2\frac{1}{3} \div \frac{5}{7}}$.
3. $\dfrac{\frac{5}{8}}{12}$.	7. $\dfrac{8 \times \frac{3}{4}}{\frac{3}{4} \times 8}$.	11. $\dfrac{\frac{2}{3} \times 6\frac{1}{5}}{6\frac{1}{5} \div \frac{5}{6}}$.
4. $\dfrac{\frac{3}{4} \text{ of } \frac{4}{7}}{\frac{5}{8} \text{ of } \frac{9}{10}}$.	8. $\dfrac{\frac{4}{5} + \frac{2}{3}}{\frac{4}{5} - \frac{2}{3}}$.	12. $\dfrac{\frac{3}{4} \times 9\frac{2}{3}}{9\frac{2}{3} \div \frac{7}{8}}$.

166. To Find the Part which One Number is of Another.

1. 3 is what part of 7?

ANALYSIS. 1 is $\frac{1}{7}$ of 7, hence 3 is $\frac{3}{7}$ of 7.

2. 5 is what part of 12? 6, of 17? 11, of 22? 5, of 20? 8, of 24? 9, of 36? 17, of 12? 19, of 7?

The part which one number is of another is always expressed by a fraction, of which the number that is the part is the numerator, and the other the denominator.

Make a rule for finding the part that one integer is of another.

3. $\frac{2}{3}$ is what part of 5?

ANALYSIS. $5 = \frac{15}{3}$. $\frac{2}{3}$ is the same part of $\frac{15}{3}$ that 2 is of 15. 2 is $\frac{2}{15}$ of 15; hence $\frac{2}{3}$ is $\frac{2}{15}$ of 5.

4. $\frac{3}{4}$ is what part of 8? $\frac{4}{5}$ is what part of 12? $\frac{5}{6}$ is what part of 10? $\frac{1}{7}$ is what part of 9? $\frac{2}{11}$ is what part of 8? $\frac{7}{12}$ is what part of 6?

5. $\frac{10}{11}$ is what part of 4? $\frac{10}{13}$ is what part of 14? $\frac{17}{18}$ is what part of 126?

6. $2\frac{1}{3}$ is what part of 6? $3\frac{2}{5}$ is what part of 4? $5\frac{5}{6}$ is what part of 10?

Change the mixed numbers to improper fractions.

Make a rule for finding the part that a fraction is of an integer.

7. $\frac{4}{5}$ is what part of $\frac{5}{6}$?

ANALYSIS. $\frac{4}{5} = \frac{24}{30}$. $\frac{5}{6} = \frac{25}{30}$. $\frac{24}{30}$ is the same part of $\frac{25}{30}$ that 24 is of 25. 24 is $\frac{24}{25}$ of 25; hence $\frac{4}{5}$ is $\frac{24}{25}$ of $\frac{5}{6}$.

What was done in the above problem? Make a rule for such cases.

8. $\frac{2}{3}$ is what part of $\frac{4}{5}$?

9. $\frac{1}{7}$ is what part of $\frac{2}{3}$?

10. $\frac{5}{12}$ is what part of $\frac{1}{2}$?

11. With each of the following pairs of numbers, find the part which the first is of the second, and give the results rapidly.

(1) $\frac{1}{3}$, $\frac{1}{2}$.	(9) $\frac{2}{5}$, $\frac{5}{7}$.	(17) $7\frac{3}{10}$, $8\frac{2}{5}$.
(2) $\frac{1}{2}$, $\frac{1}{3}$.	(10) $\frac{5}{6}$, $\frac{7}{8}$.	(18) $8\frac{1}{6}$, $10\frac{4}{7}$.
(3) $\frac{2}{3}$, $\frac{1}{4}$.	(11) $4\frac{2}{5}$, $5\frac{3}{4}$.	(19) $12\frac{3}{10}$, $15\frac{1}{3}$.
(4) $\frac{1}{4}$, $\frac{2}{3}$.	(12) $\frac{1}{6}$, $8\frac{1}{2}$.	(20) $19\frac{1}{5}$, $14\frac{3}{5}$.
(5) $\frac{1}{5}$, 6.	(13) $\frac{1}{8}$, 9.	(21) $21\frac{1}{4}$, $25\frac{3}{4}$.
(6) $\frac{3}{4}$, 8.	(14) 9, $\frac{1}{8}$.	(22) $\frac{13}{25}$, $\frac{9}{10}$.
(7) 8, $6\frac{1}{2}$.	(15) $\frac{4}{7}$, $\frac{5}{6}$.	(23) $\frac{19}{30}$, $\frac{19}{20}$.
(8) $6\frac{1}{2}$, 8.	(16) $5\frac{2}{3}$, $4\frac{3}{4}$.	(24) $22\frac{1}{12}$, $45\frac{1}{24}$.

Show how the following general rule is derived:

To find the part that one number is of another, divide the number expressing the part by the number of which it is a part.

167. To Find a Number when a Specified Part of it is Given.

Illustrative Problem. 15 is $\frac{3}{8}$ of what number? Since 15 is $\frac{3}{8}$ of the required number, $\frac{1}{8}$ of that number is $\frac{1}{3}$ of 15. $\frac{1}{3}$ of 15 is 5. $\frac{8}{8}$ of the required number is 8 fives, which are 40. Hence 15 is $\frac{3}{8}$ of 40.

Find the number of which

1. 45 is $\frac{5}{9}$.
2. 48 is $\frac{3}{4}$.
3. 36 is $\frac{9}{16}$.
4. 72 is $\frac{18}{25}$.
5. 75 is $\frac{25}{31}$.
6. 84 is $\frac{7}{16}$.
7. 90 is $\frac{18}{19}$.
8. 125 is $\frac{25}{63}$.
9. 324 is $\frac{18}{65}$.
10. 441 is $\frac{21}{80}$.
11. $\frac{3}{4}$ is $\frac{7}{8}$.
12. $\frac{2}{3}$ is $\frac{5}{6}$.
13. $3\frac{3}{4}$ is $\frac{5}{24}$.
14. $7\frac{4}{5}$ is $\frac{13}{25}$.
15. $18\frac{3}{4}$ is $\frac{3}{16}$.
16. $41\frac{2}{3}$ is $\frac{5}{12}$.
17. $46\frac{2}{3}$ is $\frac{7}{15}$.
18. $\frac{7}{11}$ is $\frac{5}{18}$.
19. $91\frac{2}{3}$ is $\frac{11}{24}$.
20. $23\frac{3}{4}$ is $\frac{19}{48}$.
21. $34\frac{4}{11}$ is $\frac{21}{64}$.

22. A has $\$3\frac{1}{4}$ and B $\$7\frac{4}{5}$. A's money is the same part of B's that B's is of C's. How much has C?

23. A farmer has $15\frac{5}{8}$ acres of meadow and 40 acres of oats. The part which the meadow is of the oats-field is the same that the latter is of the corn-field. How many acres of corn has he?

24. A house cost $\frac{4}{3}$ as much as the lot. Both cost $896. Find the cost of each.

25. The area of North America is 6,446,000 square miles. This is about $\frac{5}{9}$ of the area of Africa. What is the approximate area of the latter?

26. The area of Australasia is 3,288,000 square miles. It is about what part as large as North America? as Africa (11,514,000)? as South America (6,837,000)?

27. The annual expenditure of England per capita for military purposes is $3.72, and for education is 70 cents per capita. The latter is what part of the former? (Approximate.)

28. The distance from one corner of a field to another corner is 770 yards; this is $\frac{7}{16}$ of a mile. How many feet are there in a mile?

29. A certain farm contains 340 acres which are $\frac{17}{32}$ of a section. How many acres are there in a section?

30. A merchant sold goods to the amount of $316.80 on Monday; this was $\frac{9}{10}$ of his sales on Tuesday, which were

$\frac{5}{7}$ of his sales on Wednesday. What was the aggregate of his sales for the three days?

31. In a school-room there are three windows each 9 feet $\times\ 3\frac{1}{2}$. If the lighting area is $\frac{3}{20}$ of the floor area, what is the latter? If the room is 30 feet wide, what is its length?

168. MISCELLANEOUS PROBLEMS.

Oral Exercises.

(Use no written work in the solution of these problems.)

1. Find the l. c. m. of 4, 5, 6; of 8, 12, 20; of 5, 15, 30, 40; of 4, 6, 8, 10, 12, 15; of 12, 15, 18, 20, 30.

2. Find the l. c. m. of 12, 18, 24; of 15, 30, 40, 45; of 18, 24, 30, 36; of 60, 90, 105, 120; of 80, 120, 160, 240.

3. Change to whole or mixed numbers $\frac{18}{7}$, $\frac{29}{5}$, $\frac{46}{7}$, $\frac{85}{12}$, $\frac{125}{19}$, $\frac{160}{24}$.

4. Change to improper fractions $9\frac{2}{5}$, $10\frac{5}{11}$, $12\frac{4}{7}$, $15\frac{5}{8}$, $18\frac{2}{3}$, $21\frac{1}{5}$.

5. Add $\frac{1}{3}$ and $\frac{1}{4}$; $\frac{2}{5}$ and $\frac{2}{7}$; $\frac{3}{4}$ and $\frac{3}{8}$; $\frac{5}{9}$ and $\frac{5}{7}$; $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$; $\frac{2}{5}$, $\frac{1}{6}$, $\frac{3}{10}$, and $\frac{7}{15}$.

6. $\frac{4}{5}-\frac{1}{3}=?$ $\frac{2}{3}-\frac{2}{5}=?$ $\frac{3}{8}-\frac{3}{9}=?$ $\frac{5}{12}-\frac{1}{7}=?$

7. Multiply $\frac{5}{8}$ by 4; $\frac{7}{12}$ by 24; $\frac{8}{15}$ by 45; $\frac{6}{11}$ by 44; $\frac{9}{17}$ by 8.

8. Find $\frac{5}{8}$ of 16; $\frac{7}{12}$ of 36; $\frac{5}{11}$ of 10; $\frac{7}{12}$ of 13; $\frac{14}{15}$ of 7.

9. Find $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{5}{8}$ of $\frac{16}{25}$; $\frac{7}{10}$ of $\frac{5}{12}$; $\frac{4}{11}$ of $\frac{9}{13}$.

10. Multiply $\frac{5}{6}$ by $\frac{2}{7}$; $2\frac{1}{4}$ by $3\frac{1}{3}$; $6\frac{1}{2}$ by $2\frac{1}{4}$; $\frac{5}{7}$ by $\frac{9}{12}$; $\frac{10}{11}$ by $\frac{7}{9}$.

11. Divide 4 by $\frac{1}{6}$; 7 by $\frac{1}{8}$; 10 by $\frac{1}{5}$; 12 by $\frac{1}{11}$; 6 by $\frac{2}{5}$; 7 by $\frac{3}{10}$; 8 by $\frac{5}{9}$; 6 by $\frac{11}{12}$; 15 by $\frac{7}{10}$; 14 by $\frac{2}{3}$; 18 by $\frac{3}{4}$; 20 by $\frac{5}{6}$; 10 by $\frac{6}{7}$; 9 by $\frac{10}{11}$; 8 by $\frac{11}{12}$; 7 by $\frac{11}{15}$.

12. Divide $\frac{10}{11}$ by 5; $\frac{12}{13}$ by 6; $\frac{8}{15}$ by 16; $\frac{9}{17}$ by 18; $\frac{12}{13}$ by 36; $\frac{16}{17}$ by 48; $\frac{17}{18}$ by 51; $\frac{29}{30}$ by 87; $\frac{26}{27}$ by 39; $\frac{24}{25}$ by 72.

13. Divide 1 by $\frac{1}{8}$; 1 by $\frac{1}{9}$; 1 by $\frac{1}{13}$; 1 by $\frac{2}{5}$; 1 by $\frac{3}{11}$; 1 by $\frac{29}{30}$; 1 by $\frac{49}{55}$; 1 by $\frac{97}{100}$.

If one be divided by any fraction, what will the quotient be?

14. Divide $\frac{2}{3}$ by $\frac{4}{3}$; $\frac{5}{8}$ by $\frac{5}{9}$; $\frac{7}{8}$ by $\frac{7}{16}$; $\frac{10}{11}$ by $\frac{20}{33}$; $3\frac{1}{5}$ by $6\frac{2}{3}$.

15. $\frac{5}{6} + \frac{4}{5} = ?$ $\frac{5}{6} - \frac{4}{5} = ?$ $\frac{5}{6} \times \frac{4}{5} = ?$ $\frac{5}{6} \div \frac{4}{5} = ?$ $\frac{4}{5}$ is what part of $\frac{5}{6}$? $\frac{5}{6}$ is what part of $\frac{4}{5}$?

16. 7 is what part of 13? 8, of 19? 11, of 44? 18, of 27? $\frac{2}{3}$ is what part of 6? of 9? of 15? $\frac{7}{8}$ is what part of 4? of 7? of 10? of 12? of 15?

17. $\frac{2}{5}$ is what part of $\frac{4}{7}$? of $\frac{5}{12}$? of $\frac{8}{9}$? of $\frac{12}{13}$? $\frac{7}{10}$ is what part of $\frac{7}{8}$? of $\frac{9}{10}$? of $\frac{11}{12}$? of $2\frac{1}{5}$? of $3\frac{3}{4}$?

18. $14\frac{2}{5}$ is $\frac{8}{9}$ of what number?

19. $\frac{2}{3}$ of 15 is $\frac{5}{8}$ of what number?

20. $\frac{3}{4}$ of $\frac{4}{5}$ of 15 is $\frac{2}{3}$ of $\frac{1}{2}$ of 9 times what number?

21. $\frac{6}{7}$ of 21 is $\frac{2}{9}$ of what number?

22. $\frac{10}{11}$ of $1\frac{13}{20}$ is $\frac{3}{40}$ of 5 times what number?

23. John has $\frac{19}{20}$ of a dollar; William has $\frac{4}{5}$ as much, and this is 16 cents more than $\frac{3}{8}$ of Henry's money. How much has Henry?

24. John lost $\frac{2}{7}$ of his marbles, and has 15 left. How many had he at first?

25. Mary's money is $\frac{3}{7}$ of Laura's, and both have 90 cents. How much has each?

26. A's farm is $\frac{7}{8}$ of $\frac{2}{5}$ of B's. A and B together own 81 acres. How many acres has each?

27. John is 7; Fred 9: (1) The difference in their ages is what part of John's age? (2) Of Fred's age? (3) John's age is what part of Fred's age? (4) John's age is what part of Fred's age less than Fred's age? (5) Fred's age is what part of John's age more than John's age?

NOTE. In concrete problems, or where a fraction expresses the relation between two numbers, "of" follows the fraction. If one number is a frac-

tion of a second number more or less than the second number, the "of" phrase is frequently omitted; thus Jane's age is $\frac{1}{5}$ less than Mary's means, "Jane's age is $\frac{1}{5}$ of Mary's less than Mary's."

28. A raised 150 bushels of potatoes, which was $\frac{1}{6}$ less than what B raised. How many did B raise?

29. James has $84, which is $\frac{1}{3}$ less than B's money. How much has B?

30. A walked 120 miles, which is $\frac{1}{5}$ more than B walked, and $\frac{1}{5}$ less than C walked. How far did B and C walk?

31. 63 is $\frac{1}{8}$ more than what number? It is $\frac{1}{8}$ less than what number?

32. The time past noon is $\frac{1}{3}$ of the time till midnight. What o'clock is it?

33. The time till midnight is $\frac{1}{5}$ of the time past noon. What o'clock is it?

34. The time till midnight is $\frac{3}{5}$ of the time past noon. What o'clock is it?

At what hour is the time till midnight $\frac{3}{5}$ of the time past 3 A.M.?

35. How many cubic inches in a brick $8'' \times 4'' \times 2\frac{1}{4}''$? How many half-inch cubes in a two-inch cube?

36. Arrange the fractions $\frac{5}{8}$, $\frac{11}{18}$, $\frac{7}{12}$, $\frac{13}{24}$, in order of magnitude.

37. $\frac{8}{9}$ of water is oxygen. What weight of oxygen in a gallon of water weighing $8\frac{1}{3}$ pounds?

38. $\frac{1}{5}$ of air is oxygen. How much oxygen in a cubic foot of air weighing 525 grains?

39. If 15 gold pens cost $25, what is the cost of 3 gold pens?

40. If 21 sheep are worth $56, what are 3 sheep worth?

QUERY. Is it necessary to find the cost of one sheep?

41. If 24 men consume a barrel of flour (196 pounds) in 2 weeks, how many pounds do 3 men consume?

42. If 32 horses in 3 days consume 60 bushels of oats, how many bushels do 20 horses consume in one day?

43. If 12 bushels of oats are worth $9\frac{3}{8}$ bushels of corn, 10 bushels of oats are worth how much corn?

44. If 8 barrels of flour cost $\$25\frac{3}{8}$, what do 11 barrels cost?

45. If at noon on a certain day a 15-foot pole casts a 12-foot shadow, what length of shadow is cast by a pole 32 feet long?

46. A pine block $3'' \times 3'' \times 3''$ weighs 5 ounces. What is the weight of a similar block $3'' \times 4'' \times 6''$?

47. A can dig a ditch in 6 days. What part of it can he dig in one day?

B can do the same work in 4 days; what part can both dig in one day working together?

48. A and B can trim $\frac{8}{20}$ of a hedge in one day. In what time can they trim $\frac{1}{20}$ of the hedge? $\frac{20}{20}$ of the hedge?

49. A can do a piece of work in 7 days. A and B can do the same work in 4 days. In what time can B do the work alone?

50. A can mow a field in 5 days, and B can mow it in 6 days. In how many days can they mow it, working together?

51. A can perform a certain work in 8 days, B in 10 days, and C in 12 days. In what time can the three perform it, working together?

52. Three pipes, A, B, and C, fill a cistern in 3 hours. A alone fills it in 8 hours; B in 12 hours. In what time can C alone fill it?

53. Pipe A can fill a cistern in 10 hours; pipe B in 12 hours; pipe C empties it in 15 hours. When all three pipes are open, how long is the cistern in filling?

54. A can saw a cord of wood in $\frac{1}{2}$ a day, B in $\frac{1}{3}$ of a day. In what time can both saw it working together?

QUERY. How much does each saw in a day?

55. John can spade a garden in $\frac{3}{4}$ of a day, Charles in $\frac{4}{5}$ of a day. In what time can they do it, working together?

56. If $\frac{3}{8}$ of a yard of cloth cost $\$1\frac{4}{5}$, what will $\frac{7}{12}$ of a yard cost?

57. If to $\frac{3}{4}$ of the cost of an article $2 be added, the result will be $\frac{5}{6}$ of the cost; what is the cost?

58. $\frac{3}{7}$ of the distance from Chicago to Elgin is $\frac{9}{19}$ of the distance from Chicago to Aurora. Elgin is 4 miles farther from Chicago. How far is each from Chicago?

ANALYSIS.
$\frac{3}{7}$ of dist. to E = $\frac{9}{19}$ of dist. to A.
$\frac{1}{7}$ of dist. to E = $\frac{3}{19}$ of dist. to A.
Dist. to E = $\frac{21}{19}$ of dist. to A.
Dist. to A = $\frac{19}{19}$ of dist. to A.
Difference of distances = $\frac{2}{19}$ of dist. to A.
Difference of distances = 4 miles.
$\frac{2}{19}$ of dist. to A = 4 miles.
$\frac{1}{19}$ of dist. to A = 2 miles.
$\frac{21}{19}$ of dist. to A = 42 miles, dist. to E.
$\frac{19}{19}$ of dist. to A = 38 miles, dist. to A.

59. If $\frac{3}{5}$ of the distance from C to D is $\frac{4}{7}$ of the distance from E to F, and if the sum of the two distances is 82 miles, what is the distance from C to D? from E to F?

60. A house and lot cost $5,200. The lot cost $\frac{1}{12}$ as much as the house. What did each cost?

61. $\frac{12}{13}$ is $\frac{6}{5}$ of what number?

62. What is the cost of $6\frac{1}{3}$ yards of cloth at $\$2\frac{1}{2}$ a yard? of $4\frac{3}{8}$ yards at $\$2\frac{2}{3}$ a yard? of $7\frac{3}{7}$ yards at $\$1\frac{3}{4}$ a yard?

63. How many pounds of material can be bought for $\$5\frac{1}{4}$ at $\$\frac{3}{8}$ a pound? for $\$4\frac{2}{7}$ at $\$\frac{5}{14}$ a pound? for $\$8\frac{2}{5}$ at $\$\frac{7}{10}$ a pound? for $\$10\frac{1}{2}$ at $\$\frac{2}{3}$ a pound?

64. If A had $4 and lost $\$\frac{2}{5}$, what part of his money did he lose? if he had $5 and lost $\$\frac{3}{4}$? if he had $\$6\frac{1}{2}$ and lost $4? if he had $\$12\frac{3}{4}$ and lost $7? if he had $\$10\frac{4}{5}$ and lost $\$6\frac{1}{2}$?

65. What is the cost of $\frac{1}{2}$ a yard and $\frac{1}{3}$ of a yard at $\$\frac{4}{5}$ a yard? of $\frac{2}{3}+\frac{2}{5}$ at $\$\frac{5}{8}$? of $\frac{3}{4}+\frac{3}{7}$ at $\$\frac{7}{11}$? of $\frac{1}{4}+\frac{2}{5}$ at $\$\frac{5}{6}$?

66. A grocer bought equal lots of eggs at 9 cents per dozen and 10 cents per dozen. He sold them at 12 cents per dozen, clearing 60 cents. How many dozen did he buy?

67. Sarah's age is $\frac{2}{3}$ of Mary's and $\frac{3}{4}$ of Ruth's. The sum of their ages is 46 years. How old is each?

NOTE. Find an expression for each in sixths of Sarah's age.

WRITTEN PROBLEMS.

1. The product of three numbers is $124\frac{2}{3}$. Two of the numbers are $7\frac{1}{5}$ and $8\frac{1}{4}$. What is the third?

2. What number divided by $\frac{3}{4}$ of $\frac{7}{12}$ equals $132\frac{5}{6}$?

3. What number diminished by $253\frac{7}{15}$ leaves $84\frac{23}{35}$?

4. What is the l. c. m. of 60, 125, 180, 225, 250?

5. Bought $8\frac{1}{2}$ yards of cloth at $\$3\frac{1}{3}$ a yard, $12\frac{2}{3}$ yards at $\$2\frac{3}{4}$, and $18\frac{1}{6}$ yards at $\$4\frac{1}{2}$. How many bushels of corn at $26\frac{2}{3}$ cents a bushel will pay the bill?

6. $15 \times \dfrac{8}{3\frac{3}{4}} \times \dfrac{5\frac{2}{7}}{9\frac{1}{4}} \times 1068 = ?$

7. What is the cost of 28 pounds of butter at $37\frac{1}{2}$ cents a pound, 84 bushels of corn at $43\frac{3}{4}$ cents a bushel, 135 bushels of oats at 25 cents a bushel, 160 bushels of rye at $62\frac{1}{2}$ cents a bushel?

8. A man performed $\frac{5}{18}$ of his journey the first day, $\frac{5}{12}$ of it the second day, $\frac{2}{9}$ of it the third day, and had $17\frac{2}{3}$ miles left. What was the length of his journey?

9. A man left $\frac{2}{7}$ of his estate to his eldest son, $\frac{3}{11}$ of it to his second son, and the remainder to his third son. The share of the second was \$520 less than that of the third. What was the value of the estate? What was each son's share?

10. Bought a farm of $324\frac{2}{5}$ acres of land for $15,646. If the house and a house-lot of 12 acres were counted at $3,150, what price per acre was paid for the rest?

11. What number is that $\frac{1}{7}$ of which exceeds $\frac{1}{8}$ of it by 112?

12. What number is that $\frac{2}{3}$ of $\frac{2}{5}$ of which is 291 less than $\frac{3}{4}$ of $\frac{5}{8}$ of it?

Owning $\frac{2}{11}$ of a farm, I sold $\frac{1}{5}$ of my share. If what I had left was worth $219, what would the whole farm be worth at the same rate?

14. I bought a house and lot for $5,784, paying $\frac{7}{17}$ as much for the lot as for the house. How much did I pay for each?

15. $\frac{3}{4}$ of A's farm equals $\frac{2}{7}$ of B's. Together they have 551 acres. How many has each?

16. A and B can do a certain piece of work in $12\frac{2}{3}$ days. They worked together $6\frac{4}{5}$ days; A then left, and B finished the work in $8\frac{4}{5}$ days. In how many days could each do the work alone?

17. How many bushels of grain can be bought for $$1,260\frac{3}{4}$$ at $31\frac{1}{4}$ cents a bushel? at $56\frac{1}{4}$? at $66\frac{2}{3}$? at $87\frac{1}{2}$?

18. If $56\frac{1}{4}$ acres of land cost $1,200, what is the price per acre?

19. How long will 225 pounds of meat last a crew of 5 men, at the rate of $1\frac{7}{8}$ pounds per day for each man?

20. A man's crop of oats weighed 66,677 pounds. He sold it for $31\frac{1}{4}$ cents a bushel. Counting 32 pounds for a bushel, what was the value of his crop?

21. A crop of wheat weighing 90,144 pounds brought $1,252. Counting 60 pounds to a bushel, what was the price per bushel for wheat sold?

22. $\frac{2}{7} + \frac{8}{15} + \frac{9}{13} - \frac{4}{7}$ of $\frac{8}{15} = ?$

23. Bought $8\frac{1}{2}$ yards of broadcloth at $\$4\frac{1}{4}$ a yard, $5\frac{1}{3}$ yards of cassimere at $\$2\frac{1}{2}$ a yard, $5\frac{2}{3}$ yards of silk at $\$2\frac{1}{2}$ a yard, and paid for all with corn at $41\frac{2}{3}$ cents a bushel. How many bushels were required?

24. What is the l. c. m. of 824, 936, 1020?

25. A man owned $\frac{3}{8}$ of a factory. He sold $\frac{1}{8}$ of his share. He gave $\frac{1}{5}$ of the remainder to his daughter, $\frac{1}{4}$ of what then remained to his son, and sold $\frac{1}{3}$ of the remainder for \$14,000. What was the value of the factory? What was the daughter's share? the son's share? What was the value of what he had left?

7 **26**. Find the sum, difference, and product of $4\frac{2}{7}$ and $6\frac{5}{11}$.

27. Find the quotient arising from dividing the sum of $8\frac{7}{13}$ and $5\frac{3}{7}$ by their difference.

28. A can do a piece of work in 15 days, B in 12 days, and C in 10 days. A works 2 days, B 3 days, and C 3 days. In what time can A and B finish the job by working together?

29. Thomas can dig a ditch in $3\frac{3}{4}$ days, Richard in $6\frac{2}{3}$ days, Harry in $4\frac{4}{5}$ days. In what time can all complete it working together?

30. Divide $\frac{5}{8}$ of $6\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{7}{15}$ of $12\frac{3}{4}$.

31. $\dfrac{6}{8\frac{1}{3}} \times \dfrac{4\frac{1}{2}}{5} \times \dfrac{\frac{5}{7}}{\frac{5}{12}} \times 16\frac{2}{3} \times \frac{9}{10}$.

32. A cistern having a capacity of $88\frac{1}{2}$ barrels contained $63\frac{9}{10}$ barrels. After $15\frac{7}{15}$ barrels were pumped out, the cistern was what part full?

33. A had a journey of $43\frac{7}{12}$ miles to perform. After traveling $36\frac{1}{3}$ miles, what part of the journey remained?

34. 23 is what part of $48\frac{1}{2}$? of $61\frac{4}{5}$? of $58\frac{5}{9}$?

35. $\frac{4}{11}$ is what part of 87? of 169? of $46\frac{2}{15}$? of $83\frac{7}{10}$?

36. $7\frac{3}{8}$ is what part of $15\frac{5}{8}$? of $28\frac{3}{4}$? of $72\frac{7}{8}$?

40

7 **37.** Divide 76 into two such parts that $\frac{3}{4}$ of the first shall equal $\frac{5}{8}$ of the second. 30

38. Divide 417 into three such parts that the first shall be $\frac{7}{12}$ of the second, and the third shall be $1\frac{1}{5}$ of the second. 105 180 132

39. How is the value of a proper fraction affected by adding the same number to both terms? Why?

40. How is the value of an improper fraction affected by adding the same number to both terms? Why?

41. How is the value of an improper fraction affected by subtracting the same number from both terms? Why? Of a proper fraction? Why?

42. What is the circumference of a wheel that makes $24\frac{4}{5}$ revolutions in 400 feet?

43. If another wheel have a tire two feet shorter, it makes how many more revolutions in the same distance?

44. It takes six hours to complete a certain journey at the ordinary rate of travel. How many hours are required to complete three fourths of the journey if the rate be increased by one half of itself?

45. An adult inspires 30 cubic inches of air at an ordinary inspiration. If they breathe 18 times per minute, how long will it take 50 adults to breathe once all the air in your school-room?

46. If each expiration vitiates a cubic foot of air, how rapidly should the air be changed in the above room?

47. How many lunar months of $29\frac{1}{2}$ days in the year of $365\frac{1}{4}$ days?

ANALYSIS. There are as many lunar months of $29\frac{1}{2}$ days in $365\frac{1}{4}$ days, as $365\frac{1}{4}$ is times $29\frac{1}{2}$. $365\frac{1}{4}$ is $12\frac{45}{118}$ times $29\frac{1}{2}$; hence there are $12\frac{45}{118}$ lunar months in the year.

NOTE. When the divisor is a mixed number, it is usually best to multiply both dividend and divisor by the denominator of the divisor. In the foregoing problem, multiplying both dividend and divisor by 4, we have $1461 \div 118$.

48. What is the daily journey of the earth in its orbit, 292,000,000 miles, allowing $365\frac{1}{4}$ days for a year?

49. Allowing $1\frac{1}{4}$ cubic feet to the bushel, what is the capacity of a bin $6' \times 8' \times 10'$.

50. A bushel of corn in the ear occupies $2\frac{1}{2}$ cubic feet. To what height must a rail-crib nine feet square be filled to hold 300 bushels?

51. Measure a wagon-box and calculate to what height 30 bushels of corn in the ear will fill it; 40 bushels of wheat.

52. A man bought a piece of property for $1,050 and sold it so as to gain $\frac{33\frac{1}{3}}{100}$ of the cost. What did he receive for it?

53. By selling a piece of property for $4,820, the seller gained $\frac{33\frac{1}{3}}{100}$ of the cost. What did it cost him?

54. Sold a horse for $75, losing $\frac{37\frac{1}{2}}{100}$ of the cost. What was the cost?

55. The sign % is used instead of the word "hundredths" or the denominator 100; read it in that way. Find 25% of 80; of 92; of 144; of 420; of 610; of 712.

56. Find 75% of 64; of 72; of 112; of 320; of 684; of 860; of 1024.

57. Find $37\frac{1}{2}$% of 40; of 112; of 280; of 576; of 1320; of 1648; of 1864.

58. Bought goods for $60 and sold them for $75. The gain is how many % of the cost?

59. 75% of 375 is $2\frac{1}{2}$% of what number?

60. Find the l. c. m. of 24, 36, 40, 48, 60.

61. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{\frac{3}{5}}{\frac{5}{8}}$.

62. Divide \$6,600 among 4 persons so that the second shall receive twice as much as the first, the third three times as much as the second, and the fourth four times as much as the third.

63. $(3\frac{1}{2} + 2\frac{5}{8}) \times (4\frac{1}{5} - 1\frac{5}{8}) \div \frac{3}{4}$ of $\frac{5}{16} = ?$

64. $2\frac{1}{5}$ is what part of $6\frac{7}{12}$? of $8\frac{2}{3}$? of $9\frac{1}{6}$? of $10\frac{3}{4}$?

65. $\frac{2}{5}$ is $\frac{5}{16}$ of what? $7\frac{2}{3}$ is $\frac{23}{30}$ of what? $\dfrac{3\frac{1}{2}}{12}$ is $\frac{7}{24}$ of what?

66. Divide $\frac{3}{4}$ of $\frac{8}{9}$ by $\frac{5}{6}$ of $\frac{1}{12}$.

67. Divide $\frac{23}{24}$ of $7\frac{1}{2}$ by $\frac{24}{25}$ of $\frac{75}{16}$ of $\frac{39}{52}$.

68. Add $17\frac{1}{3}$, $26\frac{5}{6}$, $69\frac{7}{8}$, $142\frac{11}{12}$, $317\frac{11}{15}$.

69. Add $24\frac{2}{3}$, $63\frac{5}{8}$, $96\frac{11}{16}$, $38\frac{4}{5}$, $124\frac{5}{12}$.

70. $64\frac{7}{15} - 42\frac{17}{24}$; $69\frac{19}{24} + 26\frac{35}{36}$; $315\frac{5}{19} - 278\frac{1}{3}$.

71. $84\frac{2}{5} \times 63\frac{3}{4}$; $275\frac{8}{15} \times 49\frac{2}{3}$; $\dfrac{3\frac{1}{2}}{5} \times \dfrac{2}{4\frac{2}{3}}$.

72. $12\frac{3}{8} \div \frac{4}{9}$; $64\frac{11}{12} \div 5$; $65\frac{9}{10} \div 6$; $126\frac{14}{15} \div 7$.

73. An iceman delivered a block of ice 1 foot long, $\frac{5}{8}$ of a foot wide, and $\frac{3}{4}$ of a foot thick, and charged for 50 pounds. What was the shortage in weight, a cubic foot of water weighing $62\frac{1}{2}$ pounds, ice weighing $\frac{92}{100}$ as much?

74. A steel beam is 16 feet long, $2\frac{3}{4}$ inches thick, and 14 inches wide. What is its weight, its specific gravity being 7.84 (7.84 as heavy as an equal volume of water)?

75. What is the weight of a block of limestone $6\frac{1}{2}$ feet $\times$ $1\frac{1}{4}$ ft. $\times$ $4\frac{2}{3}$ feet, its specific gravity being 2.62?

76. How many pounds of 7,000 grains will \$100,000 in ten-dollar gold-pieces weigh, each piece weighing 258 grains?

77. If A can do a piece of work in 9 days, B in 10 days, and C in 12 days, in what time can they do it, working together? If they should work together 1 day, and C should leave, in what time could A and B finish it? If they should work together 2 days, and B and C should leave, in what time could A finish it?

78. Make a receipted bill for the following items, using the name of a class-mate with your own: —

$24\frac{1}{2}$ lbs. sugar at $6\frac{1}{4}$ cts.
18 lbs. coffee at $27\frac{1}{3}$ cts.
15 bu. potatoes at $36\frac{1}{2}$ cts.
$15\frac{1}{2}$ lbs. butter at 25 cts.

79. Make a bill of the following items: A. R. Brown bought of G. G. Johnson 8 gallons oil at $12\frac{1}{2}$ cents, 14 brooms at 20 cents, 1 pkg. gold-dust 25 cents, 3 cakes soap at $8\frac{1}{3}$ cents, 5 gallons gasoline at 10 cents, 16 pounds sugar at $6\frac{1}{4}$ cents, 300 lemons at $1\frac{1}{4}$ cents, 2 pine-apples at 35 cents. Credit the bill with $5.60 paid on account and show the balance due.

80. Make a bill of the following: P. A. Coen & Son sold to Augustine & Co., February 20, 1 quart mucilage 50 cents, 1 jar paste 20 cents; March 2, 1 quart ink 50 cents; (13) 10 reams cap at $1.35; April 6, 30 pounds paper at 7 cents; (9) 2 quarts ink at 50 cents; (13) 24 quinine bottles at $4\frac{1}{6}$ cents; (17) 90 pen-wipers at $1\frac{2}{3}$ cents, 1 floor brush, $3.00; May 6, 2 gallons benzine at 20 cents; (12) 31 pounds paper at 7 cents, 5 reams cap at $1.35; (14) 5 gallons turpentine at 50 cents; (16) 5 reams cap at $1.35.

81. If $80 be paid for the labor of 12 men for 5 days, at the same rate what should be paid for the labor of 18 men for 24 days?

82. If $98 be paid for the use of $560 for 2 years and 6 months, at the same rate what should be paid for the use of $825 for 3 years and 9 months?

83. At the rate of $6\frac{1}{2}$ cents for the use of $1 for a year, what should be paid for the use of $650 for 3 years? for $875? for $1,280?

84. At the same rate what should be paid for the use of $1,460 for 3 years and 4 months? for 2 years and 5 months? for 4 years and 10 months?

85. How many bushels of corn can be bought for $1,936.40 at $23\frac{1}{2}$ cents a bushel?

86. Sold 1,836 bushels of wheat at $61\frac{3}{8}$ cents and invested the proceeds in corn at $24\frac{3}{8}$ cents; how many bushels did I buy?

87. Built a fence around a square lot 400 feet on a side. The posts were placed 8 feet apart. The boards were 16 feet long and each contained 8 feet of lumber. If the fence was 5 boards high, what was the cost of the material (not counting the nails), if the posts cost 18 cents each and the lumber cost $18.50 for 1,000 feet?

88. What is the cost of the lumber for flooring and ceiling a room 24 feet and 3 inches wide and 84 feet long, if the lumber cost $3\frac{1}{2}$ cents a square foot?

89. A field is 40 rods ($16\frac{1}{2}$ feet) wide and 80 rods long. What will it cost to plow the field at $2.25 an acre (160 square rods)?

90. The diameter of the earth is about 8,000 miles. Mt. Everest is about $5\frac{1}{2}$ miles high. On a globe 2 feet in diameter, how high should a projection be to represent this mountain's height properly?

91. If a continent 6,000 miles long be represented by a raised map 4 feet long, what horizontal distance does a foot represent? an inch? How high, according to the same scale, should Mt. Everest be?

92. How many leaves in Webster's International Dictionary? The thickness of one leaf compares how with the thickness of the book, not counting the covers? Compare this result with that obtained in Problem 90.

93. If 7 cents be paid for the use of $1 for one year, what should be paid for the use of $2,460 for 4 years, 7 months, 15 days? (Call 15 days $\frac{1}{2}$ of a month.)

169. DECIMAL FRACTIONS.

1. Define fraction, numerator, denominator, common fraction.

2. A **decimal fraction** is a fraction whose denominator is expressed by the position of the right-hand figure of the numerator with respect to the decimal point.

3. Since we employ a decimal system of notation, the denominator of a decimal fraction is a power of ten.

4. Decimal fractions are commonly called "**decimals.**"

5. A decimal whose numerator is an integer is a **pure decimal.**

6. A decimal whose numerator contains a fraction is a **complex decimal.**

7. An integer plus a decimal is called a **mixed decimal.**

8. Decimals may be both mixed and complex.

9. Pure decimals: .3, .027, .346, .0001.

10. Complex decimals: $.3\frac{1}{3}$, $.0\frac{1}{3}$, $.756\frac{2}{7}$.

11. Mixed decimals: 3.2, 7.001, 300.010.

12. Mixed complex decimals: $7.00\frac{3}{4}$, $562.0\frac{1}{4}$, $27.3\frac{5}{8}$.

170. Reading of Decimal Fractions.

1. Use "and" only in reading mixed decimals or mixed numbers.

Note also that such forms as $.\frac{1}{3}$, $.\frac{3}{4}$ are never used.

2. 300.026 = three hundred and twenty-six thousandths.

3. .326 = three hundred twenty-six thousandths.

4. $.0\frac{1}{3}$ = one-third of a tenth.

5. $7.00\frac{3}{4}$ = seven and three fourths of a hundredth.

6. $.07\frac{3}{4}$ = seven and three fourths hundredths.

7. 3.2 = three and two tenths. (Mixed decimal.)

8. 3.2 = thirty-two tenths. (Pure decimal.)

171. Names of the orders to the right of the decimal point:

First,	tenths' order.
Second,	hundredths' order.
Third,	thousandths' order.
Fourth,	ten-thousandths' order.
Fifth,	hundred-thousandths' order.
Sixth,	millionths' order.
Seventh,	ten-millionths' order.
Eighth,	hundred-millionths' order.
Ninth,	billionths' order.
Tenth,	ten-billionths' order.
Eleventh,	hundred-billionths' order.
Twelfth,	trillionths' order.

172. NUMERATION.

Read the following numbers:

1. .5, .05, .004, .0006.
2. .00008, .000009, .0000009.
3. .00000002, .000000003.
4. .000000007, .0000000001.
5. .26, .264, .3864, .029.
6. .0294, .00874, .087463.
7. .0056849, .00046928.
8. .000057006, .600870543.
9. .0080500694, .4060790843.
10. .00400014, .100010014.
11. .2002000202, .33033003.
12. $7.00\frac{3}{4}$, $.07\frac{3}{4}$, .0300, .310, 300.010.
13. $.0\frac{3}{4}$, $.00\frac{5}{6}$, $.000\frac{8}{9}$, $15.0\frac{7}{8}$, $29.00\frac{5}{7}$.

RULE.

Read the number as if it were integral, and then apply the denomination indicated by the position of the right-hand figure of the numerator.

Read mixed decimals, first, as mixed numbers, and second, as simple numbers.

Illustration. 568.00861.

As a mixed number, 568 and 861 hundred-thousandths.

As a simple number, 56 million 800 thousand 861 hundred-thousandths.

In reading, make a slight pause before giving the denomination. Read the following:

1. 29.00863, 4609.001083.
2. 200183.40062, 69.00185706.
3. 30086.0030086, 4000.004.
4. 5000000.000005, $.008\frac{1}{3}$.
5. $.05691\frac{7}{12}$, $.000\frac{3}{7}$, $.0000\frac{2}{3}$.
6. $.000006\frac{11}{24}$, .0700, 17.04200.
7. 820.007600, .4500, .07900.

173. NOTATION.

1. There will be little facility in writing decimal fractions until pupils are thoroughly familiar (*a*) with the names of the orders, and (*b*) with the number of each order, counting from the decimal point toward the right.

2. What is the number of the following orders: hundredths'? millionths'? tenths'? ten-thousandths'? ten-millionths'? hundred-thousandths'? hundred-millionths'? billionths'?

3. Name each of the following orders: 4th, 8th, 1st, 2d, 7th, 5th, 3d, 9th, 6th, 10th.

4. Steps in writing decimal fractions:

(1) Think the number of the order in which the right-hand figure of the numerator must stand.

(2) Think the number of figures in the numerator.

(3) The difference between these numbers is the number of ciphers between the numerator and the decimal point.

174. *Illustration.* Write in figures 469 ten-millionths.

1. In order that a number shall express ten-millionths, its right-hand figure must stand in the seventh order to the right of the decimal point.

2. In 469 there are three figures; hence,

3. Four ciphers must precede the numerator.

To express this number, consequently, I write: decimal point, four ciphers, 469.

175. Tell how each of the following numbers is expressed before you make any figures.

1. Write in figures 17 hundred-thousandths.

Illustration. This number is expressed by writing decimal point, three ciphers, one, and seven.

2. Write in figures ·6 tenths; 15 hundredths; 43 thousandths; 28 ten-thousandths; 467 thousandths.

3. Write in figures 29 hundredths; 921 thousandths; 7 ten-thousandths; 8 hundred-thousandths; 4 millionths; 6 billionths; 38 ten-thousandths; 4562 hundred-thousandths.

4. 419 millionths; 306 hundred-thousandths; $96\frac{1}{3}$ ten-thousandths; 8158 hundred-millionths; 59001 billionths.

5. 23006 and 40007 millionths; 29000 and 29 thousandths; 29029 thousandths; 307 million and 307 millionths; 307000307 millionths.

6. 379 tenths; 5824 hundredths; 69708 thousandths; 524896 hundred-thousandths.

7. Three million seventeen thousand eight hundred-billionths.

8. Seven hundred ten-thousandths; seven hundred ten thousandths; nine thousand two hundred-millionths; nine thousand two hundred millionths.

9. Two thirds of a millionth; five sixteenths of a ten-thousandth; eighty-nine and seven thirteenths hundred-millionths.

10. Eight and eight thousandths.

11. Twenty-three and sixty-one ten-thousandths.

176. REDUCTION OF DECIMAL FRACTIONS.

1. Reduce 8 to tenths; 24 to thousandths; .5 to hundredths; .75 to ten-thousandths; .0624 to millionths.

2. Reduce .2600 to hundredths; .050800 to ten-thousandths; 18.000 to hundredths; to tenths; 75.0630000 to millionths; to thousandths.

3. Reduce 4.6 to hundred-thousandths; reduce the resulting fraction to hundredths; 87.2 to billionths; the resulting fraction to millionths; to tenths.

4. Annexing a cipher to the numerator of a decimal multiplies the numerator by ten. But in making one more place in the numerator, it multiplies the denominator by ten, hence:

Annexing ciphers to the right of a decimal does not change its value.

177. REDUCTION OF DECIMAL FRACTIONS TO COMMON FRACTIONS.

What is the difference between a decimal fraction and a common fraction? What must be done, then, to change a decimal fraction to a common fraction? Change the following to common fractions.

1. .5.	4. .000637.	7. 3.08.	10. $.00\frac{1}{3}$.
2. .37.	5. .004826.	8. 25.0063.	11. $.00285\frac{5}{12}$.
3. .085.	6. .04725.	9. $.0056\frac{1}{4}$.	12. $18.0675\frac{7}{11}$.

RULE.

To change a decimal fraction to a common fraction: Erase the decimal point, write the denominator, and reduce the fraction to lowest terms.

What will the denominator always be before reduction?

178. ADDITIONAL PROBLEMS.

1. $.8\frac{1}{3}$, $.08\frac{1}{3}$, $.008\frac{1}{3}$.
2. $1.6\frac{2}{3}$, $.16\frac{2}{3}$, $.016\frac{2}{3}$, $.00016\frac{2}{3}$.
3. $.33\frac{1}{3}$, $.033\frac{1}{3}$, $.31\frac{1}{4}$, $.031\frac{1}{4}$.
4. $.43\frac{3}{4}$, $.0043\frac{3}{4}$, $.58\frac{1}{3}$, $.0058\frac{1}{3}$.
5. $.56\frac{1}{4}$, $.068\frac{3}{4}$, $.0066\frac{2}{3}$, $.081\frac{1}{4}$.
6. $.0\frac{1}{3}$, $.000\frac{2}{3}$, $.0\frac{3}{8}$, $.00\frac{5}{16}$.
7. $.0\frac{1}{5}$, $.00\frac{2}{11}$, $.000\frac{7}{12}$, $.000\frac{1}{6}$.

179. *Illustrative Problem.*

1. Change $\frac{3}{8}$ to a decimal fraction. This fraction is to be regarded as a problem in partition, in which the numerator is the dividend and the denominator is the divisor. $\frac{3}{8}$ of 1 equals $\frac{1}{8}$ of 3.

ANALYSIS Since $\frac{1}{8}$ of 3 is not a whole number, 3 is reduced to tenths. $3 = 3.0$. $\frac{1}{8}$ of $3.0 = .3$ with a remainder of .6. $.6 = .60$. $\frac{1}{8}$ of $.60 = .07$ with a remainder of .04. $.04 = .040$. $\frac{1}{8}$ of $.040 = .005$; hence $\frac{3}{8} = .375$.

FORM.

$$8\,)\,\underline{3.000}$$
$$\;\;\;\;.375$$

The reductions may be made at once as above.

2. How did we show that the 3 was reduced? Explain the following

RULE.

To reduce a common fraction to a decimal: Place the decimal point at the right of the numerator, annex zeros and divide by the denominator. Point off as many decimals in the quotient as there were zeros annexed.

180. PROBLEMS.

Change the following to decimal fractions.

1. $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{11}{8}$.
2. $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{1}{16}$, $\frac{3}{20}$, $\frac{7}{20}$, $\frac{1}{20}$.
3. $\frac{7}{25}$, $\frac{11}{25}$, $\frac{18}{25}$, $\frac{1}{25}$, $\frac{9}{50}$, $\frac{37}{50}$, $\frac{1}{50}$.

4. $\frac{27}{32}$, $\frac{3}{32}$, $\frac{19}{64}$, $\frac{23}{64}$, $\frac{1}{64}$, $\frac{79}{125}$, $\frac{3}{125}$.

5. $\frac{17}{40}$, $\frac{6}{75}$, $\frac{18}{200}$, $\frac{16}{400}$, $\frac{143}{400}$, $\frac{1}{250}$, $\frac{7}{1250}$.

NOTE. The foregoing fractions all reduce to pure decimals because each reduced numerator is divisible by its denominator. A study of the denominators shows that the only factors of the denominators not found in the corresponding numerators are 2 and 5. But these are the factors that are introduced into the numerator with each reduction; hence it is possible to carry the reduction far enough to make each numerator divisible by its denominator.

NOTE. Dictate many problems until this becomes plain.

6. $\frac{1}{3}$, $\frac{3}{7}$, $\frac{7}{11}$, $\frac{5}{12}$, $\frac{17}{24}$, $\frac{19}{45}$.

NOTE. If the denominator of a fraction in lowest terms contains other factors than 2 or 5, it cannot be reduced to a pure decimal. In such cases it is found, as the division continues, that a certain figure or set of figures is regularly repeated. This figure, or set of figures, is called the repetend, and the decimal in which it occurs is called a repeating, or circulating decimal.

$\frac{1}{3}$ = .33333333333, etc.
$\frac{7}{11}$ = .636363636363, etc.
$\frac{3}{7}$ = .428571428571428571428571, etc.

In such cases we may express the common fraction as:

1. A complex decimal, $.33\frac{1}{3}$, $.63\frac{7}{11}$, or $.6\frac{4}{11}$, $.428\frac{4}{7}$.

2. An approximate decimal, .3333+, .6363+, .4286−. The sign + or − is used to show that the result is incomplete, and that the true value is greater or less than the value expressed.

3. A repeating decimal, $.\dot{3}$, $.\dot{6}\dot{3}$, $.\dot{4}2857\dot{1}$ −. Dots are placed over the terminal figures of a repetend.

7. Reduce the following to each of the above-named forms, making the approximate decimals true to ten-thousandths.

$\frac{7}{18}$, $\frac{5}{9}$, $\frac{11}{36}$, $\frac{1}{14}$. $\frac{85}{96}$, $\frac{9}{28}$, $\frac{17}{35}$, $\frac{13}{22}$, $\frac{55}{99}$.

181. Another Method.

Illustrative Problem. Change $\frac{7}{8}$ to a decimal fraction.

ANALYSIS. Since the denominator of a decimal fraction is a power of ten, I multiply both terms of $\frac{7}{8}$ by some number that will make the denominator a power of ten. Every power of ten is the product of an equal

number of twos and fives. Eight is the product of three twos. If it be multiplied by the product of three fives, the result will be the product of three tens, which is the third power of ten. Multiplying both terms of $\frac{7}{8}$ by 125, the result is $\frac{875}{1000}$, or .875.

Explain the following by this method.

Reduce to decimal fractions:

1. $\frac{1}{2}$, $\frac{2}{5}$, $\frac{7}{20}$, $\frac{18}{25}$, $\frac{15}{16}$, $\frac{21}{32}$, $\frac{41}{80}$.
2. $\frac{5}{8\frac{1}{3}}$, $\frac{7}{12\frac{1}{2}}$, $\frac{11}{16\frac{2}{3}}$, $\frac{1}{6\frac{1}{4}}$, $\frac{17}{33\frac{1}{3}}$, $\frac{5}{6\frac{2}{3}}$, $\frac{2}{3\frac{1}{3}}$.

RULE.

Multiply or divide both terms of the fraction by some number that will make the denominator a power of ten. Then express the denominator by the position of the right-hand figure of the numerator with respect to the decimal point.

182. ADDITION OF DECIMAL FRACTIONS.

Define addition, sum. Numbers are written how? Why? Addition begins where? Why? Make a rule. Analyze each problem as in simple addition.

183. PROBLEMS.

1.	2.	3.	4.
.625	4.073	126.0009	4006.092
.0984	26.0084	482.1872	9.059701
.4907	59.00462	600.50983	683.086409
.00864	83.01879	17.008459	34.189357
.09769	50.00043	6.098725	1086.049078

5. Add .00862, 4.04378, 73.096, 168.00097, 49.287005, 83.460037.

6. Add 77.02081, 94.09069, 88.00799, 686.060098, 897.0609, .084858, .087857, .3060686, .76978.

7. Add .04069, .008972, .0934, .0083462, .027809, .5302681, .05003701.

3. Add 7 tenths, 56 hundredths, 93 thousandths, 329 hundred-thousandths, 8052 millionths, 42067 ten-thousandths, 43 hundredths, 98 ten-millionths.

184. ADDITION OF COMPLEX DECIMALS.

Illustrative Example. Add $3\frac{1}{8}$ tenths, $7\frac{2}{5}$ thousandths, $17\frac{2}{11}$ hundredths, $862\frac{2}{3}$ ten-thousandths.

ANALYSIS. Ten-thousandths being the lowest denomination, all numbers of higher denomination are to be reduced to ten-thousandths, unless they become pure decimals before such reduction is completed.

$$
\begin{array}{lcr}
.3\frac{1}{8} & = & .3 + .\frac{1000}{8} = .3125 \\
.007\frac{2}{5} & = & .007 + .\frac{0020}{5} = .0074 \\
.17\frac{2}{11} & = & .17 + .\frac{0200}{11} = .1718\frac{2}{11} \\
.0862\frac{2}{3} & = & .0862\frac{2}{3} \\
\hline
 & & .5779\frac{28}{33}
\end{array}
$$

NOTE. If all of the mixed decimals can be reduced to pure decimals, reduce them before addition.

1. Add $3\frac{1}{2}$ tenths, $29\frac{1}{3}$ thousandths, $56\frac{1}{4}$ thousandths, $24\frac{1}{5}$ hundredths, $183\frac{1}{7}$ thousandths, $86\frac{1}{8}$ hundredths.

2. Add $15\frac{2}{5}$ thousandths, $38\frac{1}{7}$ hundredths, $409\frac{1}{3}$ ten-thousandths, $3\frac{1}{8}$ tenths, $9\frac{1}{6}$ hundredths, $7\frac{2}{9}$ thousands, $5\frac{1}{11}$ tenths, $2\frac{2}{11}$ hundredths.

3. Add $64\frac{2}{3}$ hundred-thousandths, $53\frac{4}{9}$ hundredths, $78\frac{1}{2}$ thousandths, $86\frac{4}{5}$ ten-thousandths, $4920\frac{5}{12}$ hundred-thousandths, $6\frac{7}{12}$ tenths, $9\frac{8}{15}$ hundredths.

4. Add $423\frac{5}{12}$ millionths, $29\frac{4}{11}$ hundredths, $46\frac{1}{3}$ units, $126\frac{2}{3}$ tenths, $479\frac{7}{12}$ hundredths.

185. SUBTRACTION OF DECIMAL FRACTIONS.

Define subtraction, minuend, subtrahend, remainder. Perform and explain a problem in subtraction of simple numbers. Give the rule. Apply the same to the following problems.

NOTE. Make minuend and subtrahend of the same denomination before subtracting.

186. PROBLEMS.

1. $.0861 - .0295 = ?$
2. $.7043 - .4805 = ?$
3. $.0461 - .00356 = ?$
4. $4.02603 - .9078 = ?$
5. $26.1059 - 19.74308 = ?$
6. $461.083024 - 86.59260834 = ?$
7. $.023 - .000465 = ?$
8. $92. - .06479 = ?$
9. $400. - .00004 = ?$
10. $.00583 - .0000583 = ?$
11. $.7\frac{1}{3} - .008\frac{1}{4} = ?$
12. $.43\frac{1}{6} - .0047\frac{1}{6} = ?$
13. $.0084\frac{1}{7} - .00023\frac{9}{14} = ?$

NOTE. $.0084\frac{1}{7} = .00841\frac{3}{7} = .00840\frac{10}{7}$.

14. $42.08\frac{1}{6} - 34.0574\frac{1}{12} = ?$
15. $83.50703\frac{1}{11} - 59.0\frac{2}{3} = ?$
16. $.8035\frac{2}{9} - .045\frac{2}{7} = ?$
17. $84.0431 + 56.057 - 16.059 - 23.00845 = ?$
18. $625 - .0748 + 29.0536 - 439.00596 - .089037 = ?$
19. $.08\frac{1}{3} + .0043\frac{1}{5} - .006\frac{1}{7} = ?$
20. $5\frac{1}{8} - .06\frac{1}{4} + .009\frac{1}{16} + .00\frac{5}{32} = ?$
21. 28 thousandths — 46 ten-thousandths = ?
22. 423 millionths — 17 hundred-thousandths = ?
23. 46 tenths — 46 thousandths = ?
24. 3824 hundredths — 3824 ten-millionths = ?
25. $\frac{1}{7}$ of a tenth — $\frac{1}{7}$ of a thousandth = ?
26. $\frac{1}{3}$ of a thousandth — $\frac{1}{6}$ of a millionth = ?
27. $426\frac{1}{4}$ ten-thousandths — $38\frac{1}{2}$ hundred-thousandths = ?
28. 9251 hundredths — 4659 hundred-millionths = ?

29. $94\frac{6}{7}$ thousandths — $53\frac{1}{14}$ ten-millionths =?

30. $\frac{1}{7}$ of ten — $\frac{1}{7}$ of a hundredth =?

31. $\frac{1}{6}$ of one hundred — $\frac{1}{3}$ of a hundredth =?

187. MULTIPLICATION OF DECIMAL FRACTIONS.

1. Define all terms in multiplication.

2. What is the effect of removing the decimal point one place to the right? two places? four places? How multiply by 10? by 100? by 10000? by 1000000?

3. What is the effect of removing the decimal point one place to the left? three places? six places? How divide by 10? by 1000? by 1000000?

4. Multiply .008764 by 10, and read the result; by 100; by 1000; by 1000000; by 10000000.

5. Divide 4968.307 by 10, and read the result; by 100; by 1000; by 10000.

6. Make general rules for multiplying and dividing by powers of ten.

188. 1. Multiply .0536 by 28.

NOTE. Since the multiplicand is ten-thousandths, the product is ten-thousandths; hence, 28 times .0536 = 1.5008.

2. Multiply .824 by .01; by .001; by .1; by .0001, and read the results.

Multiplying a number by .1 is equivalent to dividing it by what divisor?

3. Multiply .497 by .39. This problem means: Find .39 of .497. How do you find .01 of .497? What will the result be? How many decimal places will it have? What do you do with this result?

4. *Explanation.* I first find .01 of .497. .01 of .497 is .00497, which is found by removing the decimal point two orders to the left, and filling the vacant orders with ciphers. .39 of .497 is 39 times .00497.

FORM.

$$\begin{array}{r} (1) \\ \left.\begin{array}{r} .497 \\ \underline{.39} \end{array}\right\} \end{array} \quad = \quad \begin{array}{r} (2) \\ \left\{\begin{array}{r} .00497 \\ \underline{39} \end{array}\right. \\ \underline{.04473} \\ \underline{.1491} \\ .19383 \end{array}$$

5. How many decimal places are there in the product? Why? Make a rule for "pointing" the product.

RULE.

In Multiplication of Decimal Fractions, multiply as in simple numbers, and point off as many decimal places in the product as there are in multiplicand and multiplier.

189. PROBLEMS.

Multiply:

1. .542 by 58.
3. .00436 by .8.
5. $6.028\frac{1}{4}$ by .072.
2. .0693 by 324.
4. .7093 by .49.
6. $28.056\frac{3}{4}$ by $.057\frac{2}{3}$.

Note. Simplify the decimal fractions.

7. $.800694$ by $.17\frac{1}{8}$.
8. $.006294\frac{1}{3}$ by $.00863\frac{5}{8}$.
9. $.0976\frac{7}{8}$ by $24\frac{5}{16}$.

Note. Change $\frac{5}{16}$ to a decimal fraction.

10. 25864 by .03972.
11. 30.6895 by 4.300906.
12. 50638 thousandths by 9026 hundredths.
13. 49060037 millionths by 207003 ten-thousandths.
14. 409 billionths by 36 millionths.
15. 5063087 ten-millionths by 6204 thousandths.
16. 29 ten-thousandths by 29 ten-millionths.
17. 48 tenths by 48 ten-thousandths.

Give the results rapidly in the following problems:

18. 9 hundredths by 7 thousandths.
19. 15 ten-thousandths by 6 thousandths.
20. 8 thousands by 9 thousandths.

21. 24 ten-thousands by 24 ten-thousandths.

22. 17 hundreds by 4 hundred-thousandths.

23. 23 tenths by 4.

24. 11 millionths by 11 thousandths.

25. 18 ten-thousandths by 4 hundredths.

26. 7 ten-millionths by 14 hundred-thousandths.

27. What is the product of tenths and thousandths? of thousandths and hundredths? of ten-thousandths and ten-thousandths? of hundreds and thousandths? of thousands and hundredths? of millions and millionths? of hundred-thousands and hundredths? of millionths and hundreds? of ten-thousandths and thousands?

28. .125 is what part of 1? of 1 ten?

29. .375 is what part of 1? .0375 is what part of .1? of 1?

30. .0025 is what part of .01?

31. .0625 is what part of .1?

32. .000875 is what part of .001?

33. .006$\frac{2}{3}$ is what part of .01?

Read each of the following as a part of 1 standing in the first order at the left of its significant figures:

34. .00075, .0025, .00125, .0000875, .033$\frac{1}{3}$, .000066$\frac{2}{3}$.

35. What is the cost of 864 bushels of oats at $0.41 per bushel?

36. What is the cost of 17 horses at $112.375 each?

37. What is the cost of 384 acres of land at $67.065 each?

38. What is the cost of 18.56 yards of cloth at $2.5625 a yard?

39. What is the cost of 29 books at $0.0625 each?

40. What is the cost of 465.375 bushels of wheat at $0.9175 per bushel?

190. DIVISION OF DECIMAL FRACTIONS.

1. Define partition, dividend, divisor, quotient, remainder. $.97254 \div 8 = ?$ Explain by partition. Follow the form in simple numbers.

2. Explain the following in the same way.

191. The divisor a whole number.

1. $.7658 \div 7 = ?$	**6.** $.042864 \div 24 = ?$
2. $.04536 \div 6 = ?$	**7.** $.008399 \div 37 = ?$
3. $.157032 \div 9 = ?$	**8.** $.010867 \div 46 = ?$
4. $71.40636 \div 12 = ?$	**9.** $4.20638 \div 55 = ?$
5. $146.0736 \div 16 = ?$	**10.** $436.0095 \div 77 = ?$

11. What is the denomination of the quotient in partition? In the preceding problems the quotients are like what? How many decimal places are there in each quotient?

12. Make a rule for pointing the quotient when the divisor is a whole number.

13. Explain the same problems by division, using the form given in simple numbers.

14. The quotient in each case is like what?

RULE.

In division of decimal fractions, when the divisor is a whole number, divide as in simple numbers, and point off as many decimal places in the quotient as there are in the dividend.

192. The divisor a decimal.

1. *Illustrative Problem.* $.875 \div .5$.

The dividend is the product of the divisor and quotient. The number of decimal places in the product equals the number in both multiplicand and multiplier. Hence the number of decimal places in the quotient is equal to the number in

the dividend minus the number in the divisor. In this case the number of decimal places in the quotient is 3 — 1, or 2.

$$.5\,)\,\underline{.875}$$
$$1.75$$

2. 87.5 ÷ .005 = ?

We annex ciphers until the dividend contains as many decimal places as the divisor. The quotient contains 3—3 decimal places.

$$.005\,)\,\underline{87.500}$$
$$17500$$

193. PROBLEMS.

3. .6241 ÷ .79.
4. 1.0276 ÷ .028.
5. 44.814 ÷ .97.
6. .39071 ÷ .0089.
7. .091512 ÷ .0124.
8. 7.5522 ÷ 2.46.
9. .153032 ÷ 00376.
10. .02336081 ÷ .00583.
11. 5.66703747 ÷ 70.83.
12. .052629096 ÷ 5470.8.
13. 183.057 ÷ .379.
14. 39.3888 ÷ .0528.
15. 5.81715 ÷ .00695.
16. .633447 ÷ .000783.
17. .7737013 ÷ .0000859.
18. 112.1021 ÷ .02453.

How does the rule apply to this problem?

19. 2099.274 ÷ .3607.
20. 26624.32 ÷ .4379.
21. 5481 ÷ .063.
22. 48760 ÷ .0092.
23. 766300 ÷ .00079.
24. 133574 ÷ .000329.
25. 24980020 ÷ .0406.
26. 8.6947 ÷ 28.
27. .46083 ÷ .37.
28. .070685 ÷ .000056.
29. 860.025 ÷ .0378.
30. 90.3864 ÷ 4.77.
31. .016458 ÷ .000963.
32. .023907 ÷ .0001839.
33. .7 ÷ 43, carry the quotient to 5 decimal places.
34. 4.6 ÷ 58, " " 6 " "
35. 63. ÷ 97, " " 4 " "
36. .1 ÷ 329, " " 6 ' "
37. 4. ÷ 586, " " 8 "

194. To divide by a power of 10, remove the decimal point as many places to the left as there are zeros in the divisor.

1. Divide 8493.7 by 10; by 100; by 10000; by 1000000.

2. Divide 49.683 by 100; by 10000; by 1000; by 10; by 1000000.

195. Division by the factors of a number.

1. Divide 8.75 by 50. $8.75 \div 10 = .875$. $.875 \div 5 = .175$.

2. Divide 12.25 by 500; by 5000; by 250; by 25000.

3. Divide 75 by 750; by 7500; by 75000.

4. If 24 boxes of fruit cost $52.32, what does each box cost?

196. REVIEW PROBLEMS.

1. Change .00875 to a common fraction.

2. Change $\frac{9}{32}$ to a decimal fraction.

3. How many square feet in a piece of ground 86.48 feet long and 39.6 feet wide?

4. If a man travel 29.6 miles a day, in how many days will he travel 1,016.088 miles?

5. Change $\frac{1}{80}$ to a decimal fraction.

6. What is the cost of 473.5 bushels of corn at .47 of a dollar a bushel?

7. At .47 of a dollar a bushel, how many bushels of corn can be bought for $222.545?

8. Add $34\frac{3}{5}$ thousandths, $568\frac{1}{2}$ hundredths, $634\frac{1}{3}$ tenths, $\frac{1}{8}$ of a hundredth, and $\frac{1}{6}$ of a ten-thousandth.

9. From $\frac{1}{7}$ of a tenth take $\frac{1}{3}$ of a thousandth.

10. Change $.00\frac{1}{8}$ to a common fraction.

11. Change $.0\frac{3}{5}$ to a common fraction.

12. Change $\frac{7}{13}$ to a decimal fraction of three orders.

13. If 97 books cost $317.675, what will each cost?

14. If 38.5 bales of cloth cost \$3,048.43, what is the price of each bale?

15. If a pair of shoes cost \$2.625, how many pairs can be bought for \$49.875?

16. At 83 cents a yard, how many yards of cloth can be purchased for \$61.005?

17. At \$.045 per pound, how many pounds of sugar can be purchased for \$1.89?

18. Define a decimal fraction. How does it differ from a common fraction?

19. Find l. c. m. of 18, 24, 36, 40, 180.

20. Define a pure decimal; a mixed decimal.

21. $23\frac{5}{6} \times 4\frac{2}{7} = ?$ $83\frac{1}{3} \times 25\frac{17}{25} = ?$ $3\frac{1}{5} \times 24.08 = ?$

22. $693\frac{3}{4} \div 7$; $826\frac{4}{5} \div 9$; $938\frac{7}{12} \div 15$.

23. Tell how to change a decimal fraction to a common fraction. Give two ways of changing a common fraction to a decimal fraction.

24. $\dfrac{3\frac{1}{2} \times 4\frac{2}{3}}{3\frac{1}{2} \div 4\frac{2}{3}} = ?$ $\dfrac{.08\frac{1}{3} \times \frac{2}{3}}{\frac{2}{3} \div .08\frac{1}{3}} = ?$

25 What common fractions can be changed to pure decimals? Explain in two ways why this is so.

26. $2\frac{1}{2}$ is what part of 15? of 20? of 31? of $6\frac{2}{3}$? of $8\frac{3}{4}$? of $18\frac{3}{4}$? of 50? of 100?

27. How many twos are there as factors in the denominator of $\frac{5}{8}$? How many successive one-place reductions of the numerator must be made to introduce these twos? How many ciphers, then, should be annexed at once? Same questions for $\frac{7}{16}$; for $\frac{19}{32}$? How about $\frac{1}{16}$? $\frac{1}{32}$?

28. Study the following fractions in a similar way.

$\frac{3}{5}$, $\frac{18}{25}$, $\frac{39}{125}$, $\frac{7}{50}$, $\frac{19}{250}$, $\frac{13}{20}$, $\frac{39}{40}$, $\frac{37}{80}$.

29. In addition of mixed decimals, what should be done before beginning to add? Answer a similar question for subtraction.

30. Loaned $824.40 with the understanding that .06½ of the amount should be paid me for its use for one year. At this rate, what sum should be paid for its use for 18 months? for 30 months? for 3 years and 5 months?

31. What is the rule for "pointing" the product in multiplication of decimals? Give the reason.

32. Sold a house and lot for $1,824.60, losing 16⅔ per cent of the cost; what was the cost?

33. What shall goods costing $624.80 be sold for to gain 37½ per cent of their cost?

34. Define measurement, and all terms. Define partition, and all terms.

35. Give and explain the rule for "pointing" the quotient when the divisor is a decimal fraction. How multiply by a power of 10? How divide by a power of 10?

36. Multiply 324.086 by 2.5648. Divide 831.2157728 by 324.086.

37. There are 2,150.42 cubic inches in a bushel. What is the capacity in bushels of a bin 48 feet long, 8 feet wide, and 12 feet high?

38. There are 231 cubic inches in a gallon. What is the capacity in 40-gallon barrels of a cubical cistern 6 × 8 feet on the bottom and 10 feet deep?

39. A brick is 8 × 4 × 2 inches. What is its volume? This is what part of a cubic foot? ⅛ of a brick wall is usually filled with mortar. How many bricks will be required to build a wall 40 feet long, 8 feet high, and 8 inches thick?

40. Find the cost of an 8-inch foundation for a building 18 feet × 24 feet, the wall to be 7 feet high, and brick $8 a thousand in the wall.

41. Two men start from the same place and travel in opposite directions, one at the rate of 3.85 miles per hour, and the other at the rate of 4.12½ miles per hour. How far apart will they be at the end of 13 hours?

42. A freight train running at an average rate of $16\frac{2}{3}$ miles an hour starts from Albany at 6 A.M. At 9 A.M. an express train starts from the same point and runs in the same direction at an average rate of $42\frac{1}{2}$ miles an hour. At what time will it overtake the freight train? How far from Albany?

197. MEASUREMENT OF THE CIRCLE.

Measure accurately with a tape-line the circumferences and diameters of five circles, such as the bottom of a pail or the head of a barrel. If the figure measured is not a true circle, take half the sum of the longest and shortest diameters as the true diameter. Divide each circumference by its diameter, getting the quotient true to hundredths, thus:

$$14\tfrac{3}{16} \div 4\tfrac{1}{2} = 14.1875 \div 4.5 = 3.15+$$

These quotients differ because of inaccuracies in measurements. Find their average. This is a close approximation to the true quotient. This quotient, $\frac{\text{circumference}}{\text{diameter}}$, is represented by the character π (called *pi*).

Hence circumference $= \pi \times$ diameter.

PROBLEMS.

1. Measure the circumferences of 5 trees and calculate the diameter of each.

2. Measure the circumferences of 5 other round objects, — ball, stove, apple, etc., — and calculate their diameters.

3. How long is the tire on a 4-foot wheel?

4. How far is a wheelman advanced by each revolution of his 28-inch wheel?

5. A bicycle is said to be geared to 70 inches when each revolution of its pedals propels it a distance equal to the circumference of a 70-inch wheel. How many revolutions of the pedals will move the wheel one mile?

NOTE. By more accurate methods the value of π has been found to 200 decimal places. Hereafter call its value $3\frac{1}{7}$ or 3.1416–.

6. My bicycle has 28-inch wheels, 7 sprockets on the rear hub, 17 sprockets at the pedal. How many turns of the pedals will carry me two miles?

7. The circumference of the earth at the equator is 24,897 miles. What is its equatorial diameter?

8. What is the diameter of a circular mile race-track?

9. The wheels of a wagon are respectively 42 inches and 49 inches. In going what distance does the fore wheel make one more turn than the hind wheel?

10. How is a circle drawn?

11. Draw a circle whose diameter is 6 inches; one whose radius is 8 inches. (The radius is $\frac{1}{2}$ of the diameter.) Calculate the circumference of each. Cover the circumference of each with a string. Measure it.

198. DEFINITIONS.

1. A **Plane** is a surface against which a straight edge will fit in all directions, as the desk-top, the blackboard.

Point out surfaces in the school-room that are not planes nor made of planes. Such are called **curved** surfaces.

2. A **circle** is a plane figure bounded by a line all points of which are equally distant from a point within called the centre.

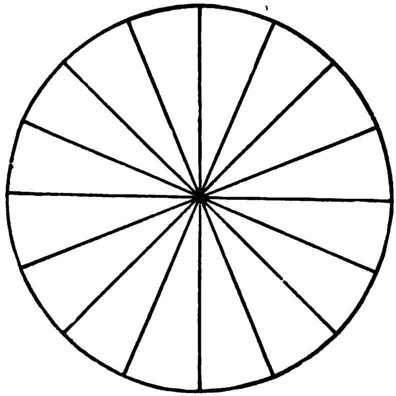
3. The bounding line is called the **circumference**. Any straight line from centre to circumference is called a **radius**. A line passing through the centre and terminating in the circumference is called the **diameter**.

FORMULÆ.

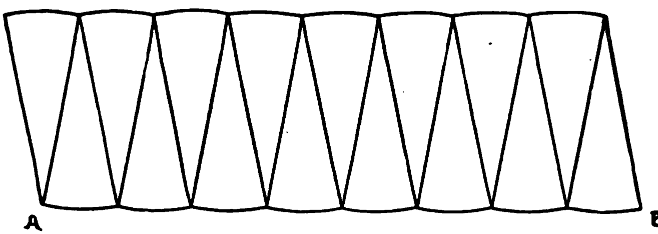
$D = 2R$. $C = \pi \times D$ or πD. $C = \pi \times 2R$ or $2\pi R$.

NOTE. When letters represent numbers, the sign of multiplication may be omitted.

199. Cut from a potato or turnip a thin circular slice. Cut it in two, and divide each semicircle into 8 equal wedges. Be careful not to cut the rind. Straighten the rind of each semicircle and fit the wedges together. The circle is now a rhombus. Its base is half the circumference, $\frac{2\pi R}{2}$, or πR.



Its altitude is R; hence its area is $\pi R \times R = \pi R^2$.



NOTE. An exponent is a figure written above and to the right of a figure or letter to show how many times the number represented by the latter is to be used as a factor; thus, $3^2 = 3 \times 3$; $4^3 = 4 \times 4 \times 4$.

PROBLEMS.

1. If R in the above diagram is 7 inches, what is the length of AB? What is the area of the circle?

2. What is the area of one face of a silver dollar?

3. The face of a watch is $1\frac{3}{4}$ inches in diameter. What is its area?

SOLUTION. $\pi R^2 = 3\frac{1}{7} \times (\frac{7}{8}'')^2 = \frac{22}{7} \times \frac{7}{8}'' \times \frac{7}{8}'' = 2\frac{13}{32}$ square inches.

4. By how many acres does a square mile exceed a mile circle?

1 mile = 320 rods. 160 square rods = 1 acre.

5. What is the length of the sweat-band in a $6\frac{7}{8}$ hat?

6. How high must a 3-inch tin-cup be made to hold one pint?

One gallon = 231 cubic inches.

7. Measure pails, tin-cups, coffee-barrels, and other cylinders and determine their capacity in cubic inches.

8. What is the volume of a new lead pencil $\frac{1}{4}'' \times 7''$?

9. Around a circular pond 500 feet in diameter is a gravel walk 30 feet wide. What is the area of the walk?

QUERY. What is the area of the entire circle, including walk and pond? the area of the pond only?

10. How many square inches of tin are needed to make a quart cup 4 inches in diameter, making no allowance for seams?

11. How many barrels of $31\frac{1}{2}$ gallons each will a cylindrical water-tank hold if 14 feet in diameter and 12 ft., 10 in. deep, inside measure?

12. Mercury is shipped from the mines in cylindrical steel bottles holding 100 pounds each. If these bottles are 4 inches in diameter, what must be their depth?

Mercury is 13.6 times as heavy as water.

13. What is the weight of a dry pine log 12 feet long and 30 inches in diameter? Specific gravity of dry pine = .48.

14. How many cubic inches in a cylindrical tile 12 inches long? Outside diameter 8 inches, inside diameter 6 inches?

15. How many square inches in the entire surface of a cylindrical block 6 inches in diameter and 6 inches high? How many cubic inches in its volume? Which is the larger, surface or volume in a $4'' \times 4''$ cylinder? in an $8'' \times 8''$ cylinder?

200. FEDERAL MONEY.

1. **Money** is that medium by which exchanges of property are ordinarily effected.

2. **Federal Money** is that system of money established by the Congress of the United States of America.

3. The unit of value is the **dollar.**

4. What are the denominations? Give the table. What is the scale?

5. A **coin** is a piece of metal on which certain characters are stamped by government authority, making it legally current as money.

6. The coins are as follows:

		WEIGHT.
One cent . .	Bronze,	48 grains Troy.
3-cent piece .	Copper and nickel,	30 " "
5-cent piece .	Copper and nickel,	73.16 " "
SILVER.		
Dime . . .	Silver and copper,	$38\frac{7}{12}$ " "
Quarter-dollar	" " "	$96\frac{9}{20}$ " "
Half-dollar .	" " "	192.9 " "
Dollar . . .	" " "	412.5 " "
GOLD.		
Quarter-eagle	Gold and copper.	
Three-dollar .	" " "	
Half-eagle .	" " "	
Eagle = $10 .	" " "	
Double-eagle .	" " "	

7. The silver and gold coins are one tenth copper.

8. The gold dollar weighs 25.8 grains Troy. To find the weights of the remaining coins, multiply this number by the number of dollars expressing the value of the coin.

9. The government also provides a currency made of paper. It consists of treasury notes, national-bank notes, gold certificates, and silver certificates.

201. BILLS AND STATEMENTS.

1. A bill is an itemized statement of indebtedness. When it includes items purchased at different times, it is usually called "a statement of account."

When payment is made, the statement is "receipted."

202. Arrange the following in bill forms:

1. On February 10, 1892, D. C. Smith bought of R. C. Rogers & Co., Rochester, N. Y., 47 rolls wall-paper, at 27 cents; 60 yards border, at 3 cents; 13 shades, at $1.72; 112 feet moulding, at 17 cents; 13 sets curtain fixtures, at 79 cents.

2. The following is a statement of W. P. Johnson's account with A. B. Cole & Co., Salem, Mass., made June 30, 1891: 1891, June 15, 2 gallons molasses, at 64 cents; June 16, 1 sack flour, $1.65; 10 pounds starch, at 7 cents; 9 pounds turkey, at 12 cents; June 21, 5 gallons oil, at 15 cents; 2 loaves bread, at 5 cents; 2 dozen eggs, at 13 cents; June 23, 24 pounds sugar, at 5 cents; June 24, ½ bushel potatoes, 40 cents; June 26, 2 pounds cheese, at 20 cents. Receipt this statement.

3. R. P. Young, in account with G. G. Johnson, Normal, Ill. 1891, December 1, 1 dozen cakes, 10 cents; 2 loaves bread, at 5 cents; 1 pound halibut, 20 cents; December 3, 25 pounds sugar, at 4 cents; 1 peck sweet potatoes, 25 cents; ½ bushel apples, 50 cents; December 4, 2 barrels kindling, at 25 cents; December 7, 3 chimneys, at 8 cents; 1 quart oysters, 35 cents; December 8, 1 pound tea, 75 cents; 1 dozen cakes, 10 cents; December 10, 2 dozen eggs, at 16 cents; December 11, 3 lemons, at 4 cents; 1 sack salt, 35 cents; December 14, 4 cans plums, at 20 cents; 1 bushel apples, $1.25; December 17, 1 peck onions, 28 cents; December 20, 3 chickens, at 25 cents; 5 gallons oil, at 15 cents; December 24, 12 pounds turkey, at 11 cents; 1 quart oysters, 35 cents; 5 bunches celery, at 4 cents.

203. ALIQUOT PARTS.

$6\frac{1}{4}$ cents is	$\frac{1}{16}$	of $1.	$33\frac{1}{3}$ cents is	$\frac{1}{3}$	of $1.	$66\frac{2}{3}$ cents is	$\frac{2}{3}$	of $1.
$8\frac{1}{3}$ "	$\frac{1}{12}$	" $1.	$37\frac{1}{2}$ "	$\frac{3}{8}$	" $1.	75 "	$\frac{3}{4}$	" $1.
$12\frac{1}{2}$ "	$\frac{1}{8}$	" $1.	50 "	$\frac{1}{2}$	" $1.	$83\frac{1}{3}$ "	$\frac{5}{6}$	" $1.
$16\frac{2}{3}$ "	$\frac{1}{6}$	" $1.	$62\frac{1}{2}$ "	$\frac{5}{8}$	" $1.	$87\frac{1}{2}$ "	$\frac{7}{8}$	" $1.
25 "	$\frac{1}{4}$	" $1.						

PROBLEMS.

1. What is the cost of 384 pounds of sugar at $6\frac{1}{4}$ cents per pound?

ANALYSIS. If the sugar were $1 a pound, 384 pounds would cost $384. Since the sugar costs $\frac{1}{16}$ of a dollar a pound, 384 pounds will cost $\frac{1}{16}$ of $384, which equals $24.

2. What is the cost of 600 articles at $8\frac{1}{3}$ cents each?
3. Of 688 articles at $12\frac{1}{2}$ cents each?
4. Of 1,272 articles at 25 cents each?
5. Of 5,865 articles at $33\frac{1}{3}$ cents each?
6. Of 575 articles at 50 cents each?
7. Of 473 articles at 10 cents each?
8. Of 972 articles at 20 cents each?

Continue this exercise until great facility is acquired.

9. What is the cost of 568 yards of cloth at $37\frac{1}{2}$ cents a yard?

ANALYSIS. If the cloth were $1 a yard, 568 yards would cost $568. If the cloth were $\frac{1}{8}$ of $1 a yard, 568 yards would cost $\frac{1}{8}$ of $568, which is $71. Since the cloth is $\frac{3}{8}$ of $1 a yard, 568 yards will cost 3 × $71, which is $213.

10. Of 480 pounds of tea at $62\frac{1}{2}$ cents per pound?
11. Of 560 articles at $87\frac{1}{2}$ cents each?
12. Of 1,272 articles at $66\frac{2}{3}$ cents each?
13. Of 2,480 articles at 75 cents each?
14. Of 42,684 articles at $83\frac{1}{3}$ cents each?

15. At $12\frac{1}{2}$ cents a dozen, how many dozens of eggs can be purchased for \$75?

ANALYSIS. At \$1 a dozen, \$75 will buy 75 dozens. At $\frac{1}{8}$ of \$1 a dozen, \$75 will buy 8×75 dozens, which equals 600 dozens.

16. At 25 cents each, how many articles can be bought for \$84? for \$125? for \$144.50? for \$328.75? for \$875.25?

17. At $33\frac{1}{3}$ cents each, how many articles can be bought for \$40? for \$95? for \$875? for \$975.33$\frac{1}{3}$? for \$1,275.66$\frac{2}{3}$?

18. At $16\frac{2}{3}$ cents each, how many articles can be bought for \$18? for \$96? for \$324.16$\frac{2}{3}$? for \$425.33$\frac{1}{3}$? for \$585.50? for \$728.66$\frac{2}{3}$? for \$2,548.83$\frac{1}{3}$?

19. At $8\frac{1}{3}$ cents each, how many articles can be purchased for 25 cents? for 50 cents? for $83\frac{1}{3}$ cents? for \$15? for \$85? for \$125.16$\frac{2}{3}$? for \$250.25? for \$324.33$\frac{1}{3}$? for \$354.41$\frac{2}{3}$? for \$681.50? for \$724.58$\frac{1}{3}$? for \$1,242.66$\frac{2}{3}$? for \$1,461.75? for \$1,595.83$\frac{1}{3}$? for \$2,568.91$\frac{2}{3}$?

20. At 50 cents each, how many articles can be bought for \$50? for \$125? for \$1,250? for \$864.50? for \$965.25? for \$1,386.75?

21. At $37\frac{1}{2}$ cents a pound, how many pounds of butter can be bought for \$15?

ANALYSIS. At \$1 a pound, \$15 will buy 15 pounds. At $\frac{1}{8}$ of a dollar a pound, \$15 will buy 8×15 pounds, which equals 120 pounds. At $\frac{3}{8}$ of a dollar a pound, \$15 will buy $\frac{1}{3}$ of 120 pounds, which is 40 pounds.

22. At $62\frac{1}{2}$ cents each, how many articles will \$125 buy? \$200? \$645? \$874.25? \$1,025.50? \$2,550?

23. At $87\frac{1}{2}$ cents each, how many articles will \$28 buy? \$42? \$91? \$126; \$357? \$826.42?

24. At $66\frac{2}{3}$ cents each, how many articles will \$8 buy? \$28? \$64? \$186? \$432.50? \$786.48?

25. Make 5 problems in which the cost of each article is 75 cents; $83\frac{1}{3}$ cents; $91\frac{2}{3}$ cents.

204. DENOMINATE NUMBERS.

1. **Measuring** is the process of finding how many times a given quantity contains another quantity called the unit of measure.

2. The unit of measure is called the **Standard Unit**, and is usually defined by law.

3. A number composed of standard or derived units employed in the measuring of magnitudes is a **Denominate Number**, as 3 bushels, 5 pounds.

4. A denominate number composed of but one kind of unit is a **Simple Denominate Number**, as 3 pecks.

5. A denominate number composed of more than one kind of unit, but which is reducible to a simple denominate number, is a **Compound Denominate Number**, as 4 bushels, 3 pecks, which is reducible either to bushels or pecks.

6. A **scale** is the statement of the number of units of each kind required to form one of the next higher kind.

NOTE. We have seen that the decimal scale is uniform. Nearly all of the scales in Compound Denominate Numbers are not uniform. While the scale in Federal Money is 10, in English Money it is 4, 12, 20.

205. MEASURES OF LENGTH.

1. A **Line** is that which has extension in only one direction.

2. Measures of length are called **Linear Measures.**

206. LINEAR MEASURE.

Table.

12 inches (in.)	= 1 foot (ft.).
3 ft.	= 1 yard (yd.).
$5\frac{1}{2}$ yd.	= 1 rod (rd.).
320 rd.	= 1 mile.

How many yards in a mile? How many feet? Remember these numbers.

(Perform without analysis.)

1. How many inches in 5 feet? 7 feet? 8 feet? 15 feet? $16\frac{1}{2}$ feet? $18\frac{2}{3}$ feet? 7.8 feet? 5.16 feet?

2. How many feet in 5 yards? 12 yards? $16\frac{2}{3}$ yards? $18\frac{1}{2}$ yards? 24 yards? $33\frac{1}{3}$ yards? 42.6 yards?

3. How many yards in 4 rods? 10 rods? 32 rods? $33\frac{1}{3}$ rods? $37\frac{1}{2}$ rods? 9.52 rods?

4. How many rods in 6 miles? 9 miles? $16\frac{1}{2}$ miles? $24\frac{2}{3}$ miles?

5. How many inches in 4 feet, 5 inches? 16 feet, 6 inches? 31 feet, 8 inches?

6. How many inches in 5 yd. 1 ft. 6 in.? 8 yd. 2 ft. 8 in.? 19 yd. 2 ft. $3\frac{1}{2}$ in.? 4 rd. 2 yd. 3 in.? 22 rd. 2 ft. 11 in.?

7. How many feet in 36 inches? 60 inches? 84 inches? 100 inches? 115 inches?

8. How many yards in 12 feet? 18 feet? 22 feet? 26 feet? 82 feet?

9. How many rods in 11 yards? 33 yards? 44 yards? $38\frac{1}{2}$ yards? $49\frac{1}{2}$ yards? $35\frac{3}{4}$ yards?

10. How many miles in 640 rods? 960 rods? 1,280 rods? 1,350 rods? 2,080 rods?

11. How many feet and inches in 65 inches? 88 inches? 131 inches? 164 inches? 236 inches?

12. How many yards, feet, and inches in 62 inches? 93 inches? 167 inches? 328 inches? 434 inches?

13. How many rods, yards, and feet in 37 feet? 62 feet? 69 feet? 86 feet?

14. How many miles, rods, and yards in 1,797 yards? 3,569 yards? 19,540 yards?

207. Define reduction. Define each kind. What is the form employed in Problem 1, above? In Problem 7? What is the general rule for reduction ascending? for reduction descending? (See Art. **17**.)

Illustrative Example.

1. Reduce 2 mi. 46 rd. 3 yd. 2 ft. 8 in.

ANALYSIS. Since in 1 mile there are 320 rods, in any number of miles there are 320 times as many rods; hence, in 2 miles there are 320 times 2 rods, which are 640 rods. 640 rods + 46 rods = 686 rods. Since in 1 rod there are $5\frac{1}{2}$ yards, in any number of rods there are $5\frac{1}{2}$ times as many yards; hence, in 686 rods there are $5\frac{1}{2}$ times 686 yards, etc.

2. Reduce 3 yd. 2 ft. 10 in. to inches.
3. Reduce 21 rd. 4 yd. 1 ft. to feet.

Reduce to feet:

4. 5 rd. 3 yd. 2 ft.
5. 8 rd. 4 yd. 1 ft.
6. 13 rd. 5 yd. 1 ft.
7. 38 rd. 2 yd.
8. 2 mi. 124 rd. 4 yd.
9. 5 mi. 312 rd. 2 ft.
10. 6 mi. 196 rd. 3 yd. 1 ft.

Reduce to inches:

11. 3 yd. 2 ft.
12. 4 yd. 1 ft. 8 in.
13. 5 yd. 2 ft. 10 in.
14. 4 yd. 2 ft. 5 in.
15. 2 rd. 4 yd. 7 in.
16. 1 mile.
17. $\frac{1}{2}$ mile.

208. Reduce to units of lower denominations:

1. $\frac{4}{7}$ of a mile.

ANALYSIS. Since in 1 mile there are 320 rods, in $\frac{6}{7}$ of a mile there are $\frac{6}{7}$ of 320 rods $= \frac{1920}{7}$ rods $= 274\frac{2}{7}$ rods. $\frac{2}{7}$ of a rod $= \frac{2}{7}$ of $\frac{11}{2}$ yard $= \frac{11}{7}$ of a yard $= 1\frac{4}{7}$ yards. $\frac{4}{7}$ of a yard $= \frac{4}{7}$ of 3 feet $= \frac{12}{7}$ feet $= 1\frac{5}{7}$ feet. $\frac{5}{7}$ of a foot $= \frac{5}{7}$ of 12 inches $= \frac{60}{7}$ inches $= 8\frac{4}{7}$ inches; hence, etc.

FORM.

$$\frac{6}{7} \text{ mi.} = \frac{6}{7} \times 320 \text{ rd.} = \frac{1920}{7} \text{ rd.} = 274\frac{2}{7} \text{ rd.}$$
$$\frac{2}{7} \text{ rd.} = \frac{2}{7} \times \frac{11}{2} \text{ yd.} = \frac{11}{7} \text{ yd.} = 1\frac{4}{7} \text{ yd.}$$
$$\frac{4}{7} \text{ yd.} = \frac{4}{7} \times 3 \text{ ft.} = \frac{12}{7} \text{ ft.} = 1\frac{5}{7} \text{ ft.}$$
$$\frac{5}{7} \text{ ft.} = \frac{5}{7} \times 12 \text{ in.} = \frac{60}{7} = 8\frac{4}{7} \text{ in.}$$

2. $\frac{5}{9}$ of a mile.
3. $\frac{3}{8}$ of a rod.
4. $\frac{7}{9}$ of a mile.
5. $\frac{7}{10}$ of a yard.
6. .375 of a yard.
7. .625 of a rod.
8. .86 of a mile.
9. .047 of a mile.
10. .253 of a rod.
11. $.08\frac{1}{3}$ of 4 miles.

NOTE. $.375 \times 3 = 1.125$ feet. $.125 \times 12 = 1.5$ inches.

209. Reduce to units of higher denominations:

1. 80526 inches.

ANALYSIS. Since there are 12 inches in 1 foot, in 80526 inches there are as many feet as there are 12's in 80526. There are 6710 12's in 80526, with a remainder of 6; hence, in 80526 inches there are 6710 feet and 6 inches. Since there are 3 feet in one yard, in 6710 feet there are as many yards as there are 3's in 6710. There are 2236 3's in 6710, with a remainder of 2; hence, in 6710 feet there are 2236 yards and 2 feet. Since in 1 rod there are $5\frac{1}{2}$ yards, in 2236 yards there are as many rods as there are times $5\frac{1}{2}$ yards in 2236 yards, or as there are times 11 half-yards in 4472 half-yards. There are 406 times 11 in 4472, with a remainder of 6; hence, in 4472 half-yards there are 406 rods, with a remainder of 6 half-yards, or 3 yards. Since there are 320 rods in a mile, etc.

2. 1253 inches.
3. 1367 inches.
4. 1598 inches.
5. 2291 inches.
6. 317 feet.
7. 461 yards.
8. 2893 inches.
9. 26459 inches.
10. 17891 feet.

210. 1. Reduce $\frac{5}{8}$ of a foot to the fraction of a rod.

ANALYSIS. 1 foot is $\frac{1}{3}$ of a yard. $\frac{5}{8}$ of a foot is $\frac{5}{8}$ of $\frac{1}{3}$ of a yard, which is $\frac{5}{24}$ of a yard. Since there are $\frac{11}{2}$ yards in a rod, $\frac{1}{2}$ of a yard is $\frac{1}{11}$ of a rod, and 1 yard is $\frac{2}{11}$ of a rod. $\frac{5}{24}$ of a yard is $\frac{5}{24}$ of $\frac{2}{11}$ of a rod, which is $\frac{5}{132}$ of a rod. Short form. $\frac{5}{8} \times \frac{1}{3} \times \frac{2}{11}$.

2. 6 inches are what part of a rod?

3. $\frac{3}{4}$ of a yard are what part of a mile?

4. 6 feet are what part of a rod? of a mile?

5. $\frac{1}{3}$ of an inch is what part of a yard? of 2 yards? of $2\frac{1}{3}$ yards?

6. 2 feet 3 inches are what part of a yard? of a rod?

7. 1 foot 9 inches are what part of a yard? of a rod?

8. 4 yards 2 feet are what part of a rod? of a mile?

9. $\frac{3}{4}$ of a foot are what part of a rod?

10. $2\frac{1}{2}$ feet are what part of a rod? $3\frac{1}{3}$ feet? $4\frac{1}{8}$ feet?

11. $\frac{5}{16}$ of a rod are what part of a mile? $2\frac{1}{2}$ rods? $3\frac{1}{3}$ rods?

12. Reduce to the fraction of a rod .12 of a foot; .015 of a foot? $.08\frac{1}{3}$ of a yard; .7 of an inch.

13. Reduce .25 of a foot to the fraction of a mile; .375 of a yard; .875 of a rod.

14. Reduce $\frac{7}{25}$ of a foot to the fraction of 2 miles; of $3\frac{1}{2}$ miles.

211. 1. Change 3 yd. 2 ft. 3 in. to the decimal of a rod.

ANALYSIS. 3 in. are $\frac{1}{4}$ of a foot. $\frac{1}{4} = .25$.
2.25 ft. = $\frac{1}{3}$ as many yds. = .75 yd.
3 75 yd. = $\frac{2}{11}$ as many rd. = .68+ rd.

2. Change 4 yd. 2 ft. 6 in. to the decimal of a rod. (Reject all terms below fourth place.)

3. Change 3 rd. 5 yd. 1 ft. 8 in. to the decimal of a mile.

4. Change to the decimal of a mile: 4 rods; 6 rods; 20 rods; 10 rods, 3 yards; 50 rods, 5 yards, 2 feet; 80 rods, 2 yards, 2 feet, 9 inches.

212. SURFACE MEASURE.

1. A **surface** is that which has length and breadth only.

2. A surface is measured by finding how many surface units it contains?

3. The **surface unit** is usually a square whose side is a linear unit.

4. A **square** has the following properties:

(*a*) It is a plane.

(*b*) It is bounded by four equal straight lines.

(*c*) Its angles are all right angles.

5. How many square inches in a square foot? square feet in a square yard? square yards in a square rod?

6. Draw a square rod on the scale of 1 inch to the yard.

7. Complete and learn the following table.

—— square inches	= 1 square foot.
—— square feet	= 1 square yard.
—— square yards	= 1 square rod.
160 square rods	= 1 acre (A).
640 acres	= 1 square mile.

In land surveys a square mile is called a section.

213. PROBLEMS.

1. Reduce 2 sq. rd. 7 sq. yd. 5 sq. ft. 86 sq. in. to square inches.

2. Reduce 3 A. 84 sq. rd. 10 sq. yd. 2 sq. ft. to a simple number.

3. Reduce 3 sections, 480 A. 125 sq. rd. to square yards.

4. Reduce 12,652 square inches to a compound number.

5. Reduce 224,725 square rods to square rods, acres, and square miles.

NOTE. The only troublesome divisor in square measure is $30\frac{1}{4}$. Observe the method in the following problem.

6. Reduce 2,480 square feet to square rods, etc.

METHOD.

2480 square feet = 275 sq. yd. 5 sq. ft.
275 square yards = 1100 fourths of a square yard.
$30\frac{1}{4}$ square yards = 121 fourths of a square yard.
1100 fourths ÷ 121 fourths = 9, with a remainder of 11 fourths.

11 fourths of a sq. yard =	2 sq. yd. 6 sq. ft. 108 sq. in.
Add first rem.	5
	3 sq. yd. 2 sq. ft. 108 sq. in.

Hence, 2480 square feet = 9 sq. rd. 3 sq. yd. 2 sq. ft. 108 sq. in.

7. Reduce 327 square yards to square rods, etc.

8. Reduce 5,873 square yards to a compound number.

9. Reduce $\frac{5}{11}$ of a square mile to lower denominations.

10. Reduce $\frac{3}{8}$ of a square rod to lower denominations.

11. Reduce .372 of an acre to lower denominations.

12. Reduce 3.75 square yards to lower denominations.

13. 36 square inches is what part of a square yard?

14. 6 A. 64 sq. rd. is what part of a section?

15. Reduce 19 A. 32 sq. rd. to the decimal of a square mile.

16. The base of the Great Pyramid of Gizeh, which is a square 764 feet × 764 feet, is what decimal fraction of a square mile?

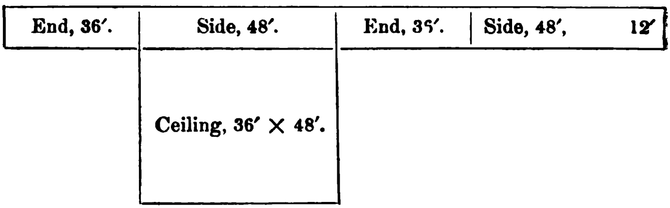
214. THE AREAS OF RECTANGULAR SURFACES.

1. What is the area of a rectangular field 40 rods wide and 92 rods long? What is its value at $63 an acre?

2. What is the cost of plastering the walls and ceiling of a room 36 feet by 48 feet, and 12 feet high, at 27 cents a square yard, no deductions being made for openings?

3. "Develop" the several plastered surfaces in Problem 2 on the scale of 6 feet to 1 inch on the blackboard, or 6 feet to $\frac{1}{4}$ inch on your tablet.

Make a similar diagram for each problem in this set.



4. What is the cost, at 26 cents a square yard, of plastering a cottage containing 6 rooms, 2 of which are 14 feet by 15 feet, 2 are 10 feet by 12 feet, and 2 are 13 feet by 15 feet, the ceilings being 10 feet high, and no allowance being made for openings?

5. What will it cost to paper the walls and ceilings of this cottage, with paper at 20 cents a roll, deducting 400 square feet for base-board, allowing for 14 windows, each 3 feet by 6 feet, and for 7 doors, each 3 feet by $8\frac{1}{2}$ feet, the paperer's charge being 20 cents a roll, and the border costing 5 cents a yard, no deductions being made for border?

NOTE. Wall-paper is 18 inches wide. A roll is 8 yards long.

6. Find cost of papering the four walls and ceiling of your school-room at 5 cents a roll, and 3 cents per yard for border for walls.

7. What is the cost of carpeting a room that is 16 feet by 19 feet, the carpet being a yard wide, and costing \$1.12$\frac{1}{2}$ a yard, if there is a loss of $1\frac{1}{2}$ yards in matching, the carpet running the longer dimension of the room?

NOTE. How many breadths are needed for this room? How much is to be turned under at one side? Make a plan of the room, using a scale of an inch for a foot, and mark the breadths.

8. How many yards of carpet, each strip being ¾ of a yard wide, the loss in matching being 6 inches in each strip except the first, will be needed for a room 22 feet long and 18 feet wide, the strips running the long way? How much if the strips ran crosswise? Make a plan of the room for each case. Which plan is more economical? Why?

9. Cost of covering your school-room with cocoa matting at 30 cents per square yard?

10. Cost of carpeting it with ingrain carpet 36 inches wide, if the "design" in the carpet is 26 inches long?

NOTE. How many times is the design repeated in one strip? How much must be cut off or turned under at the end? at the side?

11. If corn is planted in hills 4 feet apart each way, how many hills in an acre field 9 rods wide?

12. If corn is planted in hills 3′–8″ apart each way, and yields 3 ears to the hill, 100 ears to the bushel, what is the yield in the above field?

13. In a shower an inch of rain fell. How many tons to the acre?

14. The water from a roof 37½′ × 56′ is gathered into a cistern holding 200 barrels of 31½ gallons. What depth of rainfall on the roof will fill the cistern?

15. If the cistern is a cylinder 10 feet in diameter, an inch of rain from the above roof will fill it to what depth?

16. The pressure of the atmosphere in Central Illinois averages 14.4 pounds to the square inch. What is the downward pressure on the lid of a trunk 38″ × 21″?

215. SURVEYORS' MEASURE.

The *surveyors' chain*, invented by Edmund Gunter, about 1620, consists of 100 links. Its length is 4 rods.

1. How many feet in a chain?

2. How many inches in a link?

3. How many chains in a mile?

4. Make a table embodying all these facts and label it *surveyors' long measure.*

5. How many square links in a square chain? square chains in a square mile? square rods in a square chain? square chains in an acre?

6. Arrange these facts in a table and label it *surveyors' square measure.*

7. How many acres in a rectangular field 22.67 chains × 9.26 chains?

8. How many acres in a rectangular field 82 rd. 3 yd. 2 ft. 3 in. by 37 rd. 8 in.?

Note in the above problems the relative simplicity of surveyors' measure.

216. UNITED STATES SURVEYS.

1. Most of the lands of the United States west of the original thirteen States, except the tract between the Ohio and Tennessee rivers, are surveyed in accordance with the following system:

2. In each great survey district there is run a Principal Meridian and an east and west line called a Base Line. On each side of the Principal Meridian at distances of six miles are north and south lines called Range Lines, which divide the land into strips six miles wide called Ranges.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

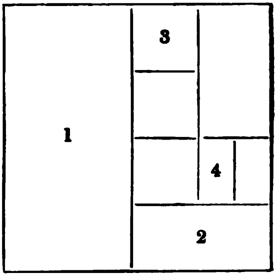
3. By east and west lines parallel to the Base Line the ranges are divided into townships six miles square. A Township is designated by giving its number and direction from the Base Line, the number and position of its range, and the name or number of the Principal Meridian.

Thus the writer is in Township 24 North, Range 2 East of the 3d Principal Meridian.

4. Townships are subdivided into 36 sections numbered thus:

The sections are divided into halves and quarters; the quarters into halves and quarters, and so on. Tracts are described thus:

SEC. 28.



1. W. ½ Sec. 28, T. 24 N. 2 E., 3d P. M.
2. S. ½ S. E. ¼, Section 28, etc.
3. N. W. ¼ N. E. ¼, Section 28, etc.
4. W. ½ N. E. ¼ S. E. ¼, Section 28, etc.

How many acres in each of these tracts?

What fraction of a section is each?

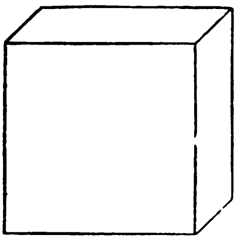
217. PROBLEMS.

Draw a 6-inch square representing a section. Mark in it each of the following tracts and tell how many acres each contains.

1. The N. E. ¼ of the N. E. ¼.
2. The S. ½ of the N. W. ¼.
3. The S. ½ of the N. W. ¼ of the S. W. ¼.
4. The E. ½ of the N. ½ of S. E. ¼ of the S. E. ¼.
5. The N. ½ of the S. E. ¼ of the S. E. ¼ of the N. W. ¼.
6. The N. W. ¼ of the S. W. ¼ of the N. E. ¼ of the N. E. ¼.
7. The N. W. ¼ of the N. W. ¼ of the N. W. ¼.
8. The N. W. ¼ of the N. W. ¼ of the N. E. ¼.
9. The N. ½ of the S. W. ¼ of the N. W. ¼ of the S. F. ¼.
10. The S. ½ of the S. ½ of the S. W. ¼ of the N. W. ¼.

218. MEASURES OF VOLUME.

1. A **Solid** is a figure having length, breadth, and thickness.



2. A **Rectangular Solid** is a solid bounded by six rectangles.

3. The rectangles are called the **faces** of the solid.

4. The intersections of the faces are called the **edges**.

5. Name five rectangular solids. Bring a rectangular solid to the class. Show the faces and the edges.

6. A **cube** is a rectangular solid whose faces are squares.

7. A **cubic inch** is a cube whose edges are each one inch.

8. Solids are measured by finding how many units of volume they contain.

9. The **units of volume** are usually cubes whose edges are linear units.

10. Complete this table:

Table of Cubic Measure.

—— cubic inches	=	1 cubic foot.
—— cubic feet	=	1 cubic yard.
128 cubic feet	=	1 cord (wood).
100 cubic feet	=	1 cord (stone).

11. Wood that is to be used for fuel is measured by the cord. The sticks are usually cut 4 feet long, and are then called "cord-wood." For convenience in measuring they are usually "corded," that is, piled 4 feet high, the length of the stick making the width of the pile. In such a pile, for every 8 feet of length there is a cord of wood. Prove that it contains 128 cubic feet.

A pile 4 feet wide, 4 feet high, and 1 foot long contains 1 cord foot. How many cubic feet does it contain?

219. PROBLEMS.

1. Reduce 4 cubic yards 15 cubic feet 964 cubic inches to cubic inches.
2. Reduce 8 cords 7 cord feet to cubic feet.
3. Reduce 864,952 cubic inches to a compound number.
4. Reduce 1,264 cubic feet to cords, etc.
5. Reduce $\frac{4}{7}$ of a cubic yard to cubic feet and cubic inches.
6. Reduce $\frac{7}{12}$ of a cord to integers of lower denominations.
7. Reduce .36 of a cord.
8. 27 cubic inches is what part of a cubic yard?
9. Change 864 cubic inches to the decimal of a cord.

220. WOOD MEASURE.

PROBLEMS.

1. How many cords of cord-wood in 3 piles of wood, each being 4 feet high, the first being 36 feet long, the second 42 feet long, and the third 73 feet long? Find its cost at $4.75 per cord.

NOTE. Where cord-wood is piled 4 feet high, there is one cord for every 8 feet of length.

2. Each of the following piles is 4 feet high. Find how many cords each contains.

(*a*) 83 feet long. (*d*) $47\frac{1}{4}$ feet long.
(*b*) 69 feet long. (*e*) 61 feet 5 inches long.
(*c*) $35\frac{1}{3}$ feet long. (*f*) 93 feet 10 inches long.

Find the cost of each pile at $5.25 a cord.

3. How much cord-wood in each of the following piles? What is the cost at $4.75 a cord?

(*a*) 26 feet long, 6 feet high.

FORM.

$$\frac{\overset{13}{\cancel{26}} \times \overset{3}{\cancel{6}} \times \$4.75}{\underset{\underset{8}{\cancel{16}}}{\cancel{32}}} = \frac{\$185.25}{8} = \$23.15\tfrac{5}{8}.$$

Explanation: The number of cubic feet in this pile is $26 \times 6 \times 4$. Since there are 128 cubic feet in a cord, the number of cords is $\frac{26 \times 6 \times 4}{128}$. Since cord-wood is 4 feet long, the four may be omitted from dividend and divisor, thus leaving 32 as a divisor.

Note that we then divide the area of the side of the pile of 4-foot wood by the area of the side of a cord.

(*b*) 54 feet long, 7 feet high.
(*c*) 61 feet long, 7 feet high.
(*d*) 42 feet 8 inches long, 6 feet 4 inches high.

NOTE. Call inches twelfths of a foot, and change mixed numbers to improper fractions, thus:

42 ft. 8 in. $= 42\frac{2}{3}$ ft. $= \frac{128}{3}$. 6 ft. 4 in. $= 6\frac{1}{3} = \frac{19}{3}$. $\frac{128 \times 19 \times \$4.75}{3 \times 3 \times 32}$.

(*e*) 86 feet 3 inches long, 7 feet 6 inches high.
(*f*) 124 feet 5 inches long, 8 feet 6 inches high.
(*g*) 97 feet 6 inches long, 5 feet 9 inches high.
(*h*) 224 feet long, 12 feet high.

221. LUMBER MEASURE.

1. The unit of lumber measure is the *board* foot, one foot long, one foot wide, one inch thick.

2. Lumber that is less than one inch thick is counted as if an inch thick. If lumber is more than inch thick, the excess is taken into account.

3. How many cubic inches in a board foot? How many board feet in a cubic foot?

4. What is the width of an inch board that contains as many board feet as it is feet long?

5. Of a two-inch plank? of a three-inch stud?

6. What is the end-area of each of the above pieces?

7. What is the end-area of a $6'' \times 8''$ sill? How many board feet in each foot of its length? What divisor have you employed?

8. How many board feet in a beam $10'' \times 12''$, 24 feet long?

9. Show the truth of the following:

$$\text{Number of board feet} = \frac{\text{thickness} \times \text{width} \times \text{length}}{12}.$$

In what units must thickness, width, and length be expressed?

Bills of lumber are regularly made out in these units.

222. PROBLEMS.

1. What is the cost of 16 sills $6'' \times 8'' \times 18'$ @ $18 per thousand feet?

FORM.

$$\frac{16 \times 6 \times 8 \times 18 \times \$18}{12 \times 1000}$$

ANALYSIS. (1) Since a number of dollars is required, I write $18, the cost of 1000 board feet. I express the cost of one board foot by writing 1000 as a divisor. I express the cost of a stick 1 foot long, 1 inch wide,

and 1 inch thick by writing 12 as a divisor. I multiply by 18 because the sill is 18 feet long; by 8 because it is 8 inches wide; by 6 because it is 6 inches thick; by 16 because there are 16 sills.

(2) The number of board feet in each sill is expressed by $\frac{6 \times 8 \times 18}{12}$. I write 16 as a multiplier because there are 16 sills, 1000 as a divisor to get number of thousand feet, then multiply by 18 because the number of dollars paid must be 18 times the number of thousand feet.

2. What is the cost of the following bill of lumber at $21 a thousand (M)?

8 sills, 8 × 10, 16 feet long; 48 studs, 2 × 4, 18 feet long; 22 joists, 2 × 10, 16 feet long; 50 rafters, 2 × 4, 14 feet long; 24 joists, 2 × 8, 16 feet long.

3. Find the cost of the following bill of lumber, at $16.50 per M:

32 common boards, 8 inches wide, 14 feet long;
65 fence boards, 6 inches wide, 16 feet long;
16 corner posts, 4 × 4, 18 feet long;
7 sills, 6 × 8, 14 feet long;
46 rafters, 2 × 6, 16 feet long;
36 joists, 2 × 8, 18 feet long.

4. Find the cost of the following bill of lumber:

86	pieces	maple flooring	$1 \times 3 \times 16$	@ $40 per M.
86	"	ash "	$1 \times 3\frac{1}{2} \times 16$	@ $36 per M.
72	"	clear pine boards	$1 \times 10 \times 16$	@ $32 per M.
24		rafters	$2 \times 6 \times 16$	@ $18 per M.
36		joists	$2 \times 10 \times 22$	@ $21 per M.

5. What is the cost of 46 planks, 2 inches thick, 10 inches wide, and 18 feet long, at $22 per M.?

6. What is the cost of 65 $2\frac{1}{2}$-inch oak planks, 12 inches wide and 16 feet long, at $36 per M.?

7. How many posts set 8 feet apart are required for wire fencing around a field 800 feet square and for two cross fences dividing the field into four equal squares? Show that the cost of these posts at $8\frac{1}{3}$ cents each is $49.75.

8. Find the cost of the lumber and posts to fence a field 40 rods by 60 rods with a 5-board fence; the posts costing 22 cents each, and placed 8 feet apart; the boards being 16 feet long and 6 inches wide, and costing $18.50 per M.

Make a plan of the field. Cut boards so as to have as little waste as possible.

9. With lumber and posts as in the preceding problem, find the cost of material, except the nails, to fence the N. ½ of N. W. ¼ of S. E. ¼ of a section.

10. Find the cost of lumber and posts to put a 4-board fence around a section of land and to make division fences separating it into quarter-sections, boards, and posts as in Problem 8.

223. MEASURES OF CAPACITY.

1. The units of **Liquid Measure** are the gallon, the quart, the pint, and the gill. The primary unit is the **gallon**, and contains 231 cubic inches. The other units are divisions of the gallon.

TABLE.

4 gills (gi.)	=	1 pint (pt).
2 pt.	=	1 quart (qt.)
4 qt.	=	1 gallon (gal.).

NOTE. The pint is divided into 16 equal parts, each of which is called an ounce. This measure is used by apothecaries. The teacher should show a 1-ounce, a 2-ounce, and a 4-ounce bottle.

In computing the capacity of tanks and cisterns, the barrel of 31½ gallons is the unit.

2 The units of **Dry Measure** are the bushel, the peck, the quart, and the pint.

The primary unit is the **bushel**, which contains 2,150⅖ cubic inches. The other units are divisions of the bushel.

NOTE. The standard bushel is 18½ inches in diameter and 8 inches deep. It contains 2,150⅖ cubic inches, or nearly 1¼ cubic feet.

TABLE.

2 pints	= 1 quart.
8 quarts	= 1 peck (pk.).
4 pk.	= 1 bushel (bu.)

3. How many cubic inches are there in the liquid quart? in the dry quart? About how many gallons are there in a cubic foot of water?

PROBLEMS.

1. Reduce 3 gal. 3 qt. 1 pt. 2 gi. to gills.

2. Reduce 17 bu. 3 pk. 5 qt. 1 pt. to pints.

3. Reduce 587 gills to a compound number.

4. Reduce 1,267 pints to a compound number.

5. Reduce to integers of lower denominations $\frac{17}{32}$ of a gallon.

6. Reduce $\frac{23}{64}$ of a bushel to integers of lower denominations.

7. $\frac{1}{4}$ of a pint is what part of a bushel?

8. $\frac{2}{3}$ of a gill is what part of a gallon?

9. Reduce .625 of a bushel to integers of lower denominations.

10. Reduce 1 gill to the decimal of a gallon.

224. WEIGHT.

1. **Weight** is the measure of the downward pressure of bodies at or near the surface of the earth.

2. There are four systems of weight. These are **Avoirdupois**, **Troy**, **Apothecaries'**, and **Metric**.

3. The standard from which the units of the first three are derived is the Troy pound of the mint.

NOTE 1. For the metric system, see Appendix.

4. The Troy pound is divided into 5,760 equal parts called grains. 7,000 grains equal a pound avoirdupois, and 5,760 the pound apothecaries'.

5. Troy weight is used in measuring gold, silver, precious stones, jewels, etc.

6. Apothecaries' weight is used in mixing medicines.

7. Avoirdupois weight is used in measuring ordinary articles of merchandise.

8. Avoirdupois Weight.

TABLE.

16 oz. = 1 lb.
2000 lb. = 1 ton (T.).

NOTE 1. The long ton is used in the United States custom-houses and in the Eastern States in weighing coal and iron.

28 lb. = 1 quarter (qr.).
4 qr. = 1 hundred weight (cwt.).
20 cwt. = 1 ton.

NOTE 2. The relation of Avoirdupois weight to Troy weight may be seen by comparing the following table with the Troy table:

$\frac{1}{16}$ of 7000 grains = $437\frac{1}{2}$ grains = 1 oz. av.
$\frac{1}{16}$ of $437\frac{1}{2}$ grains = $27\frac{11}{32}$ grains = 1 dram av.

NOTE 3. $62\frac{1}{2}$ lb. Avoirdupois = 1000 oz. = the weight of a cubic foot of distilled water.

9. Troy Weight.

TABLE.

24 gr. = 1 pennyweight (pwt.).
20 pwt. = 1 oz.
12 oz. = 1 lb.

10. Apothecaries' Weight.

TABLE.

20 gr. = 1 scruple (℈).
3 scruples = 1 dram (ʒ).
8 drams = 1 ounce (℥).
12 ounces = 1 pound (℔).

Note. The Troy pound is little used. Gold and silver bullion are sold by the ounce; gold ornaments by the pennyweight; jewels by the *carat* (3.2 grains).

The word *carat* is also used in the sense of *twenty-fourths* in stating the purity of gold. Gold 14 carats fine is $\frac{14}{24}$ gold, $\frac{10}{24}$ alloy.

The metric system is rapidly displacing Apothecaries' weight in pharmacy.

11. Comparison of Weights.

1 lb. Troy = $\frac{5760}{7000}$ = $\frac{144}{175}$ of 1 lb. Avoirdupois.
1 oz. Troy = $\frac{480}{437\frac{1}{2}}$ = $\frac{192}{175}$ of 1 oz. "

The ounce and pound Apothecaries' equal the ounce and pound Troy, respectively.

225. PROBLEMS.

1. How many ounces in 8 lb. 7 oz. av.? in 37 lb. 13 oz.? in 548 lb. 15 oz.? in $\frac{3}{4}$ of 1 lb.? in $\frac{5}{8}$ of 1 T.? in .325 of a long ton?

2. How many grains in 2 lb. 5 oz. 11 pwt. 16 gr. Troy? in 3 lb. 8 pwt.? in $\frac{1}{12}$ of 1 lb.? in .0875 of 1 lb.?

3. How many grains in 2 lb. 3 oz. 1 sc. 17 gr. Apothecaries'? in $\frac{7}{16}$ of 1 lb.? in .28 of 1 oz.?

4. Change the following simple numbers to compound numbers: 568 oz. av.; 2825 gr. Troy; 6827 gr. Apothecaries'; 174 sc.; 869 pwt.

5. What is the cost of 2 tons of sugar at $3\frac{1}{2}$ cents a pound? of a 3-ounce silver watch-case at 8.6 cents a pennyweight? of 2 oz. 5 gr. of quinine at $\frac{3}{4}$ of a cent a grain?

6. 2 lb. 12 oz. is what part of 1 T.?

7. Reduce 6 grains Troy to the decimal of a pound.

8. 2 sc. 12 gr. is what part of 2 lb. 5 dr.?

9. Reduce 4.28 lb. Troy to numbers of lower denominations.

10. $\frac{5}{24}$ of 1 lb. Apothecaries' =?

11. 12 oz. is what part of 1 T.?

12. Change 6 gr. to the decimal of a Troy pound.

13. Change 15 lb. Avoirdupois to Troy weight.

14. Change 24 lb. Troy to Avoirdupois weight.

15. Change 22 lb. 13 oz. Avoirdupois to Troy weight.

16. Change 18 lb. 7 oz. Troy to Avoirdupois weight.

17. What are you worth if you are worth your weight in gold coin? in silver dollars?

18. "A pint is a pound the world around." Is this statement exactly true for water?

19. The Orloff diamond (194¾ carats) weighs how many ounces Avoirdupois?

20. What is the value of the gold in an 18-carat watch-case weighing 3 ounces Troy? 1 oz. gold = $20.67.

NOTE. The grains are usually measured by weight. The weight of the bushel is determined by law. These laws are not absolutely uniform in the several States, except in the case of wheat. The following is the number of Avoirdupois pounds for a bushel in the great majority of cases: Indian corn, 56; oats, 32; rye, 56; wheat, 60.

21. How many bushels in 8,640 lbs. of wheat? In 7,924 lbs. of corn? In 5,872 lbs. of oats? In 10,240 lbs. of rye?

226. Apothecaries' Fluid Measure.

60 minims (m.)	= 1 fluid drachm (fl. ℨ).
8 fluid drachms	= 1 fluid ounce (fl. ℥).
16 fluid ounces	= 1 pint (O.).
8 pints	= 1 gallon (cong.).

Compare this table with Apothecaries' Weight.

227. PROBLEMS.

1. How many 20-minim doses of laudanum in a fluid ounce?

2. How many 3-ounce bottles of perfumery may be filled from one gallon?

3. **A stationer bought 5 gallons of ink for $8.00. He bought 2-ounce bottles at 10 cents per dozen, filled them with ink, and sold them at 5 cents each. What was his gain?**

228. ENGLISH MONEY.

4 farthings (qr.)	= 1 penny (d.).
12 pence	= 1 shilling (s.).
20 shillings	= 1 pound (£).

£ = *libra* = pound. d. = *denarius*, Latin for "penny." qr. = *quadrans* = fourth.

NOTE. The Troy pound of silver was originally coined into 240 silver pennies of 24 grains (1 pennyweight) each. A cross was stamped so deep that the penny was readily broken into fourths (farthings).

The present value of the pound sterling is $4.8665. The gold coin of this value is called the sovereign. The shilling is coined of silver; the penny and half-penny of copper. The guinea (21 s.) and crown (5 s.) are no longer coined. English gold coins are 22 carats fine.

PROBLEMS.

1. Reduce £7 15 s. 7 d. 1 qr. to farthings.
2. Reduce £28 9 d. to pence.
3. Reduce 586 d. to a compound number.
4. $\frac{8}{15}$ of £1 = what?
5. £ 7.048 = what?
6. 7 d. 2 qr. is what part of £1?
7. Change 15 s. 9 d. to the decimal of a pound.
8. By how much does the quarter dollar exceed the shilling in value?

229. FRENCH MONEY.

The *franc* (worth 19.3 cents in U. S. money) is the unit. The scale is decimal.

TABLE.

10 millimes (m.)	= 1 centime (c.)
10 centimes	= 1 decime.
10 decimes	= 1 franc.

230. GERMAN MONEY.

The unit is the *mark*, or *reichsmark*, worth 23.8 cents in U. S. money.

TABLE.

100 pfennige (pf.) = 1 mark (RM.).

The 5-franc piece has been proposed as an international coin nearly equal to the dollar, to 4 shillings, and to 4 marks.

1. What part of the dollar must be taken out to make it conform to the 5-franc piece?

2. What part of 4 shillings?

3. The weight of the 4 marks must be increased by what part of itself?

231. CIRCULAR MEASURE.

1. An **arc** is any portion of a circumference.

2. The unit of arc measurement is the **degree** (°). A degree is $\frac{1}{360}$ of a circumference. What part of the circumference is an arc of 90°? 60°? 45°? 270°? 30? 15°? $22\frac{1}{2}$°?

NOTE. The arc and the degree that measures it must be portions of the same circumference.

3. An arc of 90° is called a **quadrant**; an arc of 60° a **sextant**; an arc of 30° a **sign**.

4. The degree is divided into 60 equal parts called **minutes** (′); the minute into 60 equal parts called **seconds** (″).

5. Make a table setting forth these facts and label it Table of Circular Measure.

6. An **angle** is the difference in direction between two lines proceeding from the same point, called the **vertex**.

7. If the four angles formed by two intersecting straight lines are equal, each angle is called a **right angle**.

8. An angle larger than a right angle is an **obtuse angle**, an angle smaller than a right angle is an **acute angle**.

9. The unit of angular measurement is called a **degree.** This degree is one ninetieth of a right angle.

NOTE. If, with the vertex of a right angle as a center, and any radius whatever, a circumference be described intersecting the sides of the angle, the intercepted arc is a quadrant (90°). Hence a right angle is called an angle of 90°. It is evident that any acute angle is as many ninetieths of a right angle as its intercepted arc is of a quadrant. For this reason the term "degree" is applied both to the ninetieth of the right angle and the ninetieth of the quadrant.

PROBLEMS.

1. Reduce 68° 45′ 36″ to seconds.
2. Reduce 46824″ to a compound number.
3. Add 42° 17′ 26″, 51° 48′ 51″, 7° 56′ 48″, 12° 46′ 28″.
4. What is the difference between 29° 12′ 42″ and 16° 46′ 25″?
5. Multiply 12° 7′ 18″ by 16.
6. Divide 49° 17′ 24″ by 12.
7. 5″ is what part of a degree?
8. $\frac{7}{16}$ of 1° = ?

232. LONGITUDE AND TIME.

1. Find in your atlas a map of the world in hemispheres. What are the lines called that extend from the top of the map to the bottom? What are they for? What are the cross lines called? What are they for?

What is a *prime meridian?* What two prime meridians are used in your geography?

2. Find a map of the United States. Find the longitude of the following places with reference to Greenwich and to Washington City:

(1) Cape Cod, Mass.	(4) Denver, Col.
(2) Erie, Pa.	(5) Leavenworth, Kan.
(3) Washington, D. C.	(6) Memphis, Tenn.

3. The longitude of a place is its distance in degrees, minutes, and seconds, east or west of a prime meridian. It is measured on the arc of a parallel, or of the equator.

4. The difference of longitude of two places on opposite sides of the prime meridian is the sum of their respective longitudes.

5. If A is 10 miles east of B, and C is 12 miles west of B, then A is how many miles east of C? How found? If A is 5 miles west of B, and C is 21 miles west of B, then C is how many miles west of A? How found? Apply these illustrations in the following problems?

The Greenwich meridian is referred to in the following:

233. TABLE.

Paris	2° 20′ 22″ E.
London	0° 5′ 38″ W.
New York	74° 0′ 3″ W.
Boston	71° 3′ 30″ W.
Chicago	87° 35′ 0″ W.
New Orleans	90° 3′ 28″ W.
San Francisco	122° 26′ 15″ W.
Berlin	13° 23′ 43″ E.
St. Petersburg	30° 16′ 0″ E.
Pekin	116° 26′ 0″ E.
Calcutta	88° 19′ 2″ E.
Pittsburg	80° 2′ 0″ W.
St. Louis	90° 12′ 11″ W.
Cincinnati	84° 26′ 0″ W.
Rome	12° 27′ 14″ E.
Honolulu	157° 52′ 0″ W.
Sydney	151° 11′ 0″ E.

PROBLEMS.

1. What is the difference between the longitude of Paris and that of New York?

2. Of Berlin and of London?
3. Of Chicago and Calcutta?
4. Of Sydney and Honolulu?
5. Of St. Petersburg and of St. Louis?
6. Of Rome and of Cincinnati?

How find the difference of longitude of two places on opposite sides of the prime meridian? Why?

7. Of Boston and of Chicago?
8. Of London and of New Orleans?
9. Of Pekin and of Calcutta?
10. Of Pittsburg and of San Francisco?

How find the difference of longitude of two places on the same side of the prime meridian? Why?

Longitude to Time.

234. It is noon at any place when the sun is on its meridian. Before it is noon again at that place the earth must make about one revolution on its axis. The time occupied in making this revolution is divided into 24 hours, and is called one day. During this time the entire circumference of each parallel has passed under the sun. Since each circumference contains 360°, $\frac{1}{24}$ of 360°, or 15°, passes under the sun each hour, $\frac{1}{60}$ of 15°, or 15′, each minute, and $\frac{1}{60}$ of 15′, or 15″, each second.

When it is noon at any place, what time is it 15° east of that place? 30° E.? 45° E.? 90° E.? 180° E.? 15° west? 30° W.? 45° W.? 90° W.? 120° W.? 180° W.? 15′ E.? 15′ W.? 30° 30′ E.? 45° 45′ W.? 60° 30′ 15″ E.? 90° 45′ 30″ W.?

235. Prove that the following statements are true:

A difference of 15° in the longitude of two places makes a difference of one hour in their time.

A difference of 15′ in the longitudes of two places makes a difference of one minute in their time.

A difference of 15″ in the longitude of two places makes a difference of one second in their time.

PROBLEMS.

1. What is the difference in time of two places whose difference of longitude is 36° 42′ 30″?

ANALYSIS. Since a difference of 15° in the longitude of two places makes a difference of 1 hour in their time, a difference of 36° of longitude makes a difference of 2 hours in their time, with a remainder of 6°. 6° = 360′. 360′ + 42′ = 402.′ Since a difference of 15′ in the longitude of two places makes a difference of 1 minute in their time, a difference of 402′ of longitude makes a difference of 26 minutes of time, with a remainder of 12′ of longitude. 12′ = 720″. 720″ + 30″ = 750″. Since a difference of 15″ of longitude makes a difference of 1 second of time, a difference of 750″ of longitude makes a difference of 50 seconds of time. Therefore, a difference of 36° 42′ 30″ of longitude makes a difference of 2 hr. 26 min. 50 sec. in time.

Find the difference of time when the difference of longitude is:

2.	94° 17′ 45″.	**5.**	64° 0′ 50″.	**8.**	82° 31′ 30″.
3.	112° 48′ 15″.	**6.**	48′ 45″.	**9.**	59° 59′ 48″.
4.	6° 56′ 46″.	**7.**	150° 12′ 42″.	**10.**	128° 19′ 18″.

Time to Longitude.

236. If the time at A is an hour later than at B, what is their difference of longitude? if 2 hours later? if 5 hours later? if 1 minute later? 5 minutes? 10 minutes? 1 second? 5 seconds? 20 seconds? A is east or west of B? How do you know?

Change the word *later* to *earlier*, and ask the same questions.

237. Prove the truth of the following statements:

A difference of an hour in the times of two places shows a difference of 15° in their longitudes.

A difference of a minute in the times of two places shows a difference of 15′ in their longitudes.

A difference of a second in the times of two places shows a difference of 15″ in their longitudes.

PROBLEMS.

1. The time in one town is 2 hr. 35 min. 22 sec. earlier than in another. Which is farther east? How many degrees, etc.?

ANALYSIS. Since the time is earlier in the first town than in the second, the sun will not reach its meridian until it has passed the meridian of the other; it is, consequently, farther west. Since a difference of 1 second in the time of two places shows a difference of 15″ in their longitude, a difference of 22 seconds in their times shows a difference of 330″ of longitude, which equals 5′ 30″. Since a difference of 1 minute in the times of two places shows a difference of 15′ in their longitude, a difference of 35 minutes in their times shows a difference of 525 in their longitude. $525' + 5' = 530' = 8° 50'$. Since a difference of 1 hour in the times of two places shows a difference of 15° in their longitude, a difference of 2 hours in their times shows a difference of 30° in their longitude. $30° + 8° = 38°$. Their difference in longitude is 38° 50′ 30″.

Why begin with the lowest denomination?

In the following problems A and B represent places the difference of whose times is given. Find their difference of longitude, and tell which is farther east.

2. 4 hr. 25 min. 15 sec. A's later.

3. 8 hr. 6 min. 20 sec. B's later.

4. 6 hr. 40 min. 18 sec. A's earlier.

5. 9 hr. 52 min. 3 sec. B's earlier.

6. 1 hr. 59 min. 59 sec. A's later.

7. 12 hr. B's earlier.

8. 2 min. 3½ sec. B's later.

9. 11 hr. 24 sec. A's earlier.

10. 10 hr. 31 min. 29 sec. A's later.

Make a rule for each of the two general processes.

11. When it is noon at Paris, what is the time at St. Petersburg? at San Francisco?

12. When it is 6 A. M. at London, what is the time at New York? at Cincinnati? at Rome? at Honolulu? at Sydney?

13. When it is 35 minutes past 3 P. M. at Berlin it is 34 min. $41\frac{4}{15}$ sec. past 8 P. M. at a second city. Find the name of the city in the table.

14. When it is 4 P. M. at Chicago it is 40 min. 35 sec. past 1 P. M. at a second city. Find its name in the table.

15. A ship's chronometer indicates that the time at Greenwich is 25 minutes past 3 P. M. By observations the captain ascertains that it is noon where the ship is. What is the longitude of the ship?

NOTE. The teacher may form many problems from the table of longitudes.

238. A Shorter Method.

Since a difference of 15° in the longitude of two places makes a difference of one hour in their times, a difference of 1° in their longitude makes a difference of 4 minutes, and a difference of 1′ a difference of 4 seconds in their times. The two sets of facts may be combined for rapid oral work.

Illustration. Difference of longitude 49° 36′ 21″.

ANALYSIS. A difference of 45° makes a difference of 3 hours. A difference of 4° makes a difference of 16 minutes. A difference of 30′ makes a difference of 2 minutes. A difference of 6′ makes a difference of 24 seconds. A difference of 21″ makes a difference of $1\frac{2}{5}$ seconds. Combining results, the difference of time is found to be 3 hr. 18 min. $25\frac{2}{5}$ sec.

FORM.

49°	36′	21″
3	16	24
	2	$1\frac{2}{5}$
3 hr.	18 min.	$25\frac{2}{5}$ sec.

Solve Problems 2–10 (page 177) by this method.

239. Apply the same facts to the method of finding difference of longitude when difference of time is given.

Illustration. Difference of time 5 hr. 39 min. 50 sec. A difference of 4 seconds shows a difference of 1′, and a difference of 4 minutes a difference of 1°.

ANALYSIS. A difference of 5 hours shows a difference of 75°. A difference of 36 minutes (9 fours of minutes) shows a difference of 9°, and of 3 minutes a difference of 45′. A difference of 48 seconds shows a difference of 12′ and of 2 seconds a difference of 30″. Combining results, the difference of longitude is 84° 57′ 30″.

FORM.

5 hr.	39 min.	50 sec.
75°		
9°	45′	
	12′	30″
84°	57′	30″

Solve Problems 2–15 in the last set by this method.

240. THE INTERNATIONAL DATE LINE.

Travellers across the Pacific Ocean westward set their time forward a day on crossing the 180th meridian. Islands in the equatorial portion of the Pacific were colonized by Europeans coming from the east with the trade winds, and have the same reckoning as the American continent. Australia, New Zealand, and the neighboring islands originally colonized by the Dutch have the time of Asia, one day in advance. On many charts is shown the International Date Line separating these lands. It passes through Behring's Strait, thence southwest east of Japan, but west of the Philippines, thence east, southeast, and south to the east of New Zealand. Prior to 1867 this line passed east of Alaska.

241. TIME MEASURES.

60 seconds (sec.)	make	1 minute (min.).
60 minutes	"	1 hour.
24 hours	"	1 day.
7 days	"	1 week.
365 days	"	1 common year.
366 days	"	1 leap year.
100 years	"	1 century.

1. If the year is not the last in the century, its number must be divisible by 4 to make it a leap year. If it is the closing year of a century, it is not a leap year unless its number is divisible by 400.

2. The months containing 30 days are April, June, September, and November.

3. The months containing 31 days are January, March, May, July, August, October, and December.

4. February contains 28 days in a common year, and 29 days in a leap year.

PROBLEMS.

1. How many seconds are there in a day?

2. How many hours are there in a common year? in a leap year?

3. Change $\frac{5}{7}$ of a common year to days, hours, minutes, and seconds.

4. 15 minutes is what part of September?

5. Which of the following are leap years? 1866, 1880, 1500, 2000, 1894, 1892.

6. Find the time from June 12, 1881, to March 5, 1884.

METHOD. Count by years as far as possible, then by calendar months, then the remaining days; thus, from June 12, 1881, to June 12, 1883, is 2 years. From June 12, 1883, to February 12, 1884, is 8 months. There are 17 days left in February and 5 in March, hence the time is 2 years, 8 months, and 22 days.

NOTE. A calendar month beginning with the first day of the month completes the month; a calendar month beginning with any other day ends with the next preceding day in the following month. The periods Oct. 1–31, June 10–July 9, Jan. 31–Feb. 28, are calendar months.

7. Find the time:

(1) From December 5, 1881, to June 16, 1886.

(2) From May 5, 1879, to September 20, 1883.

(3) From August 16, 1888, to June 24, 1892.

(4) From January 21, 1886, to November 7, 1893.

8. How many days from January 23 to July 29, common year?

NOTE. 8 + 28 + 31, etc.

9. How many days from October 16, 1890, to June 3, 1891?

242. THE CALENDAR.

1. There are three natural time units: the year, the month, and the day.

2. The natural day from midnight till midnight is not of uniform length. The mean solar day is the average of all the days in the year.

3. The month, the period from one new moon to the next, equals $29\frac{1}{2}$ days nearly.

4. The year, the period between two successive vernal equinoxes, equals 365.2422 days.

5. A calendar is an adjustment of these natural time units for civil purposes. A lunar calendar makes the month a leading unit. Months are alternately of 29 and 30 days. The year of 12 months or 354 days is called a lunar year. Since the lunar year is 11 days too short, extra months (7 in 19 years) must be added from time to time.

6. The lunar year is still used in Mohammedan countries.

7. The Julian calendar, established by Julius Cæsar in 46 B.C., provided a common year of 365 days and, every fourth year, a leap year of 366 days. The months beginning with March were alternately of 31 and 30 days. Later August was given an additional day at the expense of February, and October and December were made months of 31 days instead of September and November.

8. The average Julian year of 365.25 days exceeds the true solar year by .0078 day or $\frac{1}{128}$ day. Gregory XIII. in 1582 corrected as much of the excess as had accumulated since 325 A.D. by decreeing that the 5th of October should be the 15th. He provided that every year that is divisible by 4, and not by 100, is a leap year. Of the century years, only those divisible by 400 are leap years. The Gregorian calendar thus provides 97 leap years in 400 years.

9. The Gregorian calendar was not adopted by the nations of northern Europe until 1700. It was adopted in England in 1752. Russia still uses the Julian calendar.

243. Questions.

1. What years since 1582 have been leap years in Russia and not in Italy?

2. Christmas in Russia comes how many days later than with us?

3. By what fraction of a day does the average Gregorian year exceed the true year?

4. In how many years will the excess in the Gregorian calendar amount to one day?

5. What is the length of the true year in days, hours, minutes, and seconds?

244. MISCELLANEOUS TABLES.

1. 12 ones = 1 dozen.
12 doz. = 1 gross.
12 gross = 1 great gross.
20 ones = 1 score.

2. 24 sheets of paper = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = bale.

Mariners' Measure.

3. 6 feet = 1 fathom.
120 fathoms = 1 cable-length.
80 cable-lengths = 1 mile.

245. ADDITION.

Define addition, sum. Tell how the numbers are written for addition.

1. What is the sum of the following numbers:

1 mi.	182 rd.	4 yd.	1 ft.	7 in.
2	309	5	2	9
5	169	3	0	4
8	274	4	2	11
15	0	3	2	6

2. A man travelled 23 mi. 186 rd. on Monday, 19 mi. 295 rd. 4 yd. on Tuesday, 36 mi. 83 rd. 5 yd. on Wednesday, 19 mi. 317 rd. 2 yd. on Thursday, 28 mi. 297 rd. on Friday, and 34 mi. 168 rd. on Saturday; how far did he travel in the six days?

3. Add:

3 sq. yd.	5 sq. ft.	68 sq. in.
5	7	124
7	6	99
2	8	136
4	2	79

4. Add:

2 A.	121 sq. rd.	17 sq. yd.
3	139	24
7	86	19
12	117	28

5. Add:

110 sq. rd.	26 sq. yd.	4 sq. ft.	83 sq. in.
129	14	7	132
147	29	6	59
153	17	2	126
88	21	8	94

6. Add:

2 cu. yd.	19 cu. ft.	824 cu. in.
5	24	1232
6	16	714
12	21	936
17	26	1532
2	15	1129

7. Add:

5 cd.	6 cd. ft.	14 cu. ft.
12	4	9
6	7	12
8	2	11
27	5	15

8. Find the sum of 5 cd. 7 cd. ft., 6 cd. 9 cd. ft., 12 cd. 4 cd. ft., 9 cd. 4 cd. ft.

9. Sold 3 cd. 5 cd. ft. 12 cu. ft., 12 cd. 14 cu. ft., 9 cd. 5 cd. ft. 15 cu. ft. How much in all?

10. Find the sum of 2 gal. 3 qt. 1 pt., 5 gal. 2 qt. 1 pt., 7 gal. 1 qt., 10 gal. 1 pt., 6 gal. 2 qt. 1 pt.

11. Find the sum of 4 bu. 3 pk. 5 qt., 6 bu. 2 pk. 7 qt., 12 bu. 1 pk. 6 qt.

12. Add 2 lb. 8 oz. 14 pwt. 18 gr., 3 lb. 7 oz. 12 pwt. 10 gr., 5 lb. 10 oz. 8 pwt. 17 gr., 12 lb. 15 pwt. 21 gr. Troy.

13. Add 1 lb. 3 oz. 5 dr. 2 sc. 16 gr., 2 lb. 7 oz. 6 dr. 14 gr., 8 oz. 2 sc. 18 gr., 5 dr. 2 sc. 9 gr. Apothecaries'.

14. Made 4 purchases in London, costing respectively £4 8 s. 7 d., £7 15 s. 9 d., £5 18 s. 8 d., and £10 7 s. 6 d. What was the amount expended?

15. What is the difference in latitude of Boston (42° 21′ 24″ N.) and Rio de Janeiro (22° 54′ S.)?

16. A man bought a quantity of broadcloth for £17 9 s.; of silk for £23 11 d.; of cotton goods for 18 s. $9\frac{1}{4}$ d.; of linen goods for £29 15 s. $11\frac{3}{4}$ d.; of groceries for 17 s. $8\frac{1}{4}$ d.; of boots and shoes for £31 19 s. $5\frac{1}{2}$ d. What did he pay for all?

17. Add $\frac{3}{7}$ of a mile and $\frac{3}{7}$ of 8 rods.

18. Add $\frac{7}{16}$ of a square yard and .37 of a square rod.

19. Add $2\frac{2}{3}$ weeks, $5.33\frac{1}{3}$ days, 6.375 hours.

20. Add £21, $\frac{3}{7}$ of £1, .27 of £1.

246. SUBTRACTION.

Define subtraction, minuend, subtrahend, remainder. Tell how the numbers are written for subtraction. Give the rule. What is the proof?

1. The distance from A to B is 12 mi. 83 rd. 3 yd. 2 ft., and from A to C is 8 mi. 117 rd. 5 yd. 1 ft.; the first distance is how much greater than the second?

2. From 4 mi. 68 rd. 2 yd. 1 ft. 4 in. take 2 mi. 97 rd. 4 yd. 2 ft. 9 in.

3. From 27 sq. yd. 6 sq. ft. 83 sq. in. take 16 sq. yd. 8 sq. ft. 141 sq. in.

4. From 5 A. 83 sq. rd. 13 sq. yd. 5 sq. ft. 67 sq. in. take 2 A. 98 sq. rd. 29 sq. yd. 7 sq. ft. 110 sq. in.

5. From 2 sec. 512 A. 73 sq. rd. take 1 sec. 538 A. 95 sq. rd. 26 sq. yd.

6. Find the difference of the following:

£17	11 s.	7 d.	2 qr.
12	15	11	3
4	15	7	3

7. Find the difference between 15 cu. yd. 18 cu. ft. 1276 cu. in. and 7 cu. yd. 23 cu. ft. and 1528 cu. in.

8. Bought 600 lb. of sugar. Sold 124 lb. 6 oz., 73 lb. 13 oz., 48 lb. 9 oz., 173 lb. 14 oz. How much was left?

9. Bought 624 cd. of wood. Sold 75 cd. 7 cd. ft., 116 cd. 14 cu. ft., 124 cd. 5 cd. ft. 12 cu. ft., 283 cd. 4 cd. ft. 10 cu. ft. How much was left?

10. What is the difference between 8 lb. Apothecaries' weight, and 5 lb. 7 oz. 4 dr. 2 sc. 16 gr.?

11. A is in long. 124° 42′ 36″ E., and B is 67° 49′ 24″ E. What is their difference in longitude?

12. From a cask containing 38 gallons the following amounts were drawn: 4 gal. 3 qt. 1 pt., 7 gal. 2 qt., 12 gal. 1 qt. 1 pt., 8 gal. 3 qt. 1 pt. How much was left in the cask?

13. From $\frac{7}{8}$ of 1 rd. take .63 of 1 rd.

14. From the sum of $\frac{7}{15}$ of 3 lb. and .35 of 2 lb., take the sum of $\frac{2}{3}$ of 5 oz. and .64 of 2 lb. Apothecaries'.

15. Started to walk 124 miles. Went the first day 18 miles, 74 rods; the second day, $\frac{5}{8}$ of 23 miles; the third day .28 of 95 miles. What distance remained?

16. Find difference in time between January 21, 1895, and July 28, 1848; between August 12, 1876, and May 10, 1890.

17. From 12 lb. 9 oz. 7 pwt. 11 gr. take 7 lb. 10 oz. 15 pwt. 18 gr.

18. How long a time from the battle of Bunker Hill to the firing on Fort Sumter?

19. A cylindrical cistern is 10 feet deep and has a diameter of 8 feet. What is its capacity in gallons? ($\pi = \frac{22}{7}$.) In barrels? ($31\frac{1}{2}$ gallons.) Being $\frac{5}{8}$ full, the following amounts were drawn out: $1\frac{1}{2}$ barrels, $2\frac{1}{4}$ barrels, 3.25 barrels, 6.325 barrels. How many barrels were left in the cistern?

SUGGESTION. $\frac{1728 \times 22 \times 16 \times 10 \times 2}{7 \times 231 \times 63} =$ barrels.

20. From $\frac{7}{8}$ of 1 cord take .16 of 3 cords.

21. A man paid £11 12 s. $8\frac{1}{4}$ d. for a wagon. He gave the merchant a £20 note. What change should he receive? Give the rule for subtraction of simple numbers.

22. A man having 34 cords of wood sold to one man 5 cd. 7 cd. ft. 12 cu. ft., to another 15 cd. 14 cu. ft., and to a third 8 cd. 5 cd. ft. How much had he left?

247. Multiplication.

Define multiplication, multiplicand, multiplier, product. Give the rule for multiplication of simple numbers. What is the denomination of the product? Be able to give the analysis of reduction at each step.

1. Multiply £7 12 s. 9 d. 3 qr. by 9.

FORM.

£7	12 s.	9 d.	3 qr.
			9
63	108	81	4) 27 (6
5	7	6	24
68	20) 115 (5	12) 87 (7	3
	100	84	
	15	3	

2. Multiply 3 mi. 25 rd. 4 yd. 2 ft. 8 in. by 12.

3. Multiply 8 cu. yd. 13 cu. ft. 124 cu. in. by 24.

4. A pile of wood is 4 feet wide, 4 feet high, and 27 feet long. How many cords in 15 such piles? What is it worth at $4.50 a cord?

5. A ship sails from N. Y., longitude 74° 0′ 3″ W., and makes an average daily easting of 9° 24′ 36.″ What is her longitude at the end of 7 days?

6. 27 cans hold an average of 10 gal. 3 qt. 1 pt. How much do they all contain?

7. A bin is 8 feet wide, 12 feet long, and 7 feet high. How many bushels of shelled corn will it hold? How much will 18 such bins hold?

8. 36 men worked an average of 12 d. 7 hr. and 30 min. How much money will pay them, at $1.25 a day, counting 10 hours to the day?

9. Find the cost to the druggist of 36 prescriptions of quinine, each containing 24 grains, if quinine cost 42 cents for an avoirdupois ounce ($437\frac{1}{2}$ grains).

10. Sold 28 loads of oats, each containing 74 bu. 3 pk., at $16\frac{2}{3}$ cents a bushel. What was the amount received?

11. Bought a city lot containing $\frac{2}{3}$ of an acre at $2.25 a square yard. What did it cost?

12. Multiply 15 hr. 24 min. 38 sec. by 42.

13. Multiply 16° 17′ 22″ by 76.

14. Multiply 128 lbs. 7 oz. by 56.

Division.

248. Define partition, divisor, dividend, quotient, remainder. What terms are alike? What kind of a number is the divisor?

249. Define measurement, divisor, dividend, quotient, remainder. Which terms are alike? What kind of a number is the quotient?

1. If £18 12 s. 3 d. 3 qr. be divided equally among 7 persons, what will each receive?

FORM.

£	s.	d.	qr.
7) 18 (£2	12	3	3
14	80	12	4
4	7) 92 (13 s.	7) 15 (2 d.	7) 7 (1 qr.
	91	14	7
	1	1	

2. How many articles at £2 7 s. 8 d. 3 qr. each can be purchased for £40 11 s. 4 d. 3 qr.?

ANALYSIS. To simplify this problem, divisor and dividend should be reduced to the lowest denomination found in the numbers. The divisor equals 2291 qr. The dividend equals 38947 qr. There are 17 2291's in 38947; hence, 17 such articles can be purchased.

3. Divide 8 mi. 186 rd. 4 yd. by 7.

4. Divide 8 mi. 186 rd. 4 yd. by 7 yd.

5. If a field containing 71 A. 82 sq. rd. be divided into 8 equal parts, what will each part contain?

6. Bought 36 2 × 4 16-foot studs, 48 2 × 8 18-foot joists, 4 sills 8 × 8 16 feet long, at $18.50 per thousand; 1,250 feet flooring at $32; 1,460 feet sheathing at $20, and 1,480 feet siding at $33.50. If the bill were divided into 4 equal payments, what would each amount to?

7. How many paving-stones 4 ft. 4 in. long and 3 ft. wide will be needed to make a 3-foot walk 186 ft. 8 in. long?

8. How many bricks, of ordinary size, will be required to pave a court 16 feet wide and 80 ft. 8 in. long?

9. Divide 4 lb. 2 oz. 13 pwt. by 1 lb. 4 oz. 17 pwt. 16 gr.

10. Divide £19 17 s. 0 d. 2 qr. by £1 11 d.

11. If a man travel at an average rate of 4 mi. 25 rd. 5 yd. an hour, how many hours will be required to travel 175 miles?

12. Divide 24 T. 826 lbs. by 724 lbs. 8 oz.

13 Bought 15 cd. 7 cd. ft. 12 cu. ft., which was delivered in 16 loads. What was the average load?

14. If 7 hr. 15 min. 40 sec. is the average time required for a man to produce a certain article, how many such articles can be produced in 348 working hours?

15. How many cases, each holding 2 gal. 3 qt. 1 pt., will be needed to hold 48 gal. 3 qt. 1 pt.?

16. A box has a capacity of 1 bu. 3 pk. 5 qt. How many times must a laborer fill it to remove 324 bu. 2 pk. of oats?

17. A cellar is 18 ft. 6 in. by 24 ft. 4 in. and 5 ft. deep. How many loads $\frac{7}{8}$ of a cu. yd. each will the excavated dirt make?

18. A man, having 44 A. 96 sq. rd. of land, sold 5 A. 92 sq. rd. What part of the land does he still own?

19. A cubical tank, 10 feet square at the base, has a capacity of 8,000 gallons. What is its height?

20. A cubic foot of pure gold may be coined into how many dollars? Gold is $19\frac{1}{4}$ times as heavy as water.

250. MISCELLANEOUS PROBLEMS.

REVIEW.

1. Find the sum of $83.2, $632.7, $504.9, $473.3, $712.5, $190.04.

2. Find the sum of $6041.072, $4003.926, $9621.863, $7028.414, $8631.372, $36027.496, $48971.022.

3. What is the cost of 7 articles at $8.464 each?
Of 36 articles at $15.842 each?
Of 329 articles at $76.575 each?
Of 974 articles at $83.125 each?
Of 87 articles at $479.375 each?

4. If 23 barrels of flour cost $155.25, what is the price per barrel?

5. If 725 acres of land cost $49,571.875, what is the price per acre?

6. What is the difference between $7000 and $2874.664?

7. A man received $6,126.82 for his farm, $2,579.12 for his stock, and $1,966.47 for his grain. He bought a house for $3,582.96; furniture for $1,391.65; a horse for $164.25; a carriage for $164.28; and harness for $36.80. How much money did he have left?

8. A man purchased a library for $763.65½, paying an average price of $2.34¼ per volume. How many volumes did he buy?

9. Find the entire cost of the following articles: 1 desk, $28.50; 1 bookcase, $68.30; 1 half dozen chairs, $18.25; 1 rocker, $12.70; 1 bedstead, $29.50; 1 bureau, $29.58; 1 washstand, $11.76; one stove, $37.49; 1 table, $24.76; 1 lounge, $19.46.

10. At 47¼ cents each, how many bushels of corn can be purchased for $343.98?

11. A paid $491.76 for a pair of horses, and $278.97 for a carriage. How much more did he pay for the horses than for the carriage?

12. $6215.824 is how much more than $1987.948?

13. A street-car company bought 864 mules, paying $79,200 for them; what was the average price?

14. At $17.58⅓ each, how many calves can be bought for $5,697?

15. A man bought six farms. For the first he paid $6,012.07; for the second, $4,631.26; for the third, $3,712.84; for the fourth, $8,067.53; for the fifth, $7,824.86; for the sixth, 6,098.94. What did he pay for all?

16. A merchant sold a man the following articles: sugar, $1.40; coffee, 97 cents; tea, 83 cents; salt, 48 cents; flour, $1.85; apples, $2.38; potatoes, 86 cents; molasses, 85

cents. He received in payment a $20-bill. What amount of money should he return?

17. What is the entire cost of the following articles: one horse, $116.87; one buggy, $129.40; a set of harness, $28.90; a whip, $2.55; one wagon, $65.75; one blanket, $3.78; one sleigh, $36.47?

18. Change 873 yards to a compound number.

19. Change 2 ft. 3 in. to a fraction of a mile.

20. Add: 4 bu. 3 pk. 5 qt. 1 pt.; 6 bu. 2 pk. 7 qt.; 12 bu. 1 pk. 6 qt. 1 pt.; and 23 bu. 3 qt.

21. A railway train, running at the average rate of 34 mi. 68 rd. 4 yd. 2 ft. per hour, went from A to B in 9 hours. What is the distance between the two places?

22. Multiply 217 rd. 4 yd. 2 ft. 7 in. by 23.

23. The area of a floor is 25 sq. yd. 6 sq. ft. 83 sq. in. What is the entire area of 12 such floors?

24. How much land is there in 9 fields, if each contain 53 A. 47 sq. rd. 26 sq. yd.?

25. How many 4-ounce vials can be filled from 5 gal. 3 qt. 1 pt. 3 gi. of alcohol?

26. Divide 583 bu. 3 pk. 7 qt. of corn into 16 equal parts.

27. How many 40-gallon barrels of water will a cubical cistern contain that is 10 feet deep?

28. Multiply 45 A. 24 sq. rd. 18 sq. yd. by 38.

29. 37 equal quantities of land contain 37 sec. 201 A. 88 sq. rd. 23 sq. yd. 2 sq. ft. 72 sq. in. What does each contain?

30. A railway train, moving at a uniform rate, ran 307 mi. 299 rd. 3½ yd. in 9 hours. What was the rate per hour?

31. How many revolutions will a carriage wheel, whose circumference is 11 ft. 4 in., make in describing a distance of 1 mi. 125 rd. 4 yd. 10 in.?

32. Divide 41 rd. 4 yd. 10 in. by 4 yd. 2 ft. 8 in.

33. If 80 cu. yd. 4 cu. ft. 848 cu. in. of earth were removed in 28 equal loads, how much did each load contain?

34. How many piles of wood, each containing 2 cd. 75 cu. ft., can be made from 93 cd. 12 cu. ft.?

35. 3.46 miles = ?

36. Change 4 inches to the decimal of a mile.

37. 3 yd. 1 ft. 8 in. is what part of 225 rd. 4 yd.?

38. What is the sum of the following numbers?

1 mi.	182 rd.	4 yd.	1 ft.	7 in.
2	309	5	2	9
5	169	3	0	4
8	274	2	2	11
15	0	1	2	6

39. Divide 66 sq. yd. 6 sq. ft. by 2 sq. yd. 7 sq. ft.

40. How many lots, each containing 4 A. 36 sq. rd., can be formed from 50 A. 112 sq. rd.?

41. From a 40-gal. barrel of vinegar a merchant sold to one man 4 gal. 3 qt. 1 pt. 1 gi.; to a second 5 gal. 2 qt. 3 gi.; and to a third 13 gal. 1 qt. 1 pt. 2 gi. What amount was left in the barrel?

42. What quantity of oats will 15 bins contain if the capacity of each be 186 bu. 3 pk. 7 qt.?

43. Change 2 yd. 1 ft. 11 in. to a fraction of a rod.

44. $\frac{3}{7}$ of a mile = ?

45. $\frac{8}{9}$ of a rod = ?

46. How much wood is there in 24 piles of wood, each containing 9 cd. 86 cu. ft.?

47. Multiply 18 cu. ft. 724 cu. in. by 46.

48. Reduce 2 mi. 180 rd. 3 yd. 2 ft. 10 in. to inches.

49. How many feet in 321 rd. 4 yd. 1 ft.?

50. Reduce 87889 inches to integers of higher denominations.

51. Reduce $\frac{2}{11}$ of a square mile to integers of lower denominations.

52. Reduce .028 of an acre.

53. 108 square inches is what part of a square rod?

54. 2 sq. yd. 5 sq. ft. 64 sq. in. is what part of 16 sq. yd. 2 sq. ft. 76 sq. in.? Express the result as a decimal fraction.

55. Reduce 4813 feet to integers of higher denominations.

56. Change 429 yards to a compound number.

57. An English officer bought 65 horses for his company at an average price of £21 12 s. 6 d.; what was the aggregate cost?

58. How many bushels of oats will a rectangular bin contain that is 6 ft. long, 4 ft. wide, and 5 ft. 8 in. high?

59. A farmer bought the following tracts of land, all lying in the same section:

The N. $\frac{1}{2}$ of the S. E. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$.
The S. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$.
The N. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$.

Draw a diagram of the section and show his purchase. What did it cost at $62.50 an acre?

60. Find the cost of the posts and fencing necessary to build a four-board fence around the land just described: posts being worth 22 cents each, and placed 8 feet apart, and the fencing being 1 in. thick, 6 in. wide, 16 ft. long, and costing $18.25 per M.?

61. Reduce $\frac{10}{11}$ of an acre to square rods, etc.

62. $2\frac{1}{2}$ qr. is what part of £1?

63. How many revolutions will a carriage wheel, whose circumference is 14 ft. 8 in., make in going 2 mi. 84 rd.?

64. Change 6 grains Troy to the decimal of a pound.

65. What are 7 loads of hay, each weighing 2,460 pounds, worth at $8.25 a ton?

66. Add $\frac{2}{3}$ of an acre, $\frac{2}{5}$ of a square rod, $\frac{5}{8}$ of a square yard, and $\frac{7}{12}$ of a square foot.

67. What is the value of 3 oz. 5 dr. of quinine at 3 cents a grain?

68. Reduce $\frac{7}{15}$ of a common year to days, hours, etc.

69. A man sets his watch at Chicago local time. After travelling for some time he finds that it is 1 hr. 24 min. 30 sec. faster than the local time where he is. What is his longitude?

70. $12''$ of arc is what part of a circumference?

71. What is the cost of the following piles of cord-wood at $4.75 a cord?

1 pile, 8 ft. high, 22 ft. long.
2 piles, each $6\frac{1}{2}$ ft. high, 31 ft. long.
1 pile, $9\frac{1}{4}$ ft. high and $32\frac{1}{2}$ ft. long.

72. A square cistern, whose bottom is 8 feet on a side, is 12 feet deep. How many gallons of water are there in it when it is $\frac{7}{8}$ full?

73. Change $\frac{11}{12}$ of a cubic yard to cubic feet and cubic inches.

74. What is the cost of the Brussels carpet, at $1.63 a yard, to cover the floor of a room that is 22 feet long and 19 feet wide, the strips to run the long way?

75. What change shall be made in a watch that is set to New York local time to make it agree with Chicago local time?

76. When eggs are sold at the rate of 2 for $3\frac{1}{2}$ cents, what is the cost of $4\frac{1}{3}$ gross?

77. Find the cost of the following bill of lumber at $21.50 per M.:

5 sills, 8 × 10, 18 ft. long.
36 joists, 2 × 10, 16 ft. long.
42 studs, 2 × 4, 22 ft. long.
70 boards, averaging 9 in. wide and 14 ft. long.

78. If 17 T. 15 cwt. 3 qr. 12 lb. of coal be divided equally among 12 bins, how much will each contain?

79. How many quarts of milk will a vessel hold whose capacity is 1 peck?

80. Add .44 of a common year to 29 days, 19 hr. 18 min.

81. Divide an angle of 100° 10′ 7″ into 13 equal parts.

82. Reduce 8975 grains Troy to a compound number.

83. Reduce 2 lb. 9 oz. 5 dr. 2 sc. 15 gr. Apothecaries' to grains.

84. A room is 15 feet by 20 feet with walls 12 feet high. There are 3 windows 2½ feet by 6 feet, and 2 doors 2 feet 8 inches by 8 feet 4 inches. If wall-paper cost 22 cents a roll, and border 5 cents a yard, what is the whole cost for walls and ceiling?

85. Make a bill of the following items, and receipt it:

32 pounds of sugar, at 6¼ cents.
48 yards of calico, at 8⅓ cents.
28 bushels of potatoes, at 37½ cents.
18 bushels of apples, at 87½ cents.
32 yards of cloth, at 75 cents.
3 dozen plates, at 50 cents.

86. What is the cost of 3½ reams of letter-paper, at 12½ cents a quire?

87. What is the time from March 11, 1883, to January 19, 1889?

88. Change 88537 square feet to square yards, etc.

89. How many days from January 25, 1892, to December 11?

90. A steamboat going down stream is propelled 12 miles an hour by steam, and 320 feet a minute by the current; in what time can she go 175 miles? In what time will she go the same distance up stream?

91. Reduce 25″ to the decimal of a degree.

92. To $\frac{4}{7}$ of a mile add 7569 inches, and multiply the result by 7.

93. From 8.36 bushels take $3\frac{5}{8}$ pecks, and divide the remainder by 12.

94. $2\frac{1}{4}$ quarts is what part of 3 gal. 1 pt?

95. 1 gill is what part of 2 gallons? Change the result to a decimal fraction.

96. In a pacing race two horses started together and went a mile. The time of the faster was 2 min. 6 sec., and of the other 2 min. $6\frac{1}{4}$ sec. How much was the winning horse ahead at the finish?

97. When a locomotive, having a driving wheel 5 feet and 10 inches in diameter, is running at the rate of a mile in a minute, how many revolutions do the "drivers" make in a second?

98. Sound travels at the rate of 1,120 feet a second under ordinary conditions. If the report of a gun is heard $3\frac{1}{4}$ seconds after the flash of the discharge is seen, what part of a mile is the observer from the gun?

99. Over what area may a horse graze if tied to a stake by a 50-foot rope?

100. Over what area may he graze if tied to the corner of a barn $30' \times 40'$ by a 50-foot rope? Draw diagram.

101. An experiment showed that a current of electricity passed over 7,200 miles in $\frac{2}{3}$ of a second. In how long a time would it describe the equatorial circuit of the earth?

102. Out of a sheet of paper $8'' \times 10''$ cut a circle 6 inches in diameter. What part of the paper is cut away?

103. What is the longitude of Quebec if it is 5 minutes and 42 seconds past 1 P. M., when it is noon at Chicago?

104. Water flows into a tank through three pipes. The first would fill it in $3\frac{1}{3}$ hours, the second in $4\frac{1}{2}$ hours, and the third in $5\frac{1}{4}$ hours; in what time will the three pipes fill it?

105. How many 2-inch circles equal in area 1 6-inch circle?

106. A borrower paid $347 for the use of $2,400, paying 7% of the amount loaned for its use for a year. How many years, months, and days should he keep it, counting 30 days for a month?

107. A 52-gallon oil barrel was $\frac{3}{8}$ full. 13 gallons were drawn out. What fraction of its capacity did it then contain? Change this fraction to a decimal.

108. At the time of her marriage, 8 years ago, Mrs. S. was ten years younger than her husband. Her age is now $\frac{3}{4}$ of his. What was her age at the time of her marriage?

109. In a school of 575 pupils the number of boys is $\frac{7}{18}$ of the number of girls. How many are there of each?

110. What is the cost of a city lot 80′ × 160′ if sold for as many silver dollars as can be laid upon it in a single layer and placed side by side in rows parallel to the sides of the lot?

111. What is the area of a 4-inch circle? of an 8-inch circle? Divide the latter area by the former. The first is what part of the second?

112. What is the area of a 5-inch circle? of a 10-inch circle? The second is how many times the first?

113. If the radius (R) of one circle is twice the radius (r) of a smaller circle, πR^2 is how many times πr^2?

114. How large a water-pipe is needed to carry 4 times as much water as a 3-inch pipe?

115. What is the cost of the gold in a 14-carat chain weighing 12 pennyweight at $20.67 per ounce?

116. A cubic inch of gold has been beaten so thin as to cover $\frac{1}{24}$ of an acre. What was its thickness?

117. How many layers of such gold would equal the thickness of a leaf of this book?

Note. Measure thickness of the book exclusive of covers, and divide by the number of leaves.

118. Simplify: $\dfrac{2\frac{1}{2} - 1\frac{1}{3} + 3\frac{1}{4}}{3\frac{1}{3} + 2\frac{5}{8} - 1\frac{7}{12}}$

119. An iron beam 16 ft. long, $2\frac{1}{4}$ in. wide, and 8 in. deep weighs 900 lbs. This specimen of iron is how many times as heavy as an equal volume of water?

120. What part of $\frac{2}{3}$ of $\frac{5}{8}$ is $\frac{3}{4}$ of $\frac{1}{8}$? Change the result to a decimal fraction.

121. Shingles are sold in bundles, each containing the equivalent of 250 shingles 4 inches wide. If shingles are laid $4\frac{1}{2}$ inches to the weather, how many bunches must be bought for a roof $20' \times 30'$? What is the cost of laying them at 80 cents per square?

Note. A "square" contains 100 square feet.

122. Add $\frac{5}{6}$ of an acre and $217\frac{1}{6}$ sq. rd.

123. A rug $16' \times 12'$ is placed in the middle of a floor $19' \times 15'$. What is the width and area of the uncovered strip?

124. Change the following to decimal fractions of five places: $\frac{1}{67}$, $\frac{2}{195}$, $\frac{1}{117}$.

125. 1,000,000 American eagles will coin into how many sovereigns?

126. An English immigrant changes his money, £248, into federal money. What does he receive for it?

127. Gold is $19\frac{1}{4}$ times as heavy as water. What is the weight of a cubic foot? What is its value?

128. If the top of your desk were covered with gold two feet deep, what would it weigh? What would be its value?

129. How high is a rectangular block of gold one foot square at the base and weighing one ton? What is its value?

130. The length of one degree of longitude at the 40th parallel is 53.063 miles. How far apart do two men live on this parallel whose noons are just one minute apart?

131. How long is the 40th parallel?

132. The length of a degree of longitude at latitude 45° is 48.995 miles. Calculate as accurately as you can the distance on this parallel from the Connecticut River to St. Paul, Minn.

133. Add 86512, 43972, 64829, 93517, 48695, 82724, 60982, 93728, 46479, 794736, and get a correct result in 20 seconds or less.

134. Multiply 96534 by 48967, and obtain a correct result in 50 seconds or less.

135. Divide 8497068314 by 59637, and obtain a correct result in 80 seconds or less.

136. Write the rule for the multiplication of a fraction, and illustrate it by an example, telling how *you* know that you have a correct result.

137. Write the rule for the division of a fraction by a fraction, illustrate it by an example, and explain in writing how *you* know that you have a correct result.

138. If 8 men can do a piece of work in $12\frac{1}{2}$ days of 8 hours each, in how many 10-hour days can 5 men do the same work?

The following 27 problems were used in examinations for State certificates in Indiana and Illinois:

139. At 90 cents per yard how much will it cost to carpet a room 20 by 27 feet with carpet $2\frac{1}{4}$ feet wide, allowing one foot waste on each cut for matching?

140. If $12\frac{1}{2}$ yards of dress goods will make a dress, how many yards of cambric $1\frac{3}{8}$ yards wide will be required to line one half of it? If the goods are 1 yard wide?

141. If one bushel of wheat will make 40 pounds of flour, how many barrels of flour can be made from the contents of a bin full of wheat, the dimensions of the bin being $10' \times 5' \times 4'$?

142. A can do a piece of work in $8\frac{3}{4}$ hours, A and B together can do it in $4\frac{8}{31}$ hours, and A and C can do it together in 4 hours. How many hours will it take B and C to do the work?

143. How much will it cost to plaster the walls and ceiling of a room 27 ft. long, 15 ft. wide, and 12 ft. high, at 25 cents a square yard, allowing 432 sq. ft. for doors and windows?

144. Find the circumference and area of a circle whose diameter is 2 ft. 4 in.

145. Divide 125.37 by 15.75. Solve by analysis, and show why the rule for pointing is correct.

146. A vessel sailed from a port directly on a line of latitude a certain distance, then sailed due north a certain other distance, when the captain found his chronometer forty minutes slow. In what direction had he first sailed and how many degrees?

147. A gold mine produces $420,000 in a single year. How many pounds av. did the output weigh, 23.22 grains being worth $1? Its volume?

148. Find the value of $\left(1 - \frac{426}{697} + \frac{2\frac{1}{2}}{8\frac{1}{2}}\right) \div \frac{3\frac{1}{3}}{5\frac{1}{8}}$.

149. Five men in a factory accomplish as much as eight boys. What part of a man's work does a boy do? Change this result to per cent. What per cent of a boy's work does a man do?

150. The diameter of a cylindrical tank is $10\frac{1}{2}$ feet, and its length is $30\frac{1}{2}$ feet. How many gallons will it hold?

151. $(4.4 - .00027) \times 2.1 \times .005 \div .000005$.

152. After paying $\frac{2}{3}$ of a debt and $\frac{3}{4}$ of the remainder, I owe 430.37\frac{1}{2}$ less than at first. What was the debt at first?

153. Reduce 57 A. 96 sq. rd. to the decimal of a square mile.

154. A man walks a certain distance at the rate of $4\frac{1}{3}$ miles an hour, and rides back at the rate of $7\frac{1}{2}$ miles an hour. If it takes him 8 hours to go both ways, what is the distance?

155. Find the cost of 25 pieces of scantling $5'' \times 3\frac{1}{2}''$, 16 ft. long, at \$10.25 per M.

156. When it is 4 hr. 20 min. P. M., 65° 25′ west longitude, what is the time 17° 20′ east longitude?

157. The Capitol at Washington is 751 feet long and 384 feet wide. How many acres does it cover?

158. If it cost \$120 to build a wall 40 ft. long, 14 ft. high, 1 ft. 6 in. thick, what will it cost, at the same rate, to build a wall 180 ft. long, 21 ft. high, and 1 ft. 3 in. thick?

159. A lake whose area is 45 acres is covered with ice an inch thick. Find the weight of the ice in tons, if a cubic foot weighs 920 ounces avoirdupois.

160. A can hoe a row of corn in a certain field in 30 minutes, B in 20 minutes, and C in 35 minutes. What is the least number of rows that each can hoe that all may finish at the same time?

161. A owns $\frac{3}{17}$ of a ship's cargo, valued at \$493,000; B owns $\frac{15}{29}$ of the remainder; C owns $\frac{3}{10}$ as much as A and B, and D owns the remainder. How much does each own?

162. How many square rods in a piece of land $\frac{5}{8}$ of a mile long and $\frac{3}{7}$ of a mile wide?

163. Light occupies 16 minutes and 36 seconds in crossing the earth's orbit. If the earth is 95 millions of miles from the sun what is the velocity of light?

164. $.0001 \div .00000001 = ?$

165. A man bought a horse and a carriage for \$280. $\frac{2}{5}$ of the cost of the carriage was $\frac{2}{3}$ of the cost of the horse. What was the cost of each?

Part II.

SECTION VII.

251. PERCENTAGE.

1. Percentage is a system of calculations by hundredths.

2. Per cent means hundredth or hundredths. 1 per cent is $\frac{1}{100}$; 7 per cent is $\frac{7}{100}$; $\frac{2}{3}$ per cent is $\frac{2}{3}$ of $\frac{1}{100}$.

3. Any per cent is a decimal fraction having 100 for its denominator. It may be expressed in the form of a common fraction, as $\frac{7}{100}$, $\frac{\frac{2}{3}}{100}$, $\frac{8\frac{1}{3}}{100}$; in the form of a decimal fraction, as .07, $.00\frac{2}{3}$, $.08\frac{1}{3}$, or with the per cent symbol, as 7%, $\frac{2}{3}$%, $8\frac{1}{3}$%.

4. Per cent differs from decimal fractions in general in two ways:

(1) Its denominator is always 100.

(2) This denominator may be expressed by the sign %.

252. Express the following as common fractions, and reduce to lowest terms.

1. 7%	**10.** $83\frac{1}{3}$%	**19.** $\frac{5}{12}$%	**28.** $2\frac{1}{2}$%
2. 15%	**11.** 225%	**20.** $1\frac{2}{3}$%	**29.** .25%
3. 21%	**12.** $41\frac{2}{3}$%	**21.** $3\frac{1}{3}$%	**30.** .7%
4. 36%	**13.** $56\frac{1}{4}$%	**22.** $\frac{2}{5}$%	**31.** $.08\frac{1}{3}$%
5. 25%	**14.** $83\frac{1}{3}$%	**23.** $\frac{5}{12}$%	**32.** 2.25%
6. 50%	**15.** 1000%	**24.** $\frac{10}{11}$%	**33.** $.0\frac{1}{2}$%
7. 75%	**16.** 465%	**25.** $\frac{3}{5}$%	**34.** $.00\frac{1}{4}$%
8. $38\frac{1}{2}$%	**17.** $116\frac{2}{3}$%	**26.** $\frac{1}{3}$%	**35.** $.00\frac{1}{8}$%
9. $62\frac{1}{2}$%	**18.** $\frac{3}{8}$%	**27.** $\frac{5}{6}$%	**36.** $2\frac{15}{18}$%.

RULE.

To change any per cent to a common fraction, erase the per cent sign and write 100 for a denominator.

253. Write each of the expressions in Art. **253** as a decimal fraction.

254. Express the following common fractions with the per cent sign, and also as decimal fractions:

1. $\frac{3}{100}$	**6.** $\frac{39}{100}$	**11.** $\frac{48}{100}$	**16.** $\frac{1.25}{100}$
2. $\frac{17}{100}$	**7.** $\frac{12\frac{1}{2}}{100}$	**12.** $\frac{.02}{100}$	**17.** $\frac{.25}{100}$
3. $\frac{125}{100}$	**8.** $\frac{\frac{3}{7}}{100}$	**13.** $\frac{\frac{5}{8}}{100}$	**18.** $\frac{79}{100}$
4. $\frac{62\frac{1}{2}}{100}$	**9.** $\frac{2\frac{1}{8}}{100}$	**14.** $\frac{75}{100}$	**19.** $\frac{87\frac{1}{2}}{100}$
5. $\frac{250}{100}$	**10.** $\frac{.7}{100}$	**15.** $\frac{.08\frac{1}{3}}{100}$	**20.** $\frac{.0\frac{3}{8}}{100}$

255. There is no problem in percentage that will not fall into one of three general problems. A mastery of these general problems gives the technique of the subject.

256. FIRST GENERAL PROBLEM.

To find any per cent of any number.

RULE.

Find one per cent of the number and multiply the result by the number of per cent.

257. ILLUSTRATIVE PROBLEM.

Find 18% of 624.

ANALYSIS. 18% of a number is $\frac{18}{100}$ of that number. $\frac{1}{100}$ of 624 is 6.24, which is found by making the 624 stand two orders farther to the right. $\frac{18}{100}$ of 624 is 18 times 6.24, etc.

This method can always be employed with integers or decimal fractions.

Find:

1. 7% of 824
2. 19% of 916
3. 26% of 589
4. 35% of 1230
5. $43\frac{1}{2}$% of 1584
6. $52\frac{1}{3}$% of 6825
7. 86% of 42563
8. 117% of $324\frac{1}{2}$
9. 125% of $861\frac{3}{4}$
10. 250% of 936.8
11. 1000% of 78.32
12. 17% of .4
13. 23% of .625
14. 31% of $3\frac{1}{3}$
15. $\frac{1}{5}$% of 125
16. $\frac{1}{8}$% of 324
17. $\frac{3}{8}$% of 762
18. $\frac{5}{12}$% of 1284
19. $\frac{17}{25}$% of 825
20. $\frac{10}{11}$% of 867
21. .2% of 163
22. .25% of 7826
23. $\frac{1}{4}$% of 7826
24. .125% of 5624
25. $\frac{1}{8}$% of 5624
26. $\frac{7}{16}$% of 3162
27. $\frac{1}{16}$% of 4563
28. .$06\frac{2}{3}$% of 58635
29. $\frac{1}{15}$% of 58635
30. .$06\frac{1}{4}$% of 32064
31. $\frac{1}{16}$% of 32064
32. .0625% of 24638
33. $\frac{1}{16}$% of 24638
34. $\frac{5}{8}$% of 896.24
35. .625% of 896.24
36. $\frac{5}{12}$% of 756
37. $41\frac{2}{3}$% of 756
38. $39\frac{1}{5}$% of 7824

258. *Illustrative Example.* Find 7 % of $\frac{3}{4}$.

ANALYSIS. $\frac{1}{100}$ of $\frac{3}{4} = \frac{3}{400}$; $\frac{7}{100}$ of $\frac{3}{4} = \frac{21}{400}$.

Find:

1. 15% of $\frac{4}{5}$.
2. 16% of $\frac{3}{8}$.
3. 24% of $\frac{7}{15}$.
4. 30% of $\frac{8}{11}$.
5. 42% of $\frac{18}{25}$.
6. 56% of $\frac{10}{21}$.
7. 63% of $\frac{8}{9}$.
8. 76% of $\frac{18}{19}$.
9. 72% of $2\frac{1}{3}$.
   NOTE. $2\frac{1}{3} = \frac{7}{3}$.
10. 95% of $3\frac{1}{5}$.
11. 124% of $\frac{15}{31}$.
12. 500% of $15\frac{18}{25}$.
13. $\frac{3}{4}$% of $\frac{5}{8}$.
14. $\frac{2}{3}$% of $\frac{18}{20}$.
15. $3\frac{1}{3}$% of $\frac{1}{2}$.
16. $4\frac{1}{6}$% of $\frac{24}{25}$.
    NOTE. $\frac{25}{6} \times \frac{24}{2500}$.
17. $8\frac{1}{3}$% of $\frac{48}{50}$.
18. $13\frac{1}{3}$% of $\frac{39}{40}$.
19. $18\frac{3}{4}$% of $\frac{121}{150}$.

ORAL PROBLEMS.

20. What is 4% of 60? 7% of 80? 5% of 90? 12% of 400? 11% of 900?

21. What is 6% of 25? of 12? of 120? of 1200? of 15? of 150? of 1500?

22. What is $\frac{1}{2}$% of 24? of 36? of 480? of 4800? of 72? of 7200?

23. What is $\frac{2}{3}$% of 1200? of 120? of 12? of $\frac{3}{4}$? of $\frac{6}{7}$? of $\frac{3}{5}$?

24. What is $\frac{1}{5}$% of 75? $\frac{1}{8}$% of 640? $\frac{1}{11}$% of 3300? $\frac{1}{3}$% of $\frac{1}{4}$? $\frac{3}{8}$% of $\frac{8}{3}$? $\frac{5}{8}$% of $\frac{18}{10}$?

25. What is 10% of 2,500 pounds? 16% of \$4,000? 7% of 71 miles? $\frac{2}{3}$% of 120 acres? $\frac{4}{5}$% of 2,500 bushels?

259. Problems are often simplified by changing per cent to a common fraction in its lowest terms.

Illustrative Problem. **1.** Find $37\frac{1}{2}$% of 96.

ANALYSIS. $37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{75}{2 \times 100} = \frac{75}{200} = \frac{3}{8}$. $\frac{3}{8}$ of $96 = 36$.

2. Find $12\frac{1}{2}$% of 72; of 144; of 60; of 240; of $\frac{2}{3}$; of $\frac{8}{15}$.

3. Find $62\frac{1}{2}$% of 2400; of 320; of .048; of $\frac{7}{12}$; of $\frac{8}{15}$; of 84000.

4. Find 40% of 250; 75% of $\frac{5}{8}$; $87\frac{1}{2}$% of $\frac{8}{21}$; $66\frac{2}{3}$% of .081; 25% of 16; $6\frac{1}{4}$ % of .32; $8\frac{1}{3}$% of 132; 50% of $\frac{1}{7}$; 60% of $\frac{2}{3}$.

5. Find $37\frac{1}{2}$% of 96; of 120; of 144; of $\frac{4}{5}$; of .64.

6. Find $33\frac{1}{3}$% of 27; of 81; of 122; of 650; of $\frac{6}{7}$; of .018.

7. Find $16\frac{2}{3}$% of 84; of 120; of 135; of 225; of $\frac{5}{8}$; of $\frac{7}{12}$; of .0144.

8. Find $6\frac{2}{3}$% of 45; of 80; of 140; of 328; of $\frac{7}{8}$; of $\frac{22}{45}$; of .18.

9. Find $18\frac{3}{4}$% of 160; of 324; $31\frac{1}{4}$% of 256, of 320; $43\frac{3}{4}$% of 180, of $\frac{3}{14}$.

10. Find $56\frac{1}{4}\%$ of .0288; $68\frac{3}{4}\%$ of $\frac{7}{32}$.

11. Find 20% of 165; of $\frac{3}{4}$; of .72.

12. Find 40% of 821; 80% of .096.

260. **1.** Find 7% of 325.

FIRST FORM.

$$\begin{array}{r} 325 \\ .07 \\ \hline 22.75 \end{array}$$

For analysis, review Multiplication of Decimals.

SECOND FORM.

$$\frac{7}{\underset{4}{\cancel{100}}} \times \overset{13}{\cancel{325}} = \frac{91}{4} = 22\tfrac{3}{4}.$$

Find:

2. 9% of 426.

3. 13% of 612.

4. 17% of \$725.

5. 20% of 630 bushels.

6. 23% of 1,824 miles.

7. $33\frac{1}{3}\%$ of 756 acres.

8. 125% of 67.2 rods.

9. $37\frac{1}{2}\%$ of $\frac{2}{11}$; of .0688; of 432.

10. $\frac{2}{3}\%$ of 7563; $\frac{3}{4}\%$ of 1200.

11. $37\frac{1}{2}\%$ of £24 16 s. 8 d.

12. $33\frac{1}{3}\%$ of 15 lb. 9 oz. 18 pwt.

13. 25% of 10 rd. 2 ft. 4 in.

14. 72% of 75 cwt. 75 lbs.

15. 75% of 440 sheep.

16. A cistern with a capacity of 84 barrels is $41\frac{2}{3}\%$ full. How many barrels does it contain?

17. How much is 200% of a quantity? 400%? 1000%? 250%? 75%? $37\frac{1}{2}\%$? $83\frac{1}{3}\%$? $66\frac{2}{3}\%$? $41\frac{2}{3}\%$?

18. Find 27% of \$864.50.

19. Find $6\frac{3}{5}\%$ of \$965.80.

20. What is $\frac{2}{3}\%$ of \$1,286.43?

21. What is $18\frac{3}{4}\%$ of \$1,680.48?

22. What is $\frac{1}{4}$% of $972.84?

23. What is $2\frac{1}{3}$% of 7,824 bushels?

24. A merchant bought a stock of goods for $8,324.60. The charge for transportation was $1\frac{1}{4}$% of the cost. What was the entire cost?

25. A owed B a certain sum of money. After paying him 20% of the debt, 25% of the remainder, 50% of what then remained, and $83\frac{1}{3}$% of the third remainder, what part of the debt was still unpaid?

26. What is the interest on $468.15 for one year at 7%?

NOTE. Interest is the amount paid for the use of money, and is computed at a given per cent of the amount loaned, called the principal, for one year.

27. What is the interest on $1,236.50 for two years at $6\frac{1}{2}$%?

28. What is the interest on $2,580 for 2 years and 6 months at $6\frac{1}{4}$%?

29. A farmer owns a section of land. 25% of it is meadow, $33\frac{1}{3}$% of the remainder is corn-land, $37\frac{1}{2}$ % of the remainder is pasture, 80% of the remainder is wheat-land, and the rest is oat-land.

NOTE. Make a diagram 8 inches on a side, and show the several tracts.

30. A piece of cloth containing 36 yards was found to have lost $3\frac{1}{3}$% of its length by shrinkage after sponging. How much did it lose in length?

31. A man's income is $1,500. He pays $46\frac{2}{3}$% of it for his household expenses, 20% of it for general expenses, and $13\frac{1}{3}$% of it for personal expenses. What are his expenses for the year? How much does he save?

32. In a school of 650 pupils 52% were girls. How many boys were there?

33. A schoolhouse is 98 feet long. Its width is $83\frac{1}{3}$% of its length. How wide is it?

34. In 1895 there was a shrinkage in the value of farm lands of not less than 22%. What reduction would this make in the value of a farm of 320 acres formerly worth $85 an acre?

35. A merchant bought an overstock of goods which cost him $12,860. He marked them to sell at an advance of 32 per cent. He finally sold them at a discount of 28% of the marked price. Did he gain or lose? How much?

36. The south wall of the room in which I am writing is $15' \times 26'$. It has three rectangular windows whose aggregate area is $30\frac{10}{13}\%$ of the wall. The windows are of uniform size, the width being 40% of the length. What is the area of each window? its width? its length? (Diagram.)

261. The second general problem of percentage is to find the per cent that one number is of another.

Illustrative Problem. 8 is what per cent of 15?

Two things are to be done in solving this problem.

1. We are to find what part 8 is of 15.

2. The resulting fraction is to be changed to hundredths.

The first will require a review of the method of finding the part that one number is of another.

The second will require a review of the methods of changing a common fraction to a decimal.

ANALYSIS. 8 is $\frac{8}{15}$ of 15. $\frac{8}{15} = .53\frac{1}{3} = 53\frac{1}{3}\%$.

Find what per cent the first number is of the second in each of the following pairs:

1.	4 : 8.	**8.**	5 : 30.	**15.**	5 : 35.
2.	5 : 20.	**9.**	18 : 54.	**16.**	5 : 4.
3.	6 : 8.	**10.**	42 : 63.	**17.**	14 : 12.
4.	8 : 10.	**11.**	7 : 56.	**18.**	24 : 6.
5.	11 : 20.	**12.**	15 : 24.	**19.**	$\frac{1}{4} : \frac{1}{2}$.
6.	18 : 25.	**13.**	21 : 56.	**20.**	$\frac{3}{4} : \frac{7}{12}$.
7.	21 : 30.	**14.**	3 : 36.	**21.**	$\frac{1}{3} : \frac{1}{2}$.

NOTE. $\frac{1}{3} = \frac{2}{6}$. $\frac{1}{2} = \frac{3}{6}$. $\frac{2}{6}$ is $\frac{2}{3}$ of $\frac{3}{6}$. $\frac{2}{3} = 66\frac{2}{3}\%$. Hence $\frac{1}{3}$ is $66\frac{2}{3}\%$ of $\frac{1}{2}$.

22. $\frac{2}{3} : \frac{3}{4}$. **24.** $\frac{1}{3} : \frac{5}{6}$. **26.** $\frac{1}{8} : \frac{1}{3}$.

23. $\frac{4}{5} : \frac{3}{4}$. **25.** $\frac{1}{6} : \frac{1}{5}$. **27.** $.4 : .25$.

NOTE. $.4 = .40$. $.40$ is $\frac{40}{25}$ of $.25$. $\frac{40}{25} = \frac{160}{100} = 160\%$.

28. $.018 : .2$. **30.** $.007\frac{1}{2} : .03$.

29. $.024 : .1$. **31.** $2\frac{1}{2} : 3\frac{1}{3}$.

32. A boy having 20 marbles lost 3 of them. What per cent of his marbles did he have left?

33. In a school of 42 pupils, 7 were in one class, 14 in a second, 6 in a third, 12 in a fourth, and the remainder in a fifth. Give the per cent of the school in each class.

34. A man owning $\frac{2}{3}$ of a mill sold $\frac{1}{2}$ of it. What per cent of his share did he sell? What per cent of the mill did he still own?

35. A received a salary of \$125 a month. His board cost him \$20, his clothing \$5, his other expenses \$30. Each of these items is what per cent of his income? He saves what per cent?

262. WRITTEN PROBLEMS.

Illustrative Problems.

1. \$17 is what per cent of \$24?

ANALYSIS. \$17 is $\frac{17}{24}$ of \$24. $\frac{17}{24}$ is to be reduced to hundredths.

First Method. Reduce the numerator to hundredths, and divide it by the denominator.

$$\frac{17}{24} = \frac{1}{24} \text{ of } 17. \quad 17 = 17.00. \quad \frac{1}{24} \text{ of } 17.00 =$$

$$24\,)\,17.00\,(\,.70\tfrac{5}{6}. \quad 70\tfrac{5}{6} = 70\tfrac{5}{6}\%.$$

$$\begin{array}{r} 16.8 \\ \hline .20 \end{array}$$

Second Method. Do anything to the fraction that will make its denominator 100 without changing the value of the fraction. $\left(\frac{17 \times 4\frac{1}{6}}{24 \times 4\frac{1}{6}} = \frac{70\frac{5}{6}}{100}\right)$

Multiplying both terms by $4\frac{1}{6}$, $\frac{17}{24}$ is found to equal $\frac{70\frac{5}{6}}{100}$; hence, 17 is $70\frac{5}{6}\%$ of 24.

2. $\frac{2}{7}$ is what per cent of $\frac{4}{5}$?

ANALYSIS. $\frac{2}{7} = \frac{10}{35}$. $\frac{4}{5} = \frac{28}{35}$. $\frac{10}{35}$ is $\frac{10}{28}$ of $\frac{28}{35}$. Change $\frac{10}{28}$ to hundredths.

3. 2 is what per cent of 450?

ANALYSIS. 2 is $\frac{2}{450}$ or $\frac{1}{225}$ of 450. $\frac{1}{225} = \frac{1.00}{225} = \frac{.04}{9} = .00\frac{4}{9} = \frac{4}{9}\%$.

NOTE. Abundant dictation work is needed to give facility.

Method. 18 is what per cent of 37?

FORM.

```
37) 18.00 (.48 24/37.
    148
    ----
     3.20
     2.96
     ----
       24
```

Problems like this should be performed at the rate of two or three a minute. Keep the decimal point in its proper place in all of the work.

263. EXAMPLES FOR PRACTICE.

Find what per cent the first number is of the second in each of the following problems:

1. 18 : 30.
2. 14 : 50.
3. 23 : 69.
4. 36 : 81.
5. 54 : 88.
6. 69 : 52.
7. 66 : 92.
8. 72 : 80.
9. 84 : 64.
10. 91 : 28.
11. 96 : 124.
12. 29 : 37.
13. 125 : 625.
14. 140 : 720.
15. 99 : 451.
16. 328 : 1076.
17. 256 : 72.
18. 500 : 128.
19. 836 : 1000.
20. $\frac{3}{4} : \frac{7}{15}$.

FORM. $\frac{3}{4} \times \frac{15}{7} = \frac{45}{28} = 28)\ 45.00.$

21. $\frac{5}{8} : \frac{19}{20}$.
22. $\frac{5}{6} : \frac{11}{12}$.
23. $\frac{1}{8} : \frac{15}{16}$.
24. .02 : .25.
25. .15 : .125.
26. 3.5 : 7.15.
27. $2\frac{1}{3} : 5\frac{5}{8}$.
28. $7\frac{11}{12} : 24\frac{19}{24}$.
29. .0128 : .456.
30. $7\frac{4}{15} : 35\frac{3}{5}$.
31. $\frac{1}{8} : \frac{1}{15}$.
32. $\frac{1}{15} : \frac{1}{8}$.
33. $3\frac{1}{3} : 28$.
34. $24 : 70\frac{2}{5}$.
35. $\frac{5}{16} : .5625$.
36. $3.75 : 7\frac{1}{2}$.

37. \$24 : \$84.

38. \$63 : \$40.

39. 125 lbs. : 370 lbs.

40. 84 A. : 640 A.

41. 130 sheep : 1200 sheep.

42. $12\frac{2}{3}$ days : 19 days.

43. 1000 rd. : 25 rd.

44. 6 ft. : 324 ft.

45. 136 : 624.

46. 38 : 112.

47. 2 yd. 2 ft. 3 in. : 12 rd.

48. \$10.24 : \$1280.

49. 375 men : 12000 men.

50. $\frac{2}{3} : \frac{7}{8}$.

51. $3\frac{1}{4} : 7\frac{2}{5}$.

52. 3 qt. 1 pt. : 5 gal. 2 qt.

53. 40 sq. rd. : 8 A.

54. 3 yd. : 8 rd.

55. 2° 30′ : 10°.

56. \$624 : \$12.

57. A man bought a farm for \$6,250. He paid cash \$1,250. What per cent of the purchase price remained unpaid?

58. A man had 24 cd. 6 cd. ft. of wood. He sold 4 cd. 4 cd. ft. What per cent of his wood was left?

59. 25% of an article is what per cent of $\frac{3}{4}$ of it?

60. 40% of $\frac{5}{8}$ of an article is what per cent of all of it?

61. If A's money is 25% of B's more than B's, B's is what per cent of A's less than A's?

NOTE. If A's is 25% of B's more than B's, it is 125% of B's, or $\frac{5}{4}$ of B's; hence, B's is $\frac{4}{5}$ of A's. (Prove this.) If B's is $\frac{4}{5}$ of A's, it is $\frac{1}{5}$ of A's less than A's; hence, is 20% of A's less than A's. Observe that "per cent of what?" is the important question.

62. If A's money is 25% less than B's, B's is what per cent more than A's?

63. If B's money is $33\frac{1}{3}$% more than A's, A's is what per cent less than B's?

NOTE. Form problems until the process is mastered.

64. If A's money is 10% more than B's, B's is what per cent less than A's? 15% more? 20% more? 30% more? 40% more? 50% more? $37\frac{1}{2}$% more? $62\frac{1}{2}$% more? $66\frac{2}{3}$% more? $83\frac{1}{3}$% more?

65. The width of the top of your desk is what per cent of its length?

66. The width of your school-room is what per cent of its length?

67. The length of this book is what per cent of its width?

68. The thickness of this book is what per cent of its width? of its length?

69. The number of boys in your room is what per cent of the whole number of pupils in the room? It is what per cent of the number of girls?

70. The percentage of girls in your room is what per cent of the percentage of girls in the primary room?

71. What was the percentage of boys in the last graduating class in your high school? of girls?

72. What is the percentage of attendance in your room for one month if all the pupils enrolled except five were present each day, and if each of them was absent three days?

73. What is the population of the town or city in which you live? What is the enrollment in public schools? What per cent of the population is in school?

264. The third general problem of percentage is to find a number when some per cent of it is given.

Illustrative Problem. 24 is 6% of what number?

ANALYSIS. Since 24 is 6% of a required number, 1% of that number is $\frac{1}{6}$ of 24, which is 4. 100% of the required number is 100 times 4, which is 400.

Method. Find 1% of the required number by dividing the given number by the number of per cent. Multiply this quotient by 100.

Such problems may be taken out of percentage by changing the per cent to a common fraction; thus, $6\% = \frac{3}{50}$. 24 is $\frac{3}{50}$ of what?

265. ORAL PROBLEMS.

1. 48 is 10% of what? 12% of what? 25% of what? $33\frac{1}{3}$% of what? 50% of what?

2. 60 is 1% of what? $\frac{2}{3}$% of what? $\frac{3}{4}$% of what? $\frac{5}{12}$% of what?

3. $\frac{5}{8}$ is 5% of what? $\frac{5}{6}$% of what? 25% of what? 100% of what? $62\frac{1}{2}$% of what?

4. $21 is 25% less than what?

5. $18 is 91% less than what?

6. $150 is 50% more than what?

7. 60 A. is 30% of what?

8. 75 miles is 25% of what?

9. $\frac{1}{2}$ of $\frac{2}{5}$ of a yard is 20% of what?

10. $\frac{2}{5}$ of $\frac{5}{6}$ of $\frac{3}{4}$ of a bushel is $16\frac{2}{3}$% of what?

266. WRITTEN PROBLEMS.

1. $6.24 is 18% of what?

FORM.

$$\frac{\$6.24 \times 100}{18}.$$

Employ cancellation.

2. 750 rd. is 125% of what?

3. 18 gal. 3 qt. 1 pt. is $6\frac{2}{3}$% of what?

4. $62.50 is $15\frac{5}{8}$% of what?

5. 4 sq. rd. 16 sq. yd. is $5\frac{3}{4}$% of what?

6. 4 cu. ft. 428 cu. in. is $73\frac{3}{8}$% of what?

7. 67° 30′ is $18\frac{3}{4}$% of what?

8. $\frac{2}{11}$ is $\frac{3}{4}$% of what?

9. 980 A. is 51% less than what?

10. $6,820 is 241% more than what?

11. 72 men deserted from a regiment, leaving $92\frac{6}{7}$% of the whole number. How many men were there in the regiment before the desertion?

12. Paid \$63.84 for the use of a certain sum of money for one year at 7%. What was the sum?

13. Paid \$90 for the use of a certain sum of money for 2 years at the rate of 6% for a year. What sum was borrowed?

14. The interest on a certain principal for 4 years and 7 months, at 6%, is \$269.94. What is the principal?

15. \$141.57 is 18% of what number?

16. 84 rods is 125% of what number?

17. $\frac{3}{44}$ is $37\frac{1}{2}$% of what number?

18. 5 lbs. 3 oz. 6 pwt. is $33\frac{1}{3}$% of what?

19. 35 barrels is $41\frac{2}{3}$% of the capacity of a cistern. How many barrels will it hold?

20. \$315.09 is $18\frac{3}{4}$% of what number of dollars?

21. The width of a pane of glass in my window is 14 inches, which is $48\frac{8}{29}$% of the length. How long are the panes?

22. What is the area of each pane? This is $31\frac{53}{162}$% of what?

23. Find the number of which 263 is $\frac{1}{4}$%; of which 79 is $\frac{2}{3}$%.

24. 826 is $\frac{7}{11}$% of what number?

25. 964 is 500% of what number? 1000% of what number? $133\frac{1}{3}$%.

26. Sold 445.5 pounds of sugar, which was 44% of what I had left. How much had I at first?

27. Sold a farm for \$12,860. $\frac{2}{5}$ of this amount is 50% of the cost of the farm. The gain is what per cent of the cost?

28. What number increased by 20% of itself equals 126?

29. What number diminished by 20% of itself equals 126?

30. A railway train, running at an average rate of 35 miles an hour, for $2\frac{4}{5}$ hours, passes over 35% of the conductor's run. What is the length of his division?

31. What number increased by 25% of itself equals $\frac{3}{5}$?

32. What number diminished by 75% of itself equals $\frac{4}{15}$?

33. If 340 be added to a number, the result is 117% of the number. What is the number?

34. If 520 be subtracted from a number, the result is 86% of the number. What is the number?

35. A town is found to have gained 824 in population in 5 years. This is an increase of 8%. What was the population of the town 5 years ago?

36. A traveler having gone 384 miles has completed 84% of his journey. How much farther has he to go?

37. A farmer having plowed $36\frac{1}{4}$ acres finds that he has finished 56% of his field. How large is it?

38. A farmer contracted to deliver to a dealer 1,800 bushels of corn. Upon taking his grain to market he found that he had overestimated the capacity of his crib 4%. How much did it contain?

39. A merchant being obliged to vacate his room sold his stock at a discount of 11% of the cost and realized $23,568.46. What did the goods cost him?

40. In a certain school there are 168 boys, who form 42% of the whole school. How many girls are there in the school?

41. A merchant sold a suit of clothes for $28.50, which was 25% less than the marked price. This was $33\frac{1}{3}$% more than the cost. What was the cost?

42. $3,825 is 11% more than what? 15% less than what?

43. What number increased by 18% of itself equals 3,379.52?

44. What fraction diminished by 30% of itself equals $\frac{14}{25}$? $\frac{15}{32}$? $\frac{8}{15}$?

45. $\frac{2}{3} \div \frac{7}{8}$ equals 24% of what?

46. $\dfrac{\frac{5}{8}+\frac{7}{12}}{\frac{5}{8}-\frac{7}{12}}$ equals $14\frac{1}{2}$% of what number?

47. In a certain city there are 15% more Germans than Swedes. The former number 5,589, and the latter form 9% of the entire population. What is the population of the city?

48. £18 17 s. 5 d. is 7% of what?

49. A traveler, having gone 24 mi. 124 rd. 4 yd., has completed $37\frac{1}{2}$% of his journey. What distance has he yet to travel?

50. A has a tract of land containing 82 A. 120 sq. rd. This is 5% more than B's. How much has B?

51. A wholesale merchant's sales were $6\frac{2}{3}$% less in 1895 than in 1894, when they aggregated $824,960.50. What were his sales in 1895?

52. H. M. Senseney lost by fire 3,720 tons of coal. This was 62% of his stock. What was his stock?

53. H. M. Senseney carried 23% more stock at the time of the fire than Parker Bros. What amount had Parker Bros.?

54. The uninjured portion of H. M. Senseney's stock was 18% less than the amount required for a year's supply to a manufacturing establishment. How much did it consume annually?

55. A traveler starts from Boston and goes west. When he reaches Chicago he finds that he has passed over $31\frac{1}{2}$% of the longitude to be described in his journey. His destination is near what city whose longitude is given in the table?

56. 63 lb. 12 oz. avoir. is $62\frac{1}{2}$% of what?

57. A pile of wood 6 feet high, 4 feet wide, and 28 feet long is $33\frac{1}{3}$% of how many cords?

58. 15 d. 12 hr. 30 min. is 12% of what?

59. A room 18 feet long, 15 feet wide, and 10 feet high contains 75% of the number of cubic feet of another room of the same height and width. What is its length?

60. $74.88 was paid for the use of a certain sum of money for two years at the rate of 6% of the money for its use for one year. What was the principal?

267. MISCELLANEOUS PROBLEMS.

1. Find $23\frac{1}{2}\%$ of \$628.50.

2. What is the interest on \$1,296, for one year, at $5\frac{1}{2}\%$ per annum?

3. If I paid \$71.28 for the use of \$1,296, for one year, what is the rate of interest?

NOTE. Rate is the number of hundredths of the principal paid for its use for one year.

4. Paid \$225 for the use of \$1,500 for 2 years and 6 months. What is the rate?

5. The lot upon which my house is built has a frontage of 121 feet, which is $53\frac{7}{9}\%$ of its depth. What is its depth?

6. How many square feet does the above lot contain? What part of an acre is it? Change this result to per cent.

7. The house on the above lot stands back 60 feet from the street. What per cent of the lot lies in front of the house?

8. My farm is the northeast quarter of a section. How many acres does it contain? I paid \$80 an acre for it. What did it cost? It has diminished in value 8%. What is its present value?

9. For the use of the above farm my tenant pays me $\frac{1}{3}$ of the oats raised, $\frac{2}{5}$ of the corn, and \$4.75 an acre for meadow and pasture. Last year the S. E. $\frac{1}{4}$ was sowed in oats, the west $\frac{1}{2}$ was planted in corn, and the N. E. $\frac{1}{4}$ was meadow and pasture. The oats yielded an average of 51 bushels, and sold for 18 cents. The corn yielded an average of 54 bushels, and sold for 25 cents. The taxes and repairs were \$200. What per cent of its present value did it pay?

10. $17\frac{1}{2}$ is what part of 49? Change the result to per cent.

11. $\frac{2}{3}$ is what part of $\frac{8}{21}$? What per cent?

12. A man earns $15 a week. He pays $4.50 for board, 70 cents for car fare, an average of $1.25 for clothing, and $3.20 for all other expenses. How much can he save in a year? This is equal to the interest on what sum at 5%?

13. Multiply one thousand one ten-thousandths by four thousand three millionths.

14. Divide one thousand one hundred one millionths by one ten-thousandth.

15. Add the following numbers, and obtain the correct result in one minute: 34883469, 55273289, 52678979, 46864278, 54489858, 47791697, 34963248, 46815798, 68866337.

16. Arrange these problems as given below, and place the differences at the right, thus:

$$\begin{aligned}75063 - 38156 &= 36907\\ 84152 - 68237 &=\\ 91005 - 42807 &=\\ 63254 - 27809 &=\\ 83274 - 58695 &=\\ 91352 - 63806 &=\\ 74083 - 35108 &=\end{aligned}$$

Find the sum of the minuends; do the same with the subtrahends; with the remainders. To the sum of the subtrahends add the sum of the remainders. If no error has been made, what should the last sum equal?

17. A man deposited $8,650 in a bank on May 1. During the month he drew the following checks against his deposit: $650.70, $329.85, $48, $64.50, $1,540.90, $1,937.20, $76.80, $2,170.40. What was his balance on June 1?

18. The following is a copy of A's bank account for June: Balance to his credit June 1, $584.60; June 2, deposited $275.25; June 4, drew a check for $146.85; June 6, deposited $64.50; same day drew a check for $186.15; June 10, deposited $225; June 15, drew a check for $324.10, and

on June 18 for \$462.90; June 21, deposited \$240; June 25, drew a check for \$72.12. What was his balance July 1?

19. $(74 - 16) - (62 - 36) = ?$ $(74 - 16) - 62 - 36 = ?$

20. $562 + 79 - (324 + 148) = ?$
$562 + 79 - 324 + 148 = ?$

21. Bought of A 974 bushels of oats at $19\frac{1}{2}$ cents; 1,328 bushels of corn at $28\frac{1}{2}$ cents; 1,726 bushels of wheat at 58 cents. Sold him 30 acres of land at \$39 an acre. Which was indebted to the other? How much?

22. Two railway trains start at the same time from the opposite ends of a division 212 miles long. One runs at an average rate of 29 miles an hour, and the other at 24 miles. How far apart will they be at the end of 3 hours? 4 hours? 5 hours? 6 hours?

23. A stock train has 29 cars. Each car contains 19 cattle, whose average weight is 1,450 pounds. They sell for \$5.25 a hundred. What do they bring?

24. Change $\frac{8}{17}$ to a 5-place decimal.

25. Add 5 mi. 180 rd. 4 yd. 2 ft.; 16 mi. 79 rd. 3 yd. 1 ft.; 26 mi. 136 rd. 2 yd. 2 ft. 8 in.; 29 mi. 278 rd. 5 ft. 10 in.; 46 mi. 316 rd. 1 yd. 1 ft. 10 in.

26. Multiply 2 lb. 10 oz. 16 pwt. 15 gr. by 36.

27. Reduce 10 square yards to a fraction of an acre.

28. If 18 gallons of water be mixed with $22\frac{1}{2}$ gallons of grape-juice, the water is what per cent of the mixture?

29. How many barrels ($31\frac{1}{2}$ gallons) will a cylindrical cistern hold, its diameter being $8\frac{1}{2}$ feet and its depth 10 feet?

30. A delivery pipe 3 inches in diameter has what percentage of the capacity of a pipe whose diameter is $3\frac{3}{4}$ inches?

31. A man whose watch shows Chicago time finds that it is 27 min. 36 sec. slower than local time. What is his longitude?

32. How many wagon-loads of sand, each containing $83\frac{1}{3}\%$ of a cubic yard, will fill your school-room?

33. If a pile of wood 6 feet high and 4 feet wide extend across the front of your school lot, what is it worth at $4.75 a cord?

34. What is the altitude of a triangle whose area is an acre, and whose base is 10 feet?

35. A square tract of land containing 6,969.6 acres costs $80 an acre. The number of silver dollars required to pay for it will exactly cover its boundary: what is the distance around the field? What is the length of one side?

NOTE. What is the diameter of a silver dollar? What is the distance around the field in feet? What, then, is the length of one side? Prove the problem.

36. Divide 160 square rods into 18 equal parts.

37. What is the cost of plank 2 inches thick to build a walk 250 feet long and 6 feet wide at $21.50 a thousand feet?

38. A's money is 25% less than B's and 25% more than C's. If A has $324, how much has B? How much has C?

39. What will discharge a debt of $586.80 at a discount of 20% and 5%?

NOTE. The second discount is computed on the remainder of the first.

40. Change 3 inches to the decimal of a rod.

41. To what single discount is a discount of $33\frac{1}{3}\%$ and 5% equal?

42. 1049760 is the product of three factors, two of which are 216 and 15. What is the third?

43. What is the difference between $\frac{1}{3}\%$ of $1,800 and $33\frac{1}{3}\%$ of the same?

44. A school-room is $15' \times 60' \times 72'$. The ventilator is $2' \times 2'$. What must be the velocity of the air in feet per second to change the air in 8 minutes?

45. The above school-room is lighted from the long sides. If the window space is 10% of the floor space, how many $4' \times 9'$ windows are there on each side?

46. The value of a house is $87\frac{1}{2}\%$ of the value of the lot. Both cost $9,645. What is the value of each?

47. Put the following in the form of a bill, supplying names: 36 collars at $16\frac{2}{3}$¢; 6 shirts at $1.50; 12 pairs cuffs at 25¢; 15 handkerchiefs at 30¢; 8 pairs hose at 45¢; 8 ties at 35¢; 3 suits underwear at $2.50; 1 hat at $2.50; 2 pairs gloves at $1.25.

48. The width of the blackboard on one side of your school-room is what per cent of its length?

49. If a gallon measure, cylindrical in form, is 4 inches in diameter, what is its height?

50. What is the weight of a solid block of gold $1\frac{1}{2}$ feet thick that will exactly cover the top of your teacher's table? (19.4.)

51. A discount of 20% and $2\frac{1}{2}\%$ having been allowed me, my bill is $386.40. What was the original bill?

52. What is the weight of a dozen silver spoons, each weighing 3 oz. 5 pwt. 7 gr.?

53. Simplify $\dfrac{\frac{5}{18}}{\frac{1}{2} \text{ of } \frac{2}{3}} \div \dfrac{\frac{2}{3} \text{ of } \frac{5}{8}}{2\frac{1}{2} \div 6}$.

54. Write the answer to the following question: What common fractions can be changed to pure decimals?

55. Give a rule for changing a common fraction to per cent; for division of a decimal by a decimal; for changing a decimal to a common fraction; for finding the per cent which one number is of another.

56. What is the interest on $560 for 3 years, 4 months, at 7%?

57. What is the cost of an article which is sold for $225, after a deduction of 10% from the marked price, it having been marked so as to gain $33\frac{1}{3}$%?

58. An article cost $7.29. What should be its marked price to permit a discount of 10% and still gain 10%?

59. How many articles, each weighing 3 oz. 6 pwt. 20 gr., can be made from 9 lb. 8 oz. 19 pwt. 4 gr. of the same material?

60. Find the time from March 12, 1891, to Jan. 5, 1896.

SECTION VIII.

268. APPLICATIONS OF PERCENTAGE.

1. The methods of calculation taught in percentage are applied to the solution of practical problems. In all such problems the pupil must determine which of the three general problems is involved.

2. In each of the general problems three numbers are employed. They are called the Base, the Percentage, and the Rate Per Cent.

3. The **Base**, in a problem in percentage, is the number to which the other two numbers are referred. It is the answer to the question, "per cent of what?"

4. The **Rate per Cent** is the decimal fraction expressed in hundredths which shows the part which the percentage is of the base.

5. The **Percentage** is the number whose relation to the base is expressed by the rate per cent.

6. In each of the general problems two of these three numbers are given to find the third.

7. In the first general problem the base and rate per cent are given to find the percentage; in the second, the base and percentage to find the rate per cent; in the third, the percentage and rate per cent to find the base.

NOTE. A fourth number, called the Amount, is employed in some of the Applications of Percentage.

269. PROFIT AND LOSS.

1. The **Cost** of an article is the expenditure involved in its purchase or production, and is usually expressed in money.

2. The **Selling Price** of an article is the amount of money which the buyer pays the seller for it.

3. The **Profit** on an article is the excess of the selling price over the cost.

4. The **Loss** on an article is the excess of the cost over the selling price.

270. In each of the following problems show which of the general problems is involved. Be ready to tell "per cent of what" in every case.

ORAL PROBLEMS.

1. Paid $2.50 for an article, and sold it at an advance of 10%. What was my gain?

2. Bought an article for $2, and sold it for $1.75. The loss was what per cent of the cost?

3. Gained 30 cents on an article, which was 15% of the cost. What was the cost?

4. Sold an article for $12, gaining 20% of the cost. What was the cost?

271. These four problems illustrate all of the cases that arise in Profit and Loss. They are as follows:

1. **Given the cost and the rate per cent of profit or loss, to find the profit or loss, or selling price.**

2. **Given the cost and the profit or loss, to find the rate per cent of profit or loss.**

3. (*a*) **Given the profit or loss and the rate per cent of profit or loss, to find the cost, or**

(*b*) **Given the selling price and rate per cent of profit or loss, to find the cost.**

272. ORAL PROBLEMS.

1. Cost, \$12; rate of gain, 25%: find gain and selling price.

2. Cost, \$25; gain, \$8: find rate per cent of gain and selling price.

3. Rate of loss, 40%; loss, \$20: find cost and selling price.

4. Rate of gain, $12\frac{1}{2}$%; selling price, \$45: find cost and gain.

5. Cost, \$1.44; loss, 18 cents: find rate per cent of loss and selling price.

6. Rate of gain, $16\frac{2}{3}$%; selling price, \$63: find cost and gain.

7. Cost, \$96; rate of loss, $6\frac{1}{4}$%: find loss and selling price.

8. Cost, \$63; gain, \$7: find rate per cent of gain and selling price.

9. Cost, \$540; selling price, \$600: find gain and rate per cent of gain.

10. Cost, \$225; rate of loss, $11\frac{1}{9}$%: find loss and selling price.

11. Gain, \$91; rate of gain, 7%: find cost and selling price.

12. Cost, \$250; rate of gain, 8%: find gain and selling price.

13. Cost, \$36; selling price, \$42: find gain and rate per cent of gain.

14. Selling price, \$52; cost, \$48: find gain and rate per cent of gain.

15. Selling price, \$560; rate of loss, 20%: find cost and loss.

16. Rate of gain, 30%; selling price, \$390: find cost and gain.

17. Gain, \$81; rate of gain, $11\frac{1}{9}\%$: find cost and selling price.

18. Loss, \$21; cost, \$63: find rate per cent of loss and selling price.

19. Cost, \$1,800; rate of gain, $66\frac{2}{3}\%$: find gain and selling price.

20. Selling price, \$3,000; cost, \$1,800: find gain and rate per cent of gain.

21. Loss, \$250; selling price, \$750: find cost and rate per cent of loss.

22. Gain, 46 cents; rate of gain, 23%: find cost and selling price.

23. Cost, 84 cents; rate of gain, $7\frac{1}{7}\%$: find gain and selling price.

24. Gain, 6 cents; rate of gain, $7\frac{1}{7}\%$: find cost and selling price.

273. WRITTEN PROBLEMS.

1. A man invests \$2,680, and makes a profit of $23\frac{1}{4}\%$. How many dollars did he gain?

2. A farmer raised in a single year 6,400 bushels of oats, 6,250 bushels of corn, and 42 tons of hay. He sold the oats at $37\frac{1}{2}$ cents a bushel, the corn at 40 cents, and the hay at \$6 a ton. The landlord received $\frac{2}{5}$ of the value of the crop, which was 7% of the value of the farm. What was the farm worth?

3. A house, costing \$12,250, was destroyed by fire. The insurance company paid the owner \$10,500. He lost what per cent of his investment?

4. A merchant marked his goods an advance of 25% above the cost. The market declining, he reduced the selling price 10%, and made a profit of \$1,280 on his sales. What was the cost of the goods sold?

5. If $\frac{7}{12}$ of an article be sold for what the whole cost, what is the rate per cent of gain?

6. A man bought a quantity of apples at 75 cents a bushel. If there was a waste of 12%, at what price must he sell the remainder to make 25% in the transaction?

7. A merchant sold 450 yards of cloth at a profit of 24%, realizing a profit of $243. Find the cost and selling price per yard.

8. Find the selling prices of the following articles so that they may bring a profit of 30%: tea, costing 50 cents; coffee, costing 30 cents; shoes, costing $2; cloth, costing $1.50; eggs, costing 15 cents; butter, costing 20 cents.

9. If a span of horses, costing $465, be sold for $620, what is the rate per cent of gain?

10. If the purchaser sold the same horses for $520.80, what was the rate per cent of loss?

11. A field, containing 125 acres, yielded 6,562½ bushels of corn. The succeeding year the crop diminished $4\frac{16}{21}$%. What was the yield per acre the second year?

NOTE. Explain the following method of solution:

$$\frac{65625 \text{ bu.} \times 2000}{10 \times 125 \times 2100} = ?$$

12. A merchant paid $1,458 for some goods. What must he ask for them that he may fall 10%, lose 10% of the selling price, and gain 10% in the transaction?

NOTE. Tell "per cent of what" in each case.

Explain this form: $\frac{\$1458 \times 11 \times 10 \times 10}{10 \times 9 \times 9} = ?$

13. Sold $\frac{2}{3}$ of an article for $\frac{3}{4}$ of the cost of the whole. What was the rate per cent of gain?

14. Sold $\frac{3}{5}$ of an article for $\frac{1}{2}$ of the cost of the whole. Was there a gain or a loss? Find the rate per cent.

15. Cost, $8,625; gain, $2,125: find rate per cent of gain.

16. Selling price, $9,850; cost, $10,000: find rate per cent of loss.

17. 50 acres of oats yielded an average of 45 bushels last year. This year the aggregate yield from the same field is 2,500 bushels. What is the per cent of gain?

18. A merchant bought 350 bushels of potatoes at 45 cents. He lost 15% of them. For how much a bushel must he sell the remainder to gain 25% of the cost of the whole?

19. Bought a bankrupt stock for $3,280. Sold it for $4,100, and discounted the bill 5% for cash. What was the gain per cent?

20. In our school there was a total enrollment last year of 560. This year it amounted to 610. What is the per cent of gain?

21. Cost, $28.60; rate of gain, 23%. Find selling price without finding the gain.

22. Cost, $824.60; gain, $206.15. Find per cent of gain and selling price.

23. Gain, $2,400; rate per cent of gain, 16. Find selling price without finding cost.

24. Selling price, $860; rate of profit, 25%. Find the profit without finding the cost.

25. Selling price, $428.50; rate of loss, 15%. Find the cost.

26. A farmer raised 6,824 bushels of grain. By fertilization he increased the yield $8\frac{3}{4}$%. What was the increased yield?

27. Owing to a storm, a railway train was obliged to reduce its average speed from 48 to 40 miles an hour. What was the per cent of reduction? The ordinary time for running over the division was 4 hrs. 24 min. How far behind time would the train be upon reaching its destination?

28. The gross gain of A's business for one year was \$12,860, which is 25% of his investment. He paid for rent \$2,000; for insurance \$100; for clerk hire \$4,000. Counting his own services at \$2,000, what per cent of his investment did he gain?

29. Sold a tract of land for \$8,403.90 which cost \$9,810. What was the per cent of loss?

30. Invested the proceeds of the above sale in corn, which was sold at an advance of 20%. Was there a gain or a loss in the whole transaction? What per cent?

NOTE. In the following problems obtain per cent approximately.

	Cost Price.	Selling Price.	Gain.	Loss.	Per Cent.
31.	\$824	\$1050	?		?
32.	?	\$1068	\$124		?
33.	\$1580	?	\$312.50		?
34.	?	?		\$21.60	15
35.	?	\$2175		\$240	?
36.	\$3812	\$3500		?	?
37.	?	?	\$14.50		$3\frac{1}{3}$
38.	?	\$4840	\$650		?
39.	?	?		\$88.60	$12\frac{1}{2}$
40.	\$6000	?	?		$5\frac{1}{8}$
41.	?	\$79.60		?	25
42.	?	?	\$29.80		48
43.	\$45.20	\$51	?		?
44.	?	\$4850.80		\$212.20	?
45.	?	?		\$68.40	$14\frac{2}{7}$
46.	\$3874.80	?	?		$37\frac{1}{2}$
47.	?	\$5160	\$1180		?
48.	?	?		\$225	$6\frac{2}{3}$

274. COMMISSION.

1. **Commission** is compensation paid by a person or persons to a person or persons for performing certain business transactions.

2. The party for whom the business is transacted is called the **principal.**

3. The party performing the service for the principal is called an **agent,** or **commission merchant.**

4. The services performed by agents or commission merchants are of two kinds:

a. Those in which money comes into their hands to be remitted to their principals.

b. Those in which money is expended for their principals.

5. Agents may receive money for their principals by collecting debts due them, or by selling property for them and receiving the proceeds. They may expend money for them by making purchases for them, or by paying their obligations.

6. Commission is one of the applications of percentage because the agent's compensation is some per cent of the amount collected or expended.

7. As the agent renders two kinds of service, there will be two kinds of problems in Commission: the "selling or collecting" problems, and the "purchasing or paying" problems. The former are simple and easily solved; the latter are more difficult.

8. The Commission is the amount which the agent receives for his services. The Proceeds are the difference between the Base and Commission.

9. **The important thing to remember: The Base in commission problems is the amount actually collected or expended for the principal.**

275. PROBLEMS.

1. A commission merchant sold 12,560 bushels of corn for me at 36 cents. His commission was 2%. What was his commission, and what amount should he remit?

Which of the general problems of percentage is this?

2. A man sent to his agent $1,050 with which to purchase books. After deducting his commission at 5%, how much did he expend in books? What was his commission?

NOTE. Do not make the mistake of calculating the commission on $1,050. This is not the amount to be expended for books.

ANALYSIS. The amount to be expended for books is 100% of itself. The commission is 5% of that amount. Their sum is 105% of the amount to be invested in the books. But their sum is $1,050; hence, $1,050 is 105% of the amount to be thus invested. 1% of the investment is $\frac{1}{105}$ of $1,050, which is $10. 100% of the investment is 100 × $10, which is $1,000. $1,050 − $1,000 = the commission.

FORM.

$$\frac{\$1050 \times 100}{105} = \$1000\text{; or, } \frac{\overset{50}{\$\cancel{1050}} \times 20}{\cancel{21}} = \$1000$$

Which of the general problems is this?

NOTE. It is desirable, often, to find the commission alone. In the above problem the amount sent is $\frac{105}{100}$ of the amount of the purchase. The commission is $\frac{5}{100}$ of the amount of the purchase. The commission, therefore, is $\frac{5}{105}$, or $\frac{1}{21}$ of the amount sent.

3. Find the commission on a sale of $2,150 at $2\frac{1}{2}$%.

4. On $3,184.36 at $3\frac{1}{3}$%.

5. On $46,912.60 at $1\frac{3}{8}$%.

6. On $236,124 at $\frac{5}{8}$%.

7. On $875,635 at $\frac{8}{11}$%.

8. Amount sent, $2,530; rate of commission, $2\frac{1}{2}$%. Find amount paid out.

9. Amount sent, $3,642; rate of commission, $3\frac{1}{3}$%. Find amount invested.

10. Amount sent, $8,156; rate of commission, 4%. Find amount invested.

11. Amount sent, $2,482.36; rate of commission, 2%. Find amount invested.

12. In Problems 3, 4, 5, 6, find the proceeds of the sales without finding the commission.

13. In Problems 8, 9, 10, find the commission without finding the amount invested.

14. An agent's commission for selling a house and lot was $240. The proceeds of the sale were $7,760. What was the rate per cent of the commission?

15. An agent's commission on a sale was $324.60; the rate of commission was 3%. What was the amount of the sale?

16. A sent $1,250 to an agent with which to buy a town lot and pay his commission at 2½%. What was the cost of the lot? What was his commission?

17. An agent received a consignment of corn amounting to 2,864 bushels. He sold it at 39 cents a bushel. He paid freight, $75.92, and charged 2¼% commission. What was his commission, and what amount did he remit to the principal?

18. An agent sold 3,150 bushels of wheat at 92 cents. The net proceeds of the sale were $2,836.41¾. What was the rate per cent of his commission?

19. Sold cotton on a commission of 4%. The commission amounted to $384.48. Invested the net proceeds in wheat, less a commission of 3%. What was the amount of the second commission?

20. Sold the N. ½ of the S. E. ¼, and the S. ½ of the N. E. ¼ of a section of land, at $72½ an acre. My commission was 3½%. What was the amount of my commission? Find the net proceeds.

21. A principal sent to his agent 324 barrels of flour, with directions to invest the proceeds of the sale in wheat, after deducting his commission of 2½% for selling, and 2¼% for

buying. The flour sold for \$6.25 per barrel. How many bushels of wheat at $83\frac{1}{3}$ cents could the agent buy?

22. What is an agent's commission for collecting \$1,264.50 at $1\frac{3}{8}\%$? What amount should be sent to the principal?

23. An agent is to pay a debt of \$836 for a merchant. What amount should the merchant remit, if the agent's commission is $\frac{1}{4}\%$?

24. An agent's commission on an investment at $1\frac{5}{8}\%$ was \$52. What amount should be sent him to cover investment and commission?

25. What is the commission on a sale of \$954.80 at 3%? If the proceeds include an investment and the commission on it at 2%, what is the commission? If the purchase were sold at an advance of 20%, and the agent's commission for selling were 3%, what amount should be remitted to the principal?

276. COMMERCIAL DISCOUNT.

1. Manufacturers and wholesale dealers usually issue schedules of the prices of their goods. These schedules are called **price lists**.

2. These price lists do not usually give the selling price, but give **bases** upon which **discounts** are made.

3. The deduction from the list price of goods is called **Commercial Discount**. It is usually computed in per cent.

4. As Commercial Discounts are made for different reasons, as the amount purchased, the time of payment, more than one discount is sometimes allowed.

5. When more than one discount is allowed, the first is computed upon the list price, the second upon the remainder, and so on.

6. The amount of a bill after all discounts are withdrawn is called the **net amount**.

277. PROBLEMS.

1. Bought a piano listed at $950. Discount 45%, and 5% for cash. What was the net amount?

2. If the list price had been $880, and the discounts 48% and 6%, what would the net amount have been?

3. Rented a piano, listed at $1,080, at $6 a month, with the agreement that the rent should be applied to the purchase price if I decided to buy it. Kept it 8 months, and bought it at a discount of 40, 8, and $4\frac{1}{2}$. What did I pay in addition to the rent?

4. Find the net amount of a bill of $1,524.60, the discounts being 16 and 4.

5. Find the net amount of a bill of $825, the discounts being 10, 4, and 2.

6. Which would you prefer, a single discount of 20%, or a discount of $16\frac{2}{3}$ and 4?

7. Which is greater, a single discount of 25%, or a discount of 20 and $6\frac{1}{4}$?

8. Find the net amount of a bill of $650, discounted at $18\frac{3}{4}$%.

9. What must be the second discount on the above bill to give the same net amount if the first discount were $12\frac{1}{2}$%?

10. Find the net amount of a bill of $2,500, discounted at 20, 10, and 5.

11. If the first and second discounts in the above bill were 25 and $6\frac{2}{3}$, what must the third be to give the same net amount?

12. The net amount of a bill discounted at 15 and 10 is $722.16. What is the gross amount of the bill?

13. Find net amount of a bill for $1,260.50 discounted at $8\frac{1}{3}$%.

14. What is the bill which, discounted at $5\frac{1}{4}$%, yields a net amount of $648.09?

15. The bill is $728.50; the net amount, $684.79. What is the rate of discount?

16. The amount of the bill is $800; the net amount, $608; the first discount, 20%. What is the second discount?

Find the net amount of the following bills:

17. $325.56 at 18 and 3.

18. $464.92 at 21 and 6.

19. $681.20 at 15, 5, and 3.

20. $760.10 at 20, 8, and 2.

21. $1,241.10 at 12, 8, and 4.

22. 200 yards of calico at 6 cents; 225 yards of muslin at 7 cents; 50 yards of broadcloth at $3.50; 80 yards of silk at $1.40; 95 yards of flannel at 39 cents; 72 yards of cashmere at 98 cents; 140 yards of gingham at 9 cents. The discounts were 16 and 5.

23. 30 sacks of flour at 86 cents; 120 pounds of coffee at 28 cents; 60 pounds of butter at 24 cents; 50 pounds of lard at 7 cents; 36 pounds of tea at 54 cents; 300 pounds of sugar at 6 cents; 75 pounds of cheese at 11 cents. Discounts, 20 and 3.

24. 864 bushels of corn at $26\frac{1}{2}$ cents; 1,040 bushels of oats at 18 cents; 1,500 bushels of wheat at 63 cents; 980 bushels of rye at 46 cents; 836 bushels of potatoes at 38 cents: 17 tons of timothy at $6.75. Discounts, $18\frac{3}{4}$ and 5.

25. To what single discount is a discount of 20 and 5 equal? 25 and 10? $16\frac{2}{3}$ and 20? 25, 10, and 4?

278. STOCKS, BONDS, BROKERAGE.

1. Commercial enterprises are often undertaken by associations consisting of a number of individuals.

2. If these individuals enter into an agreement by contract, the association is called a **Partnership**, and each individual is called a **Partner**. If they organize by the

election of such officers as a president, secretary, treasurer, and board of directors, and secure a charter, the association is called a **Corporation**, or **Stock Company.**

3. The Charter of a corporation determines its name, its object, the number of shares that shall comprise its capital stock, the method of managing its business, etc.

4. **Charters** are laws passed by legislative bodies, or are issued in accordance with law by some state officer.

5. Railroad, steamboat, or manufacturing corporations are illustrations of stock companies.

6. The shares are usually one hundred dollars each, and the document which certifies that a person owns one or more of these shares is a **Stock Certificate.** Such certificates are often called **Stock**, or **Stocks.**

7. The aggregate amount of the shares of a corporation is its capital stock.

8. The market value of stock is the amount for which it will sell. It is usually indicated by the number of dollars for which a single 100-dollar share will sell. Thus, when stock is quoted at 108, the price is $108 for a 100-dollar share.

9. The par value of stock is the number of dollars for which the certificate calls.

10. When the market value exceeds the par value, the stock is at a **Premium.** When the two values are equal, the stock is at **Par.** When the market value is less than the par value, the stock is at a **Discount.**

11. When a corporation is prosperous, its income exceeds its expenses; it then has a **Net Income.**

12. A **Dividend** is a division of the net income among the stockholders.

13. An **Assessment** is a sum levied upon the stockholders to meet expenses not otherwise provided for.

14. Dividends and assessments are always computed at some per cent of the par value.

15. Stock certificates yield an income to their owners only when dividends are declared.

16. **Bonds** are notes issued by corporations and secured by mortgages on its capital stock. They bear a specified rate of interest.

17. The most common forms of bonds are government bonds, railroad or city bonds, etc.

18. A Broker is a person whose business is the purchase and sale of stocks and bonds.

19. The broker's commission is called **Brokerage,** *and is reckoned upon the par value of the stocks or bonds.*

How does it differ from the compensation of the commission merchant?

20. What is the discount or premium when stocks or bonds are sold at 94? 62? 108? $43\frac{1}{2}$? $217\frac{1}{4}$? $65\frac{1}{3}$? $186\frac{2}{5}$? $17\frac{1}{8}$? $329\frac{4}{5}$?

21. What is the market price when stocks or bonds are sold at 7% discount? $16\frac{1}{3}$% premium? 100% premium? $43\frac{1}{3}$% discount? 260% premium? 1000% premium? $\frac{1}{2}$% discount? $16\frac{2}{3}$% discount?

22. Be ready to recognize each of the general problems of percentage in the following problems.

279. PROBLEMS.

1. What is the cost of 25 railroad shares at 92, brokerage $1\frac{1}{8}$%?

SHORT FORM. $\frac{\$100 \times 25 \times 187}{200}$.

2. What is the income from the above stock, if it yields an annual dividend of $4\frac{1}{2}$%?

SHORT FORM. $\frac{\$100 \times 25 \times 9}{200}$.

3. A broker bought for his principal 16 United States 500-dollar bonds, bearing 4% interest, at 102.

(*a*) If the brokerage was 2%, what was the entire cost?

(*b*) What was the income from the bonds?

(*c*) The first year's income was what per cent of the entire cost? Show short forms for *a* and *b*.

4. A bought 40 shares of bank stock at 140. If the bank declared semi-annual dividends of 4%, what was the annual income from the stock? What rate per cent of his investment did he receive annually?

5. A invests $36,000 in railroad stock at 90. It yields 2% semi-annual dividends. What is his annual income from the stock?

SHORT FORM. $\dfrac{\$36000 \times 100 \times 4}{90 \times 100}$.

6. Find the brokerage on the following purchases:
28 shares bank stock at 125, brokerage $1\frac{3}{8}$%.
148 shares railroad stock at 76, brokerage $2\frac{1}{2}$%.
86 city bonds, brokerage 3%.

NOTE. Brokerage is reckoned on what?

7. What is the rate per cent of income from 4% bonds bought at 80? From 6% bonds bought at 120? From 5% bonds bought at 105? From $4\frac{1}{2}$% bonds bought at 90?

Find the cost of the following shares:

8. 300 Atchison, Topeka, and Santa Fé R. R. at 14, brokerage $\frac{1}{8}$%.

9. 200 Chesapeake & Ohio at $14\frac{7}{8}$, brokerage $\frac{1}{8}$%.

10. 8,075 Chicago Gas Co. at $57\frac{5}{8}$, brokerage $\frac{1}{10}$%.

11. 16,340 Chicago, Milwaukee, & St. Paul R. R. at $75\frac{1}{4}$, brokerage $\frac{1}{10}$%.

12. 5,948 Chicago, Rock Island, & Pacific R. R. at $63\frac{3}{8}$, brokerage $\frac{1}{4}$%.

13. 276 Lake Erie & Western R. R., preferred, at $70\frac{1}{8}$, brokerage $\frac{1}{8}$%.

14. 1,300 Illinois Central R. R. at 93, brokerage $\frac{1}{8}\%$.

15. 600 Philadelphia & Reading R. R. at $13\frac{5}{8}$, brokerage $\frac{1}{10}\%$.

16. 9,164 Chicago, Burlington, & Quincy at $72\frac{3}{8}$, brokerage $\frac{1}{8}\%$.

17. 1,154 Western Union Telegraph Co. at $82\frac{5}{8}$, brokerage $\frac{1}{7}\%$.

18. 250 Wabash R. R. at $6\frac{1}{8}$, brokerage $\frac{1}{4}\%$.

19. 624 N. J. Central R. R. at 102, brokerage $\frac{1}{10}$.

20. The total cost of 8700 shares Missouri Pacific is \$185,745, brokerage $\frac{1}{10}\%$. What is their rating?

21. How are Louisville and Nashville shares quoted when 4,365 shares cost me \$171,762.75, brokerage $\frac{1}{10}\%$?

22. How many shares of bank stock at 140, brokerage $\frac{1}{8}\%$, can be purchased for \$676,243.25?

23. At what rate must 6% bonds be purchased to yield annually 5% of the investment?

NOTE. The question is, 6% of the par value is 5% of what per cent of the par value; or. 6 is 5% of what?

$$\text{Form.}\quad \frac{6 \times 100}{5}.$$

24. At what rate must 4% bonds be bought to yield annually 5% of the investment? $4\frac{1}{2}\%$ bonds to yield 6%? 7% bonds to yield 5%? 8% bonds to yield $5\frac{2}{3}\%$?

25. Which is the better investment, 5% bonds at 95, or 6% bonds at $102\frac{1}{2}$, if the brokerage on the first is $1\frac{1}{2}\%$, and on the second $1\frac{2}{3}\%$?

$$\text{Form.}\quad \frac{5}{96\frac{1}{2}} = \frac{10}{193} = 10.00 \div 193 = ?$$

26. Tell which is the better investment in each of the following cases:

(*a*) $4\frac{1}{2}\%$ bonds at 92, brokerage $1\frac{1}{8}\%$, or $5\frac{1}{2}\%$ bonds at 106. brokerage $1\frac{3}{4}$.

(*b*) 8% bonds at 124, brokerage $2\frac{1}{4}$%, or $6\frac{1}{2}$% bonds at 110, brokerage $1\frac{7}{8}$%.

(*c*) 3% bonds at 62, brokerage $\frac{7}{8}$%, or 10% bonds at 185, brokerage $1\frac{3}{8}$%.

27. A has a farm of 240 acres, which yields him an annual rental of $\$5\frac{1}{4}$ an acre. A real estate agent sells it for $75 an acre, charging him 3% commission. Reserving $43.33, A invests the net proceeds in insurance stock at 83, brokerage $\frac{1}{3}$%. His annual income is increased $203. What is the rate per cent of the semi-annual dividends?

28. A sells 42 100-dollar school bonds bearing 7% interest, at 5% premium. The brokerage was $39. He sent the net proceeds to a Denver broker to invest in silver-mine stock at 45% premium, brokerage 2%. This stock yielded a semi-annual dividend of 12%. How much was his annual income increased? What rate per cent did the first broker charge him? What was the unused balance?

29. If bonds bought at $16\frac{2}{3}$% discount pay 10% on the investment annually, what rate of interest do they bear?

30. If stocks bought at 125 pay 8% annually, what is the rate per cent of their semi-annual dividends?

31. If 6% bonds pay annually $4\frac{2}{3}$% of their cost, were they bought at a premium or a discount? Give the rate per cent.

32. If 7% bonds yield annually $17\frac{1}{2}$% of their cost, what was the rate of discount?

33. An electric light company, whose capital stock was $50,000, earned above all expenses $3,212.50 during the first half of the year. If it passed $212.50 to the repair fund, what semi-annual dividend can it declare? What will one receive who owns $2,150 worth of the stock?

34. A manufacturing company, whose capital stock was $250,000, bought new machinery worth $4,000, and levied an assessment on its stockholders to meet the expenses. What

was the rate per cent of the assessment? What did A pay, who owned $7,800 worth of stock?

35. A milling company, whose capital stock was $30,000, declared a semi-annual dividend of $4\frac{1}{2}$%, and passed $864.50 to a reserve fund. What were the net earnings?

36. How many shares of Illinois Central can be bought for $7,087 at 7% discount, brokerage $\frac{1}{4}$%?

37. What amount must be invested in U. S. 4's, at $18\frac{1}{2}$ premium, to secure an annual income of $600, brokerage being $\frac{1}{8}$%? What per cent of the investment is the income?

38. How many bonds at $83\frac{1}{3}$, brokerage $\frac{1}{6}$%, can be purchased for $5,761.50?

39. How much must I invest in bank stock, paying a semi-annual dividend of 4%, and selling at 160, brokerage $\frac{1}{4}$%, to yield an annual income of $320?

40. Bought 75 shares R. R. stock at 28 and sold it at $30\frac{1}{4}$, brokerage $\frac{1}{8}$% in each case. What was the profit?

41. Sold shares at a discount of $1\frac{1}{2}$% which I had purchased at a premium of $\frac{1}{8}$%, losing $600, brokerage being $\frac{1}{4}$% on the sale and $\frac{1}{8}$% on the purchase. How many were there?

280. TAXES.

1. In order to pay the expenses of the government, maintain public schools and charitable institutions, build bridges and roads, supply towns with water and light, and meet other expenditures for the common good, the State collects money from its citizens.

2. Money collected for the purposes named is called a **Tax.**

A Tax is a sum of money levied by authority of law upon persons and property for public purposes.

3. Taxes may be levied by the General Government, by a State, county, city, township, or school district.

NOTE. The method of taxation employed by the General Government will be discussed under Duties and Imposts.

4. Many States levy a tax upon each voter without regard to the amount of property that he owns. Such a tax is called a **Poll Tax.** It is usually a small amount, rarely exceeding two dollars a year.

5. Property may be either Personal or Real Estate.

Personal Property consists of movables, such as money, securities, household goods, cattle, etc

Real Estate is that form of property which consists of lands and improvements put upon them.

6. A **Property Tax** is a tax assessed upon property. It may be general or special.

NOTE. An example of a special tax is a paving tax. Give other examples.

7. General taxes are usually assessed and collected as follows:

The State Legislature determines by its appropriation bills the amount to be expended for State purposes. A State officer, usually the Auditor of Public Accounts, ascertains the amount of taxable property in the State from the reports of certain officers, called assessors, who are elected by the people in townships or other districts of territory. He thus discovers what each dollar's worth of property must pay.

The same thing is done in each of the smaller political districts, — the county, town, etc.

These several rates of taxation all find their way to the proper officer, who calculates the amount of tax each individual and piece of real estate must pay, and puts it into a book. This book is given to a collector, who collects the tax and returns it to another officer called a treasurer.

281. PROBLEMS.

1. If the taxable property of a town is $568,324, and the rate is $1\frac{1}{8}$%, what is the whole amount of the tax?

NOTE. The rate is often expressed by the number of mills on each hundred dollars.

2. If the taxable property of a town is $329,864, and the tax to be raised is $4,123.30, what is the rate per cent of taxation? How many mills on each hundred dollars? on each dollar?

3. The tax in a city is $35,825. The rate is $2\frac{1}{2}$%. What is the assessed valuation?

4. The taxable property in a city is $5,864,528. The entire tax is $108,809.24. There are 4,120 persons subject to a poll tax of $1.50 each. What amount must be raised by a tax on property? What is the rate per cent? How many mills on each hundred dollars? What is A's tax, whose personal property is assessed at $964.80, his real estate at $2,160, and who pays a poll tax?

5. Calculate the tax of each of the following, the rate being $1\frac{3}{4}$%, each paying a poll tax of $1.25:

(*a*)	Personal	property,	$736.40;	real estate,	$2,483.00
(*b*)	"	"	$1,126.84;	" "	$5,812.40
(*c*)	"	"	$5,824.90;	" "	$12,960.25
(*d*)	"	"	$12,960.65;	" "	$42,875.49
(*e*)	"	"	$439.62;	" "	$2,974.38

6. Find the assessed value of property that pays $46.23 at $1\frac{2}{3}$%; that pays $75.46 at $2\frac{3}{4}$%; that pays $86.17 at $1\frac{3}{4}$%; that pays $483.60 at $1\frac{7}{8}$%, and includes one poll tax at $1.50.

7. The expenses of a school district for one year are $2,332.40. The collector's compensation is 2% of the amount collected. The taxable property of the district is $272,000.

What is the amount of the levy? What is the collector's commission? What is the rate per cent of taxation?

NOTE. Compare "buying problems" in Commission.

8. The net amount of a tax collection is $4,750.60. If the collector's commission was 2½%, what was the whole amount collected? If the rate of taxation was 21 mills on a dollar, what was the amount of taxable property?

9. Net collection, $5,824.80; collector's compensation, 2%. What was the amount of his commission? What amount did he collect?

10. Collector's commission, $348.80; rate of commission, 1¾%. What amount should he turn over?

11. Amount collected, $53,860.40; amount not collected, $1,620.12; collector's commission, 2%; rate, 13 mills on a dollar. What was the assessment?

12. The State tax is 61 mills on $100. The local tax is 18 mills on $1.00. What is the total tax on property assessed at $1,560?

13. The assessed valuation being $1,324,000, and the rate of taxation being 1¼%, what is the collector's commission at 2% if he collect all but $625 of the tax?

14. The rate of taxation is 8½ mills on the dollar. What is the acre valuation of a farm of 320 acres upon which the tax is $163.20?

282. UNITED STATES REVENUE.

1. The money necessary to meet the expenses of the United States Government is chiefly derived from a tax imposed upon goods imported from foreign countries, called **Customs or Duties**, and from a tax imposed upon certain articles manufactured within the country, called **Excise Taxes**.

2. Duties are collected at certain cities designated by law, and called Ports of Entry. Each port of entry con-

tains a custom-house. The collection of duties is under the direction of an officer called Collector of the Port, appointed by the President and confirmed by the Senate of the United States.

3. These Duties are either **Specific** or **Ad Valorem.**

Specific duties are duties assessed upon imported goods with reference to their quantity, and not to their value. Such duties are not computed by percentage.

Ad valorem duties are duties assessed upon imported goods with reference to their cost in the countries from which they are brought.

4. Importers submit an invoice which shows the cost of each article imported in the country where it was purchased.

5. A second source of revenue to the United States Government is a tax imposed upon spirituous liquors manufactured within the country.

6. A third source of revenue is the sale of public lands.

7. On certain kinds of imports an allowance called **tare** is deducted for the weight of the cases containing the goods, for breakage, leakage, etc.

283. PROBLEMS.

1. A dealer imported 65 watches invoiced at $28. What was the duty at 25%?

2. What is the duty at 15% on 15 oil paintings, averaging $2,350 each, and on seven pieces of statuary, averaging $1,750 each?

3. What is the duty on 1 gross of brushes at 40%, invoiced at $7.85 a dozen?

4. The duty on lace is 60%. What was the number of yards in an importation upon which the duty was $624, the lace being invoiced at 80 cents a yard?

5. What is the rate of duty upon musical instruments, when the duty on the following importation amounted to $1,679.40?

24 violins,	invoiced	at	$42?
7 pianos	"	"	$324?
12 flutes	"	"	$38?

6. Find the duty on the following importation:
$1,575 worth of sponges at 20%.
285 pounds of spices at 4 cents a pound.
$5,000 worth of leather at 10%.

7. What is the duty on
2,650 pounds of cheese, duty 5 cents a pound?
3,650 pounds of tin plate, duty $2\frac{2}{10}$ cents a pound?
4,200 pounds of rice, duty 2 cents a pound?

8. What is the duty at 2 cents a pound on 300 cases of starch, each containing 50 pounds?

9. What is the duty on 360 yards of silk, invoiced at $1.12, at 50% ad valorem?

10. Find the duty on 1,200 yards of Irish linen, invoiced at 35 cents, at 32% ad valorem?

11. What is the duty on one gross toilet bottles, invoiced at $3 a dozen, at $33\frac{1}{3}$% ad valorem, with an allowance of 5% for breakage?

12. What is the specific duty on 50 bags of rice, each containing 150 pounds, at 2 cents a pound, the tare being 3%?

13. What is the specific duty on 600 pounds almonds, at 5 cents a pound?

14. Find the duty on 624 yards Brussels carpet ($\frac{3}{4}$ yard wide), invoiced at 58 cents, the ad valorem duty being 40%, and the specific duty 28 cents a square yard.

15. What is the duty on 8 dozen watch movements, invoiced at $6.80 each, the duty being 25%?

16. Find the duty on 24,000 pounds of bituminous coal at 45 cents for a long ton.

17. Find duty on 6 lb. 7 oz. 8 pwt. of a drug which cost 15 cents an ounce, the duty being 33% ad valorem.

18. Find duty on 4 dozen watch-cases averaging 2 oz. 12 pwt. 18 gr., invoiced at $1.36 an ounce, the duty being 25%.

19. Find duty on 24 cases cloth, each containing 36 yards, invoiced at $1.38, duty being 55%.

20. Find duty on 368 pounds cork, duty being 15 cents a pound, and tare 11%.

284. INSURANCE.

1. Houses are liable to be destroyed by fire or tornadoes, ships to be wrecked, and persons upon whom others are dependent to be injured or to die.

2. Because of these facts companies are organized that, for a certain amount, agree to pay for property thus destroyed, to allow a person a stated amount if injured, or to pay to his heirs a fixed sum in case of his death. Such companies are called Insurance Companies.

3. **Insurance** is security against financial loss on account of the destruction of property, or by the injury or death of a person.

4. The different kinds of insurance receive their names from the dangers against which they offer insurance:

5. The common forms of insurance are:

(*a*) Fire and Lightning.
(*b*) Tornado.
(*c*) Marine.
(*d*) Accident.
(*e*) Life.

6. The company taking the risk is called the Insurer, or Underwriter.

7. The property or person, upon whom the risk is taken, is insured.

8. A **Policy** is a contract made between the insurer and the person securing the insurance.

9. The **Premium** is the amount paid to the insurer for assuming the risk.

10. The underwriters assume the risk for the length of time specified in the policy.

11. In Property Insurance the premium is estimated at a certain per cent of the amount of the risk for a specified time.

12. In Accident Insurance the premium is estimated at a certain amount for the protection afforded for a given time.

13. In Life Insurance the premium is estimated at a certain amount per year for each thousand dollars of insurance, and varies with the age of the insured.

285. PROPERTY INSURANCE.

PROBLEMS.

1. What is the premium on the following policies:

(*a*) \$2,500 for 3 years, at $\frac{1}{2}$% a year.
(*b*) \$3,650 " 5 " $\frac{1}{4}$% "
(*c*) \$4,680 " 2 " $\frac{2}{3}$% "
(*d*) \$8,250 " 1 year, at $1\frac{1}{4}$% "

2. Find the annual rate per cent of the premium on the following policies:

	Amount.	Premium.	Time.
(*a*)	\$3,600	\$60	5 years.
(*b*)	\$4,250	\$68	4 "
(*c*)	\$8,400	\$315	3 "
(*d*)	\$24,600	\$1,230	2 "

3. Find the amount insured in each of the following policies:

	Premium.	Annual Rate.	Time.
(*a*)	\$68.24	$\frac{4}{5}$%	1 year.
(*b*)	\$750	$1\frac{1}{4}$%	3 years.
(*c*)	\$872.73	$1\frac{1}{8}$%	5 "
(*d*)	\$463.40	$2\frac{1}{3}$%	2 "

4. A schoolhouse worth \$32,400 is insured for $\frac{2}{3}$ of its value at $\frac{1}{2}$% annually for 3 years. Find the premium.

5. A ship worth \$52,000 was insured for $\frac{3}{4}$ of its value at $2\frac{1}{2}$%. The cargo, worth \$8,640, was insured for $\frac{2}{3}$ of its value at 3%. What was the whole premium?

6. Paid \$290.19 for insuring a cargo worth \$13,656. What was the rate per cent of the premium?

7. My house being destroyed by fire, I received from the underwriters \$2,760, which was $\frac{2}{3}$ of the amount of the policy. Insurance was $\frac{3}{4}$ of value of house; the premium was \$66.24. Find the rate, and the value of house.

8. A factory worth \$75,000 was insured for $\frac{4}{5}$ of its value by an insurance company at $2\frac{1}{3}$%. Not wishing to carry the entire risk, the company reinsured $\frac{1}{4}$ of the risk at $1\frac{7}{8}$%, $\frac{1}{3}$ of the risk at $2\frac{1}{4}$%, and $\frac{1}{2}$ of the remainder at 2%. What was the amount of the premium remaining for the first company?

LIFE INSURANCE.

1. Find annual premium of a policy of \$8,000 at \$17.54 a thousand.

2. Find annual premium on an endowment policy of \$10,000 at \$85.60 a thousand.

3. In 5 years I have paid premiums amounting to \$472.50 on my life policy of \$5,000. What is the annual premium for \$1,000?

4. For 8 years I have made bi-monthly payment of \$7.86 as premiums on my policy in a Mutual Insurance Company. What is the aggregate? The annual premium is \$15.72 per \$1,000. What is the amount of my policy?

5. A person carried a policy for \$6,000 for 9 years, paying annual premiums of \$31.80 per \$1,000. At the end of that time he surrendered his policy for 23% of what he had paid. What did he receive?

6. A person insured his life for \$7,000 at an annual premium of \$36.40 per \$1,000. After making 7 payments he died. How much more did his heirs receive than he had paid?

7. A man carried a 20-year endowment policy for \$2,000 at an annual premium of \$44.70 per \$1,000. His dividends aggregated $16\frac{2}{3}\%$ of the premiums. How much more did he receive at the maturity of the policy than he had paid?

8. If I pay an annual premium of \$69.80 per \$1,000 on a 10-year endowment policy of \$4,500, what amount has been paid at the maturity of the policy?

286. MISCELLANEOUS PROBLEMS.

1. Change $\frac{1}{125}$ to a decimal fraction.

2. When it is 45 minutes past 8 A. M. at Buffalo, 78° 55′ W., what is the time at Salt Lake City, 112° 6′ W.?

3. Reduce $\frac{2\frac{1}{3} \times 5\frac{1}{2}}{3\frac{3}{4} \div 1\frac{7}{8}} \times \frac{\frac{3}{5} + \frac{5}{8}}{\frac{11}{12} - \frac{2}{3}}$.

4. Bought 36,824 bushels of oats at $18\frac{3}{4}$ cents and sold them at a gain of $12\frac{1}{2}\%$. What did they bring?

5. Settled a bill of \$436.50 at a discount of 16 and 5. What was the amount paid?

6. Find tax on a real estate assessment of \$5,640 and a personal property assessment of \$3,824, at 15 mills on the dollar.

7. What is the commission, at $2\frac{1}{2}\%$, on the sale of 320 acres of land at \$72.50 an acre?

8. Find the premium on a policy insuring a house worth \$4,860 for $\frac{3}{4}$ of its value at $1\frac{1}{3}\%$.

9. Divide .00864 by 2.7.

10. Bought a stock of goods for \$8,645 and sold them for \$9,748. What per cent did I gain? (Approximate.)

11. How many silver table-spoons, each weighing 1 oz. 7 pwt., in a package weighing 2 lb. 8 oz. 8 pwt.?

12. Find the cost of 8 $100-bonds, bearing $4\frac{1}{2}$% interest, which yield an annual income of $6\frac{1}{4}$% of the investment.

13. The difference of time of two places is 3 hr. 40 min. 30 sec. If the place having the later time is in 15° 24′ 12″ east lon., what is the longitude of the other?

14. What is the duty on drugs weighing 6 lb. 8 oz. 5 dr. 2 sc., invoiced at 12 cents a dram, the duty being 35%?

15. What is the area of a circular pond, whose diameter is 42 rods?

16. A commission merchant sold for me 8,624 bushels of corn at $28\frac{1}{2}$ cents. After deducting his commission at $3\frac{1}{2}$%, and paying charges amounting to $38.75, he invested the remainder in oats at $16\frac{1}{2}$ cents a bushel, first withdrawing his commission at $2\frac{1}{2}$ %. How many bushels did he buy?

17. Find the cost at $5.50 a cord of a pile of wood 8 feet high, 4 feet wide, and 36 feet long.

18. Sold a lot of goods for $650.40. After paying $2\frac{1}{2}$% commission to the auctioneer, I find that I have made a net gain of 24%. What did they cost me?

19. What is the capacity in barrels ($31\frac{1}{2}$ gallons) of a cylindrical cistern whose diameter is 10 feet and depth 12 feet? ($\pi = \frac{22}{7}$. Employ cancellation.)

20. What is the rate of income on 4% bonds bought at 16% discount?

21. How far will a train go in 7 hours, if its average speed is 36 mi. 124 rd. 4 yd. an hour?

22. What is the area of a triangular piece of land whose base is 124 rods and whose altitude is 236 rods?

23. The valuation of a town is $2,624,800. The tax levy is $32,810. Give the rate in mills.

24. What is gained by buying 24 shares of Illinois Central at $8\frac{3}{4}$% discount and selling them at $93\frac{1}{2}$, brokerage $\frac{1}{8}$% in each case?

25. A owns $\frac{2}{5}$ of a mill, and sells $\frac{3}{8}$ of his share to B, who then owns $\frac{1}{5}$ of the mill. What part of the mill did he own before his purchase? B's share is now what part of A's?

26. $4\frac{1}{2}$ inches is what decimal of a rod?

27. What is the value of 8 sills, each $10'' \times 10''$, and 16 feet long, at $19.50 a thousand feet?

28. A bill, having been discounted 25 and 5, amounts to $855. What is its face?

NOTE. Employ cancellation when possible.

29. At what advance must goods be marked so that the merchant may discount the marked price 20 and 5, and still make 14%?

30. Find premium on a policy insuring a house worth $6,000 for $\frac{3}{4}$ of its value, for a term of 5 years, the annual rate being $\frac{1}{3}$%.

31. If 15 men working 8 hours a day for 25 days can do a piece of work, how many hours a day must 18 men work to complete the same job in 20 days?

32. Divide 133574 by .000329.

33. Find the cost of lumber and posts for fencing the E. $\frac{1}{2}$ of S. E. $\frac{1}{4}$ of a section with a 4-board fence, posts costing $18\frac{1}{2}$ cents, and lumber $19.?5 per thousand.

34. How many bushels of oats will a bin contain that is 10 feet high, 8 feet wide, and 32 feet long?

35. How many shares at 83, brokerage $\frac{1}{8}$%, can be purchased for $4,571.88?

36. Sugar was selling at 20 pounds for a dollar. The price advanced $16\frac{2}{3}$%. How many pounds less could then be bought for a dollar?

37. Sent to a commission merchant \$894 to be invested after withdrawing his commission at 3%. What is the amount of his commission?

38. What must be the depth of a cylindrical vessel whose diameter is 6 inches, to hold a gallon?

39. A can do a piece of work in 7 days, B in 8 days, and C in 9 days. In what time can they do it, working together?

40. If A worked 3 days and B 4 days, in what time could C finish the work?

41. $\frac{1}{3}$ of A's money equals $\frac{3}{4}$ of B's. If A has \$65 more than B, how much has each?

42. Bought a house for \$5,680. After renting it for one year at \$41 a month, and spending \$121 for taxes and repairs, I sold it for \$6,000. What is the per cent of gain?

43. What is the commission on the following sales?

3,640 bushels of oats at 18 cents. Commission 2%.
2,500 bushels of wheat at $53\frac{1}{2}$ cents. Commission $2\frac{1}{4}$%.
3,600 bushels of corn at $27\frac{1}{2}$ cents. Commission $2\frac{1}{2}$%.

44. $\frac{3}{4}$ of $\frac{7}{12}$ is what per cent of $\frac{5}{6} \div \frac{3}{8}$?

45. How many cubic yards of earth in 150 loads averaging 24 cu. ft. 1,200 cu. in.?

46. $.023907 \div .0001839 = ?$

47. What is the area of a circular field whose diameter is 45.12 rods?

48. Find the cost of the Brussels carpet for a room $20' \times 24'$, the strips running the long way, 8 inches being lost on each strip in matching, and costing \$1.12 a yard.

49. A wood-house is $16' \times 16'$. If filled with wood to a height of 7 feet, what is the wood worth at \$5.50 a cord?

50. By selling an article for \$565, I gain 13%. What did it cost me?

51. Find the value of a pile of cordwood 7 feet high and 36 feet long, at \$5.25 a cord.

287. INTEREST.

1. Many persons have occasion to use money when they do not have it at hand. If they are responsible, they can obtain the money from others by guaranteeing its return and by agreeing to pay a specified sum for such service. Such persons are called **borrowers.**

2. The amount paid for such service is called **Interest.** Interest is compensation for the use of money.

NOTE. Money paid for labor is usually called *wages;* when paid for the use of houses and lands, it is called *rent,* etc.

3. The quantity of money upon which interest is paid is the **Principal.**

4. The **Amount** is the sum of the principal and interest.

5. Wages and rent are usually estimated at a specified amount for a day, a month, or a year. Time, consequently, is an essential element in calculating such compensation. Similarly, the longer the time that one uses the money of another, the larger should be the compensation.

6. The ordinary unit of time for estimating interest is one year. The **Rate of Interest** is a specified number of hundredths of the principal, as a compensation for its use for one year; hence, interest is one of the applications of percentage.

7. When money is loaned at 6%, it is understood that the compensation is 6 hundredths of the principal, for its use for one year, unless otherwise specified.

8. Interest is one of the most common of the applications of percentage. Since the time for which money is borrowed may vary from a few days to many months or years, it is the most difficult of these applications.

9. Remember that problems in interest involve :

(1) A sum upon which interest is calculated; (2) A rate of interest, — a number of hundredths of that sum for its use for a unit of time, usually a year; (3) A specified time for which the given sum is to be loaned.

288. ORAL PROBLEMS.

1. What is the interest on $800 for one year at 6%? at 8%? at 9%? at 4%? at 12%? at 10%? at 7%?

2. What is the interest on $600 for 2 years at 3%? at 5%? at $7\frac{1}{2}$%? at $4\frac{1}{3}$%?

Note. First find the interest for one year.

3. Substitute 3 years for 1 year in the first problem.

4. What is the interest on $900 for 2 years and 6 months ($2\frac{1}{2}$ years) at 6%? at 4%? at 3%? at 8%?

5. What is the interest on $400 for 3 years, 3 months, at 6%? $4\frac{1}{2}$? 5%? 8%? 3%? 10%?

289. There are many methods of calculating interest. Persons whose business requires a considerable amount of such work supply themselves with books containing "Interest Tables." With their assistance it is very easy to perform any problem in interest.

For the ordinary person one good method is all that is needed. Several methods will be presented, but proficiency in two is recommended. These are the "General Method" and the first "Six Per Cent Method."

290. WRITTEN PROBLEMS.

General Method.

Illustrative Problem. What is the interest on $750 for 2 yr. 7 mo. 15 d. at 6%?

FORM.

	$750			
	.06			
	$45.00	= interest for 1 yr.		
	2			
	$90	=	"	" 2 yr.
6 mo. = $\frac{1}{2}$ yr.	22.50	=	"	" 6 mo.
1 mo. = $\frac{1}{6}$ of 6 mo.	3.75	=	"	" 1 mo.
15 d. = $\frac{1}{2}$ of 1 mo.	1.875	=	"	" 15 d.
	$118.125	=	"	" 2 yr. 7 mo. 15 d.

RULE.

For finding interest by the general method:

1. ***Find the interest for one year at the given rate, and multiply the result by the number of years.***

2. ***Separate the months into divisors of 12, and find such a part of the interest for one year as each of these divisors is of 12.***

NOTE. It will be found more convenient sometimes to compare some of the divisors with others, as in the illustrative problem.

3. ***Separate the days into divisors of 30. Find the interest for one month, and proceed as in (2).***

NOTE. An arrangement of the days can be made sometimes similar to that of the months, as suggested in the preceding note.

291. In the following problems follow the above method. Count 30 days for a m 'th. Reject all results below tenths of mills. Find the amount in each case.

	Principal.	Rate.	Time.
1.	$480	4%	3 yr. 5 mo. 10 d.
2.	$650.40	5%	2 " 7 " 15 "
3.	$864.36	6%	4 " 8 " 6 "
4.	$1275.86	8%	1 " 9 " 5 "
5.	$1464.29	10%	2 " 1 " 3 "
6.	$2580.47	7%	3 " 2 " 10 "
7.	$3600	$4\frac{1}{2}$%	5 " 10 " 15 "
8.	$4580	$5\frac{1}{2}$%	4 " 6 " 2 "
9.	$5000	$6\frac{1}{3}$%	5 " 3 " 1 "
10.	$6840.75	$7\frac{1}{2}$%	6 " 11 " 19 "
11.	$490.92	8%	1 " 3 " 6 "
12.	$3794.08	9%	2 " 4 " 7 "
13.	$892.45	5%	3 " 4 " 8 "
14.	$1234.16	6%	2 " 3 " 12 "

NOTE. Correct solutions may differ by a few cents in the results because of different divisions of the time.

	Principal.	Rate.	Time.
15.	$5871.48	7%	4 yr. 1 mo. 20 d.
16.	$89.26	3%	6 " 8 " 25 "
17.	$13.51	5½%	5 " 10 " 16 "
18.	$1.05	6⅓%	8 " 2 " 24 "
19.	$10874.80	4%	5 " 10 "
20.	$916.15	8%	9 " 18 "
21.	$1371.49	5%	1 " 11 " 22 "
22.	$1641.04	9%	3 " 2 " 29 "
23.	$3200	10%	2 " 7 " 1 "
24.	$93.94	12%	10 " 4 "
25.	$16921.72	3½%	3 " 11 "
26.	$879.40	4%	2 " 4 " 10 "
27.	$783.12	5½%	3 " 8 " 15 "
28.	$1864.60	7%	1 " 7 " 18 "
29.	$1460.20	7%	4 " 10 " 21 "
30.	$21.25	6%	4 " 1 " 2 "
31.	$583.40	6%	5 " 3 " 16 "
32.	$1560	7½%	2 " 11 " 22 "
33.	$830.80	7½%	3 " 5 " 25 "
34.	$98.07	4⅓%	3 " 6 " 28 "
35.	$2654.90	4⅓%	6 " 7 " 7 "
36.	$3371.10	8%	5 " 5 " 11 "
37.	$640	3½%	4 " 4 " 4 "
38.	$756.50	3½%	5 " 3 " 8 "
39.	$833.12	4¼%	7 " 6 " 13 "
40.	$794.80	4¼%	8 " 8 " 7 "
41.	$1026.25	5%	9 " 1 " 14 "
42.	$1230	5%	4 " 2 " 16 "
43.	$1426.80	5½%	3 " 10 " 2 "
44.	$1862.60	6%	7 " 5 " 19 "
45.	$2045	6%	6 " 9 " 21 "

	Principal.	Rate.	Time.
46.	\$3870	$6\frac{1}{4}\%$	2 y. 11 mo. 20 d.
47.	\$4312	$6\frac{1}{4}\%$	3 " 11 " 25 "
48.	\$5680	7%	5 " 10 " 22 "
49.	\$6875	7%	6 " 24 "
50.	\$5892	7%	8 " 22 "
51.	\$66.40	7%	24 "
52.	\$83.70	7%	18 "
53.	\$168.10	8%	2 " 10 " 29 "
54.	\$234.90	8%	5 " 24 "
55.	\$361.12	9%	7 " 16 "
56.	\$432.90	10%	11 " 22 "
57.	\$563.08	10%	24 "
58.	\$839.16	10%	20 "
59.	\$748.90	12%	26 "
60.	\$2126.40	12%	12 "

292. Six Per Cent Methods.

(*a*) The Method by Aliquot Parts.

Note. An "aliquot part" of a number, as here used, is a part of a number which is expressed by a fraction whose numerator is 1.

1. Since any principal which bears interest at 6% per annum gains $\frac{6}{100}$ of itself in a year, it gains $\frac{1}{100}$ of itself in 2 months, and doubles itself in 200 months.

What part of itself will it gain in 100 months? in 50? 40? 20? 10? 2? $33\frac{1}{3}$? $66\frac{2}{3}$?

2. In computing interest, 1 month = 30 days; hence, 2 months = 60 days; hence, the interest on any principal for 60 days at 6% is $\frac{1}{100}$ of that principal.

Since 6 days is $\frac{1}{10}$ of 60 days, the interest for 6 days is $\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000}$ of the principal.

3. FACTS TO BE REMEMBERED.

Any principal at 6%

(1) Doubles itself in 200 months.

(2) Gains $\frac{1}{100}$ of itself in 60 days.

(3) Gains $\frac{1}{1000}$ of itself in 6 days.

4. *Illustrative Problem.* Find the interest on $860 for 4 yr. 7 mo. 24 d. at 6%.

FORM.

4 yr. 7 mo. = 55 mo.

		$860	
50 mo.	= $\frac{1}{4}$ of 200 mo.	$215	= interest for 50 mo.
5 "	= $\frac{1}{10}$ of 50 "	21.50	= " " 5 "
20 d.	= $\frac{1}{3}$ of 60 d.	2.866	= " " 20 d.
4 "	= $\frac{1}{5}$ of 20 "	.573	= " " 4 "
		$239.94	

Solve the following by the above method:

293. PROBLEMS.

	Principal.	Time.	Rate.
1.	$540	3 yr. 3 mo. 15 d.	6%.
2.	$650	4 " 8 " 20 "	"
3.	$1240.50	6 " 9 " 12 "	"
4.	$980.40	5 " 6 " 20 "	"

NOTE. 5 yr. 6 mo. 20 d. = $66\frac{2}{3}$ mo. = $\frac{1}{3}$ of 200 mo.

5.	$2450.72	8 yr. 4 mo.	"
6.	$6000	2 " 9 " 10 d.	"

NOTE. 2 yr. 9 mo. 10 d. = $33\frac{1}{3}$ mo. = $\frac{1}{6}$ of 200 mo.

7.	$8450	8 yr. 10 mo. 26 d.	"
8.	$1750.86	3 " 11 " 29 "	"
9.	$5826.25	4 " 9 " 19 "	"
10.	$349.46	5 " 1 " 1 "	"
11.	$850	6 " 2 " 13 "	8%.

Note. First find the interest at 6%, then divide the result by 6 and multiply the quotient by the number of per cent. Why? Or, add to the result ½ of itself. Why?

12. Prin., $1148.12. Time, 4 yr. 8 mo. 14 d. Rate, 9%.

13. Solve 1, 2, 3 at 7%.

14. Solve 4, 5, 6 at 5%.

15. Solve 7, 8 at 4½%.

16. Solve 9, 10 at 10%.

For further practice in this method use problems under Art. 291.

294. (*b*) The Method by First Finding the Interest on $1.00.

Prove the following statements:

1. The interest on $1.00 at 6% is ½ a cent a month, and $\frac{1}{10}$ of a cent for 6 days.

Method. Express the time in months and days. The interest on $1.00 at 6%, for any number of months and days, is ½ as many cents as there are months, and ⅙ as many mills as there are days.

Illustrative Problem.

Principal, $560.25. Time, 4 yr. 7 mo. 15 d. Rate, 6%.

4 yr. 7 mo. = 55 mo.

The interest on $1 for	55 mo.	= $0.275 cents.
" " " "	15 d.	= .0025 "
" " "	55 mo. 15 d.	= $0.2775 cents.

The interest on $560.25 is 560.25 × $0.2775 cents.

```
    $0.2775 cents.
     560.25
    .013875
    .05550
  16.650
 138.75
$155.469375
```

Note. Reject results below mills. Where more convenient, use first result as multiplier.

Perform Problems 1–11, Art. 293, by this method.

295. Method by changing Rate Per Cent for One Year to Rate Per Cent for the Whole Time.

1. Since in calculating interest it is common to consider 360 days as equal to a year,

Reduce the years to months, and to this result add the number of months.

Thus: 3 yr. 4 mo. = 40 mo. etc.

2. **Since there are 30 days in a month, 3 days equal $\frac{1}{10}$ of a month; hence, dividing the days by 3 reduces them to tenths of a month.**

3 yr. 7 mo. 18 d. = 43. 6 mo.; 5 yr. 9 mo. 23 d. = $69.7\frac{2}{3}$ mo.

3. Express the following periods of time as months and tenths:

(1) 2 yr.	2 mo.	7 d.		(4) 5 yr.	8 mo.	25 d.
(2) 3 "	5 "	9 "		(5) 8 "	10 "	29 "
(3) 4 "	11 "	16 "				

4. **12% a year is 1% a month; hence, if the time be expressed in months and tenths of months, the result will be the number of hundredths of the principal which the interest would be at 12% per annum. To find the corresponding number for any other rate per cent, divide this result by 12 and multiply the quotient by the number of ones in the rate per cent.**

5. **The interest upon any principal is then found by finding as many hundredths of it as the preceding result expresses.**

Illustrative Problem. Find the interest on $820 for 4 yr. 8 mo. 12 d. at 6%. 4 yr. 8 mo. = 56 mo.; 12 d. = .4 mo. The time = 56.4 mo. The interest is 56.4% of the principal at 12%, 4.7% at 1%, and 28.2% at 6%.

Note. It is sometimes more convenient to find the interest at 12% and then take such a part of the result as the given rate is of 12%.

Solve Problems 1–10, Art. **291**, by this method.

296. Review method of finding time between two dates in Compound Denominate Numbers.

NOTE. The following method is often employed: Write for the minuend the year, number of month, number of day in the month of the later date; and for the subtrahend, the corresponding numbers of the earlier date. Proceed as in subtraction of compound numbers, counting 30 days for a month.

Illustration.

Later date,	July 12, 1891 =	1891	7	12
Earlier date,	Sept. 21, 1886 =	1886	9	21
		4	9	21

The former method is preferred in this text.

PROBLEMS.

NOTE. The teacher should name the method for each problem.

1. What is the interest on $748.40, at 7%, from June 12, 1885, to Dec. 21, 1888?

2. What is the interest on $548.50, at 8%, from July 18, 1886, to May 5, 1890?

3. What is the interest on $93.80, at 5½%, from Dec. 7, 1885, to Oct. 1, 1892?

4. Find the amount of $2480, on interest at 6%, from Aug. 28, 1884, to Feb. 19, 1889.

5. Find the amount of $3728.30, on interest at 7%, from Feb. 29, 1888, to Jan. 14, 1892.

Find the interest on:

6. $469.12, at 6%, from June 8, 1887, to Aug. 15, 1890.
7. $48.16, at 7%, from Dec. 15, 1885, to May 3, 1888.
8. $907.92, at 7%, from May 7, 1890, to Sept. 19, 1892.
9. $1359.06, at 7%, from Oct. 24, 1888, to April 6, 1891.
10. $750.20, at 5%, from March 12, 1886, to Nov. 28, 1886.
11. $609.47, at 5%, from Aug. 16, 1891, to June 1, 1892.
12. $1936.82, at 4½%, from Jan. 29, 1892, to Oct. 14, 1892.

13. $207.49, at 4½%, from Nov. 3, 1883, to July 28, 1889.
14. $2382.75, at 8%, from Feb. 15, 1880, to Oct. 11, 1883.
15. $69.20, at 8%, from April 1, 1888, to Jan. 22, 1893.
16. $7512.36, at 8%, from July 30, 1888, to Dec. 9, 1888.
17. $169.17, at 7½%, from Sept. 7, 1885, to March 18, 1892.
18. $273.48, at 7½%, from Dec. 19, 1886, to Nov. 30, 1890.
19. $428.10, at 6%, from Feb. 23, 1887, to Aug. 5, 1891.
20. $491.73, at 6%, from Nov. 16, 1886, to Nov. 28, 1886.
21. $636.80, at 6%, from Jan 5, 1889, to Jan. 31, 1893.

Find the amount of:

22. $19.12, at 5⅓%, from Aug. 12, 1883, to June 10, 1887.
23. $729.13, at 5⅓%, from June 18, 1886, to Oct. 21, 1890.
24. $258.18, at 5⅓%, from Dec. 3, 1890, to May 1, 1892.
25. $371.29, at 5⅓%, from March 25, 1889, to July 29,1891.
26. $580.00, at 9%, from Jan. 16, 1885, to Sept. 10, 1889.
27. $412.31, at 9%, from Oct. 2, 1888, to March 28, 1893.
28. $7539.06, at 3½%, from Feb. 19, 1891, to Nov. 6, 1892.
29. $117.59, at 3%, from April 29, 1882, to Dec. 19, 1887.
30. $396.16, at 6%, from July 5, 1885, to Feb. 26, 1888.
31. $872.38, at 7%, from May 11, 1887, to Jan. 7, 1892.
32. $250.10, at 7½%, from Nov. 17, 1884, to Aug. 24, 1890.
33. $1536.81, at 6½%, from Sept. 23, 1889, to Jan. 17, 1893.
34. $2140.60, at 6%, from June 6, 1890, to April 23, 1894.
35. $16.31, at 8%, from Oct. 12, 1884, to March 5, 1889.
36. $589.76, at 7%, from Feb. 8, 1888, to Aug. 19, 1893.
37. $26824, at 4%, from Dec. 18, 1890, to Feb. 21, 1894.

297. The Method of finding Interest by Days.

1. Interest is sometimes computed by days, when the time is short. In such cases find the actual number of days by the following:

Method. The actual number of days from March 10 to July 15:

March,	21	days
April,	30	"
May,	31	"
June,	30	"
July,	15	"
	127	"

2. The second 6% method is most convenient for this purpose.

3. Since the interest on $1 for 6 days at 6% is 1 mill, if the number of days be divided by 6, the result will be the interest in mills on $1 at 6%. To find the interest at any other rate, find such a part of this result as the given rate is of 6%.

Illustrative Problem. Find the interest on $850, at 5%, from April 9 to July 12, same year.

TIME.		
21		$850.00
31		.015⅔
30		4.250
12		8.500
6) 94		.283
		.283
15⅔ mills = interest on $1 at 6%.		$13.316
	Less ⅙,	2.219
		$11.097

298. PROBLEMS.

	Principal.	Time.	Rate.
1.	$640.80	March 15, 1891, to June 24, 1891	6%
2.	$1254.25	June 24, 1891, to Sept. 18, 1891	7%
3.	$796.13	Aug. 12, 1891, to Dec. 27, 1891	5%
4.	$2958	May 17, 1891, to Aug. 29, 1891	4½%
5.	$4872.80	Sept. 28, 1891, to Jan. 5, 1892	8%

	Principal.		Time.	Rate.
6.	$86.97	Jan.	5, 1891, to June 16, 1891	9%
7.	$916.20	Oct.	12, 1891, to Feb. 2, 1892	10%
8.	$8712.91	Dec.	27, 1891, to April 30, 1892	7½%

299. ACCURATE INTEREST.

Since the common year contains 365 days, in counting the year as 360 days, each day is counted at $\frac{73}{72}$ of its real length as compared with a year. Hence the interest found by the 360-day method is $\frac{73}{72}$ of the "accurate" interest. To find the interest for a given number of days by the 365-day method, subtract $\frac{1}{73}$ of the result obtained by the 360-day method, and the difference will be the "accurate interest by days."

Find the accurate interest on:

1. $824.50 for 120 days at 6%.
2. $695.30 for 90 days at 7%.
3. $1275 for 180 days at 8%.
4. $2560 for 66 days at 5%.
5. $88.40 for 48 days at 6%.
6. $360 for 200 days at 7%.
7. $940 for 300 days at 6%.
8. $4875 for 96 days at 7%.

300. PARTIAL PAYMENTS.

$800. BOSTON, MASS., Jan. 8, 1892.

Six months after date, for value received, I promise to pay to James B. Rogers, or order, Eight Hundred Dollars, with interest at seven per cent per annum.

JOHN T. WALKER.

1. The above-written promise is a Promissory Note. Walker is the Maker, and Rogers the Payee. The $800 is the Principal.

2. From the foregoing, form a definition of a promissory note, and of maker and payee.

3. Promissory notes may be bought and sold like other forms of property. Some calculation is usually necessary to determine the value of a note. Why?

4. If James B. Rogers should sell the above note he would write his name on the back. This is called "indorsing the note in blank." It is an order to John T. Walker to pay it to the person who owns it at its maturity, July 8, 1892. The indorsement might indicate some person upon whose order the note is to be paid. This is a "special indorsement."

5. Under the laws of some States the above indorsements would make Rogers responsible for the payment of this note in case Walker should fail to do so. If he wishes to avoid such responsibility he adds the words "without recourse" after his name.

6. There are many forms of notes, but the differences are slight. If, instead of "six months after date," the words "on demand" occurred in the above note it would be called a Demand Note. Write one.

7. If, instead of "I promise," it read, "we or either of us promise," and two persons signed it, it would be a Joint-and-Several Note.

8. The place at which a note is to be paid is often indicated in the note. It will usually be a bank. In some States, notes, payable at banks, are not legally due until three days after their apparent maturity. Such time is called Days of Grace. Interest is charged for them if the note bears interest.

NOTE. If a note is payable to "bearer" it may be exchanged without indorsement. If made payable to a particular person, and does not contain the words "or order" or "or bearer," it is not negotiable.

9. Partial payments are sometimes made on notes and other evidences of indebtedness that bear interest. The

United States Supreme Court has prescribed a rule for finding their value at any time. It is called

301. THE UNITED STATES RULE.

I. ***The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due.***

II. ***If the payment exceeds the interest the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due.***

III. ***If the payment be less than the interest the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance as aforesaid.***

Illustrative Problem. A note of $850, bearing interest at 8%, and dated March 1, 1885, has the following indorsements:

Jan. 24, 1886.	$86.25.
March 12, 1887.	$72.00.
Jan. 5, 1888.	$153.50.
Feb. 20, 1889.	$265.80.

What was due March 1, 1890?

FORM.

	March 1, 1885			
			10 mo.	23 d.
$86.25	Jan. 24, 1886			
		1 yr.	1 "	16 "
$72.00	March 12, 1887			
			9 "	24 "
$153.50	Jan. 5, 1888			
		1 "	1 "	15 "
$265.80	Feb. 20, 1889			
		1 "		9 "
	March 1, 1890			

Principal $850

.08

$68.00 = interest for 1 yr.

6 mo. = $\frac{1}{2}$ of 1 yr.	\$34.00	= interest for 6 mo.
4 " = $\frac{1}{3}$ of 1 "	22.666	= " " 4 "
20 d. = $\frac{1}{6}$ of 4 mo.	3.777	= " " 20 d.
2 " = $\frac{1}{10}$ of 20 d.	.377	= " " 2 "
1 " = $\frac{1}{2}$ of 2 d.	.188	= " " 1 "
	\$61.008	= " " 10 mo. 23 d.
	850	= first principal.
	\$911.008	= amount, Jan. 24, 1886.
	86.25	= first payment.
	\$824.758	= new principal, Jan. 24, 1886.
	.08	
	\$65.9800	= interest for 1 yr.
1 mo. = $\frac{1}{12}$ of 1 yr.	5.498	= " " 1 mo.
10 d. = $\frac{1}{3}$ of 1 mo.	1.832	= " " 10 d.
6 " = $\frac{1}{5}$ of 1 "	1.099	= " " 6 "
	\$74.409	= " " 1 yr. 1 mo. 16 d.
Payment	\$72.00	less than interest.
	\$65.98	= interest for 1 yr.
6 mo. = $\frac{1}{2}$ of 1 yr.	\$32.99	= " " 6 mo.
3 " = $\frac{1}{2}$ of 6 mo.	16.495	= " " 3 "
15 d. = $\frac{1}{6}$ of 3 "	2.749	= " " 15 d.
9 " = $\frac{1}{10}$ of 3 "	1.649	= " " 9 "
	\$53.883	= " " 9 mo. 24 d.
	74.409	= previous interest.
	824.758	= second principal.
	\$953.050	= amount, Jan. 5, 1888.
2d payment \$72.00		
3d " \$153.50	225.50	2d and 3d payments.
	\$727.55	= 3d principal.
	.08	
	\$58.2040	= interest for 1 yr.
1 mo. = $\frac{1}{12}$ of 1 yr.	4.8503	= " " 1 mo.
15 d. = $\frac{1}{2}$ of 1 mo.	2.4251	= " " 15 d.
	\$65.4794	= " " 1 yr. 1 mo. 15 d.
	727.55	

	\$793.029	= amount, Feb. 20, 1889.
	265.80	= 4th payment.
	\$527.229	= 4th principal.
	.08	
	\$42.1776	= interest for 1 yr.
		\$3.514 = interest for 1 mo.
6 d. $= \frac{1}{5}$ of 1 mo.	.703	= interest for 6 d.
3 " $= \frac{1}{2}$ of 6 d.	.351	= " " 3 "
	\$43.231	= " " 1 yr. 9 d.
	527.229	= 4th principal.
	\$570.460	= amount due March 1, 1890.

NOTE. Payments less than the accumulated interest will rarely be made, since they do not diminish the interest-bearing portion of the debt.

1. A note whose principal is \$500, dated March 1, 1890, and bearing interest at 6%, has the following indorsements:

June 1, 1891, \$65.
Sept. 16, 1892, \$124.

What was due Jan. 1, 1894?

2. Principal, \$850. Date, May 10, 1891. Rate, 7%. Indorsements:

July 15, 1892, \$130.
June 1, 1893, \$46.
Dec. 12, 1894, \$380.

What was due May 10, 1895?

3. Principal, \$1,000. Date, Sept. 1, 1892. Rate, 8%. Indorsements:

March 12, 1893, \$75.
June 18, 1894, \$275.
March 15, 1895, \$360.

What was due Sept. 1, 1895?

4. Principal, \$1,200. Date, July 1, 1890. Rate $5\frac{1}{2}$%. Indorsements:

March 16, 1891, \$160.
June 12, 1892, \$320.
Aug. 5, 1893, \$500.

What was due July 1, 1894?

5. A note of \$1,800, bearing interest at 7%, and dated June 12, 1886, was indorsed as follows:

March 21, 1887,	\$183.50.
Oct. 12, 1888,	\$395.75.
May 10, 1890,	\$583.45.

What was due July 1, 1892?

6. \$1,250. BLOOMINGTON, ILL., Oct. 15, 1886.

Four years after date, for value received, we, or either of us, promise to pay to W. O. Davis & Co., or order, Twelve Hundred Fifty Dollars, with interest at 6% per annum.

JAMES T. RONEY.
JOHN J. CONDON.

The following statements were written across the back of this note:

"Rec'd on the within note, Dec. 1, 1887, \$358.80."
"Rec'd on the within note, Jan. 21, 1889, \$475.00."
"Rec'd on the within note, Oct. 25, 1889, \$261.50."
"Rec'd on the within note, June 15, 1890, \$91.40."

What was due Oct. 15, 1890?

7. A note for \$2,580, dated July 12, 1884, bearing 5% interest, had the following indorsements:

Jan. 1, 1885,	\$75.
May 25, 1885,	\$87.40.
Dec. 18, 1885,	\$260.
Oct. 15, 1886,	\$326.45.
June 28, 1887,	\$752.31.
Nov. 12, 1888,	\$850.

What was due July 12, 1889?

8. Principal, \$762.84. Date, Sept. 24, 1886. Rate of interest, 10%. Indorsements:

Jan. 1, 1887,	\$51.80.
Aug. 23, 1887,	\$128.
May 17, 1888,	\$125.
Oct. 28, 1889,	\$214.80.
March 13, 1890,	\$306.90.

What was due Sept. 24, 1890?

9. Principal, $8,750. Rate, 5%. Date, April 12, 1882. Indorsements:

June 20, 1883,	$1,250.
Aug. 3, 1884,	$2,560.
Dec. 25, 1885,	$3,164.86.
July 30, 1886,	$1,571.29.
Dec. 15, 1887,	$1,262.80.

What was due Oct. 12, 1888?

10. Principal, $2,350. Date, Aug. 3, 1885. Rate, 7½%. Indorsements:

Sept. 5, 1886,	$250.
Jan. 1, 1888,	$60.
July 25, 1888,	$475.
March 15, 1889,	$560.
Aug. 3, 1890,	$880.

What was still due?

11. Principal, $3,000. Date, Jan. 10, 1886. Rate, 6%. Indorsements:

March 1, 1887,	$260.
June 11, 1888,	$624.
Aug. 25, 1889,	$1,030.
May 1, 1891,	$1,250.
Jan. 10, 1892,	$280.

What was still due?

302. THE MERCHANTS' RULE.

I. ***Find the amount of the principal for the entire time.***

II. ***Find the amount of each payment from the time that it was made to the time of settlement.***

III. ***From the first amount subtract the sum of the amounts of the several payments.***

NOTE. This method allows interest on each payment for all of the time that it is in the creditor's possession. It is a perfectly fair method.

1. A note for $650, dated Jan. 10, 1894, and bearing interest at 5%, has the following indorsements: March 15,

$125; July 12, $240; Oct. 5, $85. What was due Jan. 10, 1895?

2. A note for $785.40, dated May 1, 1895, and bearing interest at 6%, is indorsed as follows: Aug. 20, $180.20; Nov. 5, $250.80; Feb. 24, 1896, $236.50. What was due May 1, 1896?

3. Principal, $892.60. Date, July 25, 1895. Rate, 7%. Indorsements: Sept. 1, $325; Nov. 19, $175.50; Jan. 12, 1896, $90; May 10, $300. What was due July 1?

4. Principal, $1,280. Rate, 8%. Date, May 1, 1894. Indorsements: July 25, $300; Sept. 10, $250; Dec. 1, $350; Feb. 10, 1895, $100. What was due April 20, 1895?

303. ANNUAL INTEREST.

1. **Annual interest** differs from compound interest in one particular: interest does not draw compound interest, but simple interest.

A problem will make the difference clear.

2. If a note provides that interest is payable annually, it means that unpaid interest at the end of any period shall draw simple interest until paid.

Illustrative Example. A note of $400 is due in 4 yr. 6 mo. The interest at 7% is payable annually. If nothing is paid until the note is due, what will the interest amount to? The interest on $400 for 4 yr. 6 mo. = $126. The $28 due at the end of the first year draws interest for 3½ years; the $28 due the second year, for 2½ years; the third, for 1½ years; and the fourth for ½ year. There will then be due, in addition to the $126, the interest on $28 for 3½ yr. + 2½ yr. + 1½ yr. + ½ yr. = the interest for 8 years = $15.68. $126 + $15.68 = $141.68.

Make a rule from the above analysis.

PROBLEMS.

	Principal.	Time.			Rate.
1.	$350	3 yr.	5 mo.	6 d.	6%
2.	$400	4 "	7 "	9 "	6%
3.	$480	2 "	8 "	12 "	7%
4.	$510.40	4 "	10 "	15 "	7%
5.	$560	5 "	6 "	18 "	7%
6.	$85.50	3 "	11 "	19 "	7%
7.	$128.20	4 "	2 "	21 "	8%
8.	$649	3 "	4 "	24 "	8%
9.	$763.50	6 "	7 "	20 "	5%
10.	$840	5 "	1 "	1 "	5%
11.	$24.10	7 "	7 "	7 "	5½%
12.	$968	6 "	6 "	6 "	6%
13.	$1070	4 "	2 "		6%
14.	$312.40	5 "	6 "	20 "	6%
15.	$2060	2 "	9 "	10 "	6%
16.	$3840	3 "	4		6%
17.	$5625	4 "	9 "	12 "	7%
18.	$96.90	5 "	8 "	15 "	4½%
19.	$4500	6 "	3 "	17 "	4½%
20.	$6970	4 "	5 "	5 "	5%
21.	$386.75	5 "	10 "	10 "	5%
22.	$193.40	6 "		6 "	5½%
23.	$4200	4 "		9 "	5½%
24.	$10000	3 "	3 "	3 "	4½%
25.	$693.42	4 "	11 "	6 "	5%

304. COMPOUND INTEREST.

1. A man borrowed $350 at 7% interest, agreeing that if the interest were not paid at the end of the first year it should be added to the principal to make a new principal for

the second year, and that the interest should be added thus each year until the debt was paid. If he paid nothing until the end of the 3 years and 3 months, how much was then due?

1st principal	$350
1st year's interest	24.50
2d principal	$374.50
2d year's interest	26.215
3d principal	$400.715
3d year's interest	28.049
4th principal	$428.764
Interest for 3 mo.	7.502
Amount	$436.266
1st principal	350
Interest	$86.26

2. **Compound interest** is interest upon a principal that is increased at regular periods by its accumulated interest.

3. Interest may be compounded at the end of any period agreed upon, instead of annually, as above.

RULE

To calculate compound interest:

1. At the end of each period increase the principal for that period by the interest accumulated during the period.

2. From the final amount subtract the first principal.

305. PROBLEMS.

Find the compound amount and interest:

1. Of $528, for 3 years, at 5%.
2. Of $1200, for 4 years, at 8%.
3. Of $1680.50, for 2 years, at 10%, compounding quarterly.

4. Of $2560, for 3 yr. 8 mo. 25 d., at 6%, compounding semi-annually.

306. TABLE,

Showing the amount of $1 at compound interest from 1 year to 10 years, at 3, 4, 4½, 5, 6, and 7 per cent.

Years.	3 per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.030000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.060900	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.092727	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.125509	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.159274	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.194052	1.265319	1.302260	1.340096	1 418519	1.500730
7	1.229874	1.315932	1.360862	1.407100	1.503630	1.605781
8	1.266770	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.304773	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.343916	1.480244	1.552969	1.628895	1.790848	1.967151

Illustrative Example. Find compound interest of $100 for 8 yr. 4 mo. 12 d., at 5%.

Amount of $1.00 for 8 yr. at 5%	$1.4774
	100
Amount of $100	$147.74
Interest of $147.74 for 4 mo. 12 d.	2.70
Amount	$150.44
	100.00
Compound interest	$50.44

With the aid of the table find the amount and compound interest:

	Principal.	Time.			Rate.
1.	$750	4 yr.	6 mo.		4½%.
2.	$5,000	9 "	7 "	10 d.	6%.
3.	$1,275.45	8 "	2 "	18 "	7%.

	Principal.	Time.			Rate.
4.	$1,640.25	4 yr.	5 mo.	24 d.	5%.
5.	$2,850.66	7 "	1 "	10 "	4%.
6.	$834	5 "	7 "		3%.
7.	$796	8 "	5 "	10 "	4%.
8.	$1,028.50	9 "	2 "	12 "	4½%.
9.	$1,725.80	3 "	10 "	15 "	5%.
10.	$960	4 "	1 "	21 "	6%.
11.	$2,480	7 "	11 "	24 "	7%.
12.	$3,812	6 "	3 "	28 "	6%.
13.	$86.95	2 "	7 "	19 "	5%.
14.	$731.25	3 "	4 "	17 "	4½%.
15.	$5,960	4 "	5 "	11 "	3%.
16.	$72.15	5 "	8 "	16 "	4%.
17.	$3,824	6 "	2 "	18 "	5%.
18.	$7,500	7 "	10 "	12 "	3%.
19.	$438.90	9 "	4 "	15 "	6%.
20.	$1,500	10 "	10 "	10 "	7%.
21.	$2,500	4 "	1 "	1 "	4½%.

	Principal.	Time.	Rate.
22.	$800	4 yr. 4 mo.	6% compounded semi-ann'ly.

NOTE. Use half the rate for double the time.

23.	$1,500	3 yr. 6 mo. 24 d.	8%	compounded semi-ann'ly.
24.	$1,850	4 " 8 " 20 "	9%	" "
25.	$2,500	2 " 5 " 12 "	12%	" quarterly.

NOTE. Use one-fourth the rate for four times the time.

26.	$3,150	3 yr. 7 mo. 18 d.	8%	compounded semi-ann'ly.
27.	$3,675	4 " 2 " 6 "	6%	" "
28.	$4,180	5 " 3 " 10 "	6%	" "
29.	$5,000	3 " 7 " 15 "	8%	" "
30.	$10,000	4 " 10 " 18 "	6%	" "

307. GENERAL PROBLEMS IN SIMPLE INTEREST.

1. We have seen that five elements have appeared in problems in interest. These are principal, interest, amount, rate per cent, and time. The relations between them are such that if any three of them be given, the other two may be found. Several different problems, consequently, may arise.

308. Problem I.

Given the principal, rate per cent, and time, to find the interest.

This problem has been discussed sufficiently.

309. Problem II.

Given the principal, interest, and time, to find the rate per cent.

Illustrative Problem. The interest on \$324.60 for 3 yr. 6 mo. 15 d. is \$91.97. What is the rate per cent?

ANALYSIS. The interest on \$324.60 for 3 yr. 6 mo. 15 d., at 1%, is $\$11.49\frac{5}{8}$. To produce \$91.97 in the same time, the rate must be as many times 1% as \$91.97 is times $\$11.49\frac{5}{8}$. \$91.97 is 8 times $\$11.49\frac{5}{8}$; hence, the required rate must be 8%.

PROBLEMS.

	Principal.	Interest.	Time.				
1.	\$480	\$66.13	3 yr.	5 mo.	10 d.	Find	rate.
2.	\$650.40	\$85.36	2 "	7 "	15 "	"	"
3.	\$864.36	\$242.89	4 "	8 "	6 "	"	"
4.	\$1,275.86	\$180.04	1 "	9 "	5 "	"	"
5.	\$1,464.29	\$306.28	2 "	1 "	3 "	"	"
6.	\$2,580.47	\$577.02	3 "	2 "	10 "	"	"
7.	\$3,600	\$951.75	5 "	10 "	15 "	"	"
8.	\$4,580	\$1,134.94	4 "	6 "	2 "	"	"
9.	\$5,000	\$1,663.38	5 "	3 "	1 "	"	"
10.	\$6,840.75	\$3,575.72	6 "	11 "	19 "	"	"

310. Problem III.

Given the principal, interest, and rate per cent, to find the time.

Illustrative Problem. The interest on $324.60, at 8%, was $91.97. What was the time?

ANALYSIS. The interest on $324.60 for 1 month, at 8%, is $2.164. To produce $91.97 at the same rate, the time must be as many times 1 month as $91.97 is times $2.164. $91.97 is 42½ times $2.164; hence, the time was 42½ months, which = 3 yr. 6 mo. 15 d.

	Principal.	Interest.	Rate.	
1.	$490.92	$49.75	8%.	Find time.
2.	$3,794.08	$803.40	9%.	" "
3.	$892.45	$149.73	5%.	" "
4.	$1,234.16	$169.08	6%.	" "
5.	$5,871.48	$1,701.10	7%.	" "
6.	$89.26	$18.04	3%.	" "
7.	$13.51	$4.37	5½%.	" "
8.	$1.05	$0.55	6⅓%.	" "
9.	$10,874.80	$193.33	4%.	" "
10.	$916.15	$58.63	8%.	" "

311. Problem IV.

Given the interest, rate per cent, and time, to find the principal.

Illustrative Problem. What principal will produce $91.97 in 3 yr. 6 mo. 15 d., at 8%?

ANALYSIS. A principal of $1.00, with the above rate and time, will produce $0.28⅓. To produce $91.97, the principal must be as many times $1.00 as $91.97 is times $0.28⅓; $91.97 is 324.6 times $0.28⅓; hence, the required principal is $324.60.

NOTE. Change divisor and dividend to thirds.

	Interest.	Rate.	Time.			
1.	$184.67	7%	3 yr.	6 mo.	9 d.	Find principal.
2.	$166.62	8%	3 "	9 "	17 "	" "
3	$35.16	5½%	6 "	9 "	24 "	" "
4.	$89.68	6%	3 "	2 "	7 "	" "
5.	$8.04	7%	2 "	4 "	18 "	" "
6.	$150.41	7%	2 "	4 "	12 "	" "
7.	$233.34	7%	3 "	5 "	13 "	" "
8.	$26.67	5%		9 "	15 "	" "
9.	$24.21	5%		8 "	15 "	" "
10.	$61.74	4½%	5 "	8 "	25 "	" "

312. Problem V.

Given the amount, rate per cent, and time, to find the principal.

Illustrative Problem. What principal will amount to $416.57 in 3 yr. 6 mo. 15 d. at 8%?

ANALYSIS. A principal of $1.00 will amount to $1.28⅓ with the above time and rate. To amount to $416.57, the principal must be as many times $1.00 as $416.57 is times $1.28⅓. $416.57 is 324.6 times $1 28⅓; hence, the required principal is $324.60.

NOTE. Observe that we first found, in Art. 309, what a 1% rate would produce; in Art. 310, what would be produced in 1 month; in Art. 311, what a $1.00 principal would yield; and in Art. 311, what a $1.00 principal would amount to. From these illustrative problems the following general statement may be derived: Perform the problem, assuming *one* of the required kind to be the required answer; then compare the result with the given number of the same kind.

PROBLEMS.

	Amount.	Rate.	Time.			
1.	$460	5%	2 yr.	3 mo.	20 d.	Find principal.
2.	$380.80	6%	3 "	4 "	15 "	" "
3.	$524.10	7%	4 "	6 "	12 "	" "
4.	$736.50	8%	1 "	9 "	16 "	" "

	Amount.	Rate.	Time.				
5.	$695.12	7%		5 mo.	24 d.	Find	principal.
6.	$874	6½%	3 yr.	11 "	19 "	"	"
7.	$926.95	4½%	5 "	2 "	21 "	"	"
8.	$1,284	5½%	2 "	7 "	6 "	"	"
9.	$2,065.48	7%	3 "	8 "	28 "	"	"
10.	$3,129.76	6%	1 "	1 "	18 "	"	"

313. PROBLEMS.

1. At what rate will $240 gain $8.96 in 6 months and 12 days?

2. In what time will $145.60 gain $42.47 at 5%?

3. What principal will gain $52.57 in 2 yr. 4 mo., at 6%?

4. What principal will amount to $132.40 in 5 yr. 7 mo. 20 d., at 10%.

5. At what rate per cent will any principal double itself in 10 years? 8 years? 12 years? 16⅔ years? 20 years? At what rate will any principal treble itself in the same periods?

6. In what time will a principal double itself at 3%? 4%? 4½%? 5%? 5½%? 6%? 7%? 8%? 9%? 10%? In what time will it treble itself at same rates per cent?

7. What was a man's investment that yielded him an income of $1,264.75 in 1 yr. 7 mo. 24 d., at 7% interest?

8. What principal will amount to $5,860.80 in 3 yr. 10 mo. 20 d., at 8% interest?

9. At what price must bonds that bear 4% interest annually be bought, to yield 6% interest on the investment?

10. Bought 4% bonds at 70. What rate per cent of interest does the investment yield?

11. Bought a piece of land for $6,825. Kept it 2 yr. 6 mo. 18 d., and sold it for $9,841.65. What is the rate per cent of interest that the investment yielded?

12. How long must $586.40 be on interest at 8% to gain $134.50?

13. What sum of money must be invested at 6% interest, compounded annually, to amount to $10,000 in 10 years?

14. What investment will yield annually $564.75 at 6%?

15. In what time will $2,500 amount to $3,650 at 4½%?

16. What sum put at interest, Jan. 1, 1886, will amount to $343.75, Feb. 1, 1888, at 7%?

Find the lacking numbers in the following problems:

	Principal.	Rate.	Time.	Interest.	Amount.
1.	$834.60	7%	?	$48.20	?
2.	$560.40	?	2 yr. 2 mo.	$54.639	?
3.	?	6%	7 mo. 15 d.	?	$350.65
4.	$2,500	8%	?	$556.67	?
5.	?	9%	90 d.	$72.55	?
6.	$150.60	?	72 d.	$2.26	
7.	$425.75	6%	?	$8.52	
8.	?	8%	93 d.	$12.60	
9.	?	7%	33 d.		$524.12
10.	$1,530.38	5½%	?	$375.00	

314. PRESENT WORTH AND TRUE DISCOUNT.

1. James K. Briggs purchased from Robert R. Stone a horse and carriage for $250, with the understanding that he was to pay for them 18 months afterward, without interest. He made the following note:

$250. ROCHESTER, N. Y., May 1, 1890.

Eighteen months after date, for value received, I promise to pay to Robert R. Stone, or order, Two Hundred and Fifty Dollars, without interest until due.

JAMES K. BRIGGS.

On the 1st day of July, 1890, Stone offered this note for sale. What should a person pay for it so that at its maturity he should receive his purchase money and interest on it at 6% per annum for 16 months?

He will receive $250 at the maturity of the note. The question is, What principal, at 6% interest per annum, will amount to $250? A principal of $1.00 will amount to $1.08 in 16 months at 6% per annum. To amount to $250, the principal must be as many times $1.00 as $250 is times $1.08. $250 is 231.48+ times $1.08; hence, the required amount is $231.48+.

The problem may be proved by finding the interest on $231.48 for the given time at 6%.

2. The **Present Worth** of a debt due at a future time, and not bearing interest, is that sum which, put at interest, at the given rate, for the given time, will amount to the debt.

3. The **True Discount** of a debt is the difference between the debt and its present worth.

RULE.

To find the present worth of a debt, divide the amount of the debt at its maturity by the amount of $1.00 for the given time at the given rate of interest.

To find the true discount, subtract the present worth from the debt.

NOTE. Observe that true discount is the interest on the present worth for the given time.

315. *Another Method.* 1. Any principal at 6% will amount to 108% of itself in 16 months. In this case the amount is $250. $250, therefore, is 108% of the present worth. 1% of the present worth is $\frac{1}{108}$ of $250. 100% of the present worth is $\frac{100}{108}$ of $250.

Which of the general problems of percentage is this?

2. Observe that by this method the discount may be found without finding the present worth.

The discount is 8% of the present worth. We have seen that 1% of the present worth is $\frac{1}{108}$ of the amount; hence, 8% of the present worth is $\frac{8}{108}$ of the amount.

PROBLEMS.

1. What is the present worth of a non-interest-bearing debt of $1,250, due in 2 yr. 3 mo. 15 d., if money is worth 6%? What is the true discount?

2. Principal, $875.40; time, 1 yr. 4 mo. 20 d.; rate, 7%. Find present worth and discount, the principal not bearing interest.

In the following problems the principal does not bear interest. Find present worth and discount.

	Principal.	Time.	Rate.
3.	$580.30	3 yr. 4 mo. 10 d.	6%
4.	$671.95	5 " 7 " 8 "	7%
5.	$2,834.25	2 " 5 " 12 "	5%
6.	$10,000	1 " 3 " 6 "	4%
7.	$383.59	2 " 11 " 21 "	8%
8.	$789.13	2 " 9 " 26 "	9%

9. What is the present worth of an interest-bearing debt, if the rate of discount is the same as the rate of interest?

10. A note dated May 12, 1889, for $1,060, due Sept. 21, 1891, and bearing interest at 5%, was discounted Oct. 15, 1890, at 6%. What was its worth?

NOTE. Remember that the amount due at the maturity of the note is the debt whose present worth is to be found.

Find present worth and true discount of the following non-interest-bearing notes.

	Principal.	Date.	Maturity.	Rate of Discount.
11.	$84.90	Jan. 1, 1890	March 12, 1892	5%
12.	$120	June 12, 1892	Oct. 5, 1894	5%
13.	$250.80	March 10, 1888	May 1, 1891	6%

	Principal.	Date.	Maturity.	Rate of Discount
14.	$360	Oct. 15, 1893	June 28, 1896	6%
15.	$480.40	Feb. 20, 1892	Aug. 1, 1895	6½%
16.	$560.70	July 28, 1890	Dec. 10, 1894	6½%
17.	$1,290	Aug. 3, 1894	May 15, 1897	7%
18.	$2,580	Nov. 19, 1893	Oct. 1, 1897	7½%
19.	$1,624	Dec. 30, 1895	July 24, 1898	7½%
20.	$3,560	April 20, 1896	Dec. 1, 1898	8%
21.	$787.50	May 1, 1896	Jan. 19, 1899	8½%
22.	$48	Sept. 21, 1896	March 5, 1899	9%
23.	$6,000	Jan. 23, 1896	April 21, 1899	9%
24.	$980	Dec. 15, 1895	June 12, 1898	10%
25.	$25	Oct. 19, 1896	Aug. 1, 1898	10%

316. BANK DISCOUNT.

1. If the note in Art. **314** had been sold to a bank, its value would have been estimated somewhat differently from the method there given. The banker would have calculated the interest on $250 for 16 months and 3 days, if days of grace are counted, and would have subtracted the result from $250. The remainder would have been the *proceeds*, or *avails*, or *cash value*, of the note.

2. The **Bank Discount** of a note not bearing interest is the interest upon the face of the note for the time from the day of discount until its legal maturity.

3. The three days that are sometimes added to the time which a note is to run are called **Days of Grace**.

4. Several of the States have done away with days of grace. In the following problems take no account of them unless they are mentioned.

PROBLEMS.

What are the bank discount and avails of the following note, if discounted at 6% on the day that it was made?

$600. CHICAGO, ILL., May 12, 1891.

Ninety days after date, for value received, I promise to pay to the First National Bank of Chicago Six Hundred Dollars, with interest at 6% per annum, after due.

BENJAMIN R. BROWN.

ANALYSIS. The interest on $600 for 93 days at 6% is 9.30; hence, the bank discount is $9.30, and the avails $600 − $9.30 = $590.70.

RULE.

To find the bank discount of a note that does not bear interest:

1. ***Find the interest on the note for the time that it is to run.***

2. ***To find the avails, subtract the discount from the face of the note.***

NOTE. Remember that the sum discounted is the amount to be paid at the maturity of the note; hence, if the note bears interest, first find the amount of principal and interest.

Similarly solve the following:

	Face of Note.	Time Discounted.	Rate of Discount.
1.	$825.00	60 days	6%
2.	$927.18	30 "	5%
3.	$264.83	45 "	5%
4.	$169.47	90 "	7%
5.	$2,968.51	4 months	5½%
6.	$417.80	5 "	8%
7.	$361.28	6 "	7%
8.	$248.65	50 days	7%
9.	$1,827.90	40 "	6½%
10.	$83.70	1 year, 3 months	6%

11. $850. BUFFALO, N. Y., July 21, 1896.
Three months after date, for value received, I promise to pay to Katharine Sedgwick, or order, Eight Hundred Fifty Dollars, at the City Bank.

WILLISTON COOK.

Discounted Aug. 15, 1896, at 7%.

12. $1,200. PHILADELPHIA, PA., Aug. 1, 1896.
Four months after date I promise to pay to Richard M. Johnson, or order, Twelve Hundred Dollars, value received.

THOMAS R. WILLIAMS.

Disconnted Oct. 15, 1896, at 6%.

13. $128.50. DETROIT, MICH., July 10, 1896.
Sixty days after date, for value received, we promise to pay to Henry R. Sunderland, or order, One Hundred Twenty-eight and $\frac{50}{100}$ Dollars.

ARTHUR G. HUNTING.
PETER T. SMALL.

Discounted Aug. 1 at 8%. Count days of grace.

14. $480. BOSTON, MASS., Sept. 1, 1896.
Three months after date, for value received, I promise to pay to Samuel S. Huston, or order, Four Hundred Eighty Dollars, with interest at 6 per cent per annum.

JOHN L. WHITNEY.

Discounted Sept. 18 at 8%. Count days of grace.

15. $1,540. AURORA, ILL., Aug. 21, 1896.
Two months after date, for value received, I promise to pay to Joseph H. Freeman, or order, Fifteen Hundred Forty Dollars.

JAMES S CAMPBELL.

Discounted Sept. 1, at $6\frac{1}{2}$%.

16. $875.00. CHICAGO, July 10, 1896.

Three months after date, for value received, I promise to pay to the Chemical National Bank Eight Hundred Seventy-five Dollars, with interest at 7% per annum after maturity.

ROBERT S. DANIEL.

Discounted at 7%, July 10.

17. $760. BLOOMINGTON, ILL., Aug. 3, 1896.

Four months after date, for value received, I promise to pay to the National State Bank, or order, Seven Hundred Sixty Dollars, with interest at seven per cent per annum after due.

JAMES L. ATWOOD.

Discounted at 7% Aug. 3.

317. Mr. A, desiring to pay a debt of $460, went to a bank to obtain the money. Since he must receive $460 from the bank, it is evident that he must make his note for more than that amount. He wishes to borrow the money for 4 months. The current rate of interest is 7%. If he should make his note for $1.00, the banker would give him the difference between $1.00 and the interest on it for 4 months and 3 days, if days of grace are counted. The interest on $1.00 at 7% for 4 months and 3 days is $23\frac{11}{12}$ mills. The avails of such a note would be $\$1.00 - \$0.023\frac{11}{12} = \$0.976\frac{1}{12}$. In order that he shall receive $460 his note must be made for as many dollars as there are times $\$0.976\frac{1}{12}$ in 460. $460 \div .976\frac{1}{12} = 471.27$; hence, the note must be made for $471.27.

Proof. The interest on $471.27 for 4 months and 3 days at 7% is $11.27+; hence, the avails of such a note would be $460.

NOTE. In calculating interest on $1.00 for short periods, the second 6% method is most convenient.

From the foregoing analysis we may form a

RULE.

To find the face of a note that will yield a given amount by bank discount:

1. Find the avails of a note for $1.00 for the given time and at the given rate, and

2. Divide the given amount by it.

Observe that there are two kinds of problems in bank discount.

(*a*) To find the avails of a note when face, time, and rate are given.

(*b*) To find the face when avails, time, and rate are given.

PROBLEMS.

For what must a note be drawn to yield:

	Amount.	Time.	Rate of Interest.
1.	$724.50	48 days	6%
2.	$826.40	60 "	5%
3.	$692.24	21 "	8%
4.	$5,860	90 "	7%
5.	$84	6 months	6%
6.	$951.28	4 "	9%
7.	$10,864.80	5 "	8%
8.	$5,712.40	80 days	7%
9.	$438.76	50 "	6%
10.	$11,372.12	48 "	5%

318. EXCHANGE.

1. A, in Normal, Ill., desired to pay Wm. Dulles, Jr., in New York, $100. It would not be safe to send the money in a letter. To send it by express would be expensive. This is what he did. He went to the bank and purchased an order upon another bank in New York to pay the $100 to the order of Wm. Dulles. [See next page.] The Normal bank keeps money on deposit in the New York bank, and

The above is a fac-simile (except in size) of the draft that was actually employed in transmitting $100 from Normal, Ill., to Wm. Dulles, Jr., in New York. On receipt of this draft, Mr. Dulles wrote his name on the back, and presented it for payment. Such an order is called a Draft, or Bill of Exchange.

sells its orders to people who wish to pay bills in Eastern cities.

2. A **Draft** is an order made by one party upon a second to pay a designated sum to the order of a third party.

3. The party issuing the draft is the **Maker**, the party upon whom the order is drawn is the **Drawee**, and the party in whose favor it is drawn is the **Payee**.

4. In the draft shown on the preceding page, W. H. Schureman & Co. is the maker; United States National Bank, the drawee; and Wm. Dulles, Jr., the payee.

5. **Exchange is a process of paying obligations due at distant places by transmitting drafts.**

6. The terms *par*, *discount*, and *premium* are applied to drafts in the same way as to bonds.

319. PROBLEMS.

1. What is the cost of a draft on New York for $650, when exchange is at $\frac{1}{8}$% premium?

What is the cost of a draft for:

2. $185.40 on Chicago at $\frac{1}{10}$% premium?
3. $467.32 on New Orleans at $\frac{1}{8}$% premium?
4. $1,875.12 on Boston at $\frac{1}{4}$% premium?
5. $2,694.38 on Pittsburg at $\frac{1}{10}$% discount?
6. $10,850 on Philadelphia at $\frac{1}{8}$% premium?
7. $791.13 on New York at $\frac{1}{4}$% discount?
8. $2,874.93 on St. Louis at $\frac{1}{8}$% discount?
9. $167.91 on Cincinnati at $\frac{1}{8}$% premium?
10. $16,824.57 on San Francisco at $\frac{1}{8}$% premium?
11. If exchange is at $\frac{1}{10}$% premium, how large a draft will $1,001 buy?

ANALYSIS. If exchange is at $\frac{1}{10}$% premium, a draft for $1.00 will cost $1.001. $1,001.00 will buy a draft whose face is as many times $1.00 as $1,001.00 is times $1.001.

12. If exchange is $\frac{1}{4}\%$ discount, how large a draft will \$467.50 buy? \$328.37? \$572.20?

13. How large a draft will \$5,824.75 buy, if exchange is $\frac{3}{8}\%$ premium? $\frac{2}{3}\%$ discount? $\frac{1}{3}\%$ premium? $\frac{3}{8}\%$ discount?

14. How large a draft will \$325.50 buy, if exchange is $\frac{1}{8}\%$ discount? $\frac{3}{4}\%$ discount? $1\frac{1}{2}\%$ premium? $\frac{2}{3}\%$ premium?

15. Find the cost of a draft for \$650.40, at $1\frac{2}{3}\%$ premium; at $1\frac{2}{3}\%$ discount; at $\frac{1}{8}\%$ premium; at $\frac{1}{8}\%$ discount.

320. **1.** The drafts with which we have been dealing are called **Sight Drafts**, because payable when presented. A second form is sometimes used, called **Time Drafts**, payable a specified time "after sight;" that is, after they have been shown to the drawee and he has stamped "accepted," with the date of acceptance, and has written the name of the drawee on the face.

2. Time drafts differ from sight drafts in no essential respect except in the designation of the time which they are to run. If the words "Thirty days after sight" preceded "Pay to the order of," in the draft given, it would be a time draft.

3. It is evident that such a draft would not ordinarily be at par, since the purchaser cannot have it cashed by the drawee until its maturity. If sight exchange is at par, such a draft is worth its face, less bank discount for the time it is to run.

PROBLEMS.

1. What is the value of a 90-day draft for \$800, exchange being $\frac{1}{8}\%$ premium, and the current rate of interest being 6%? (With grace.)

ANALYSIS. The interest on \$800 for 93 days, at 6%, is \$12.80. If exchange were at par, the draft would be worth $\$800 - \$12.80 = \$787.20$. Since exchange is at $\frac{1}{8}\%$ premium, it is worth $\frac{1}{8}\%$ of $\$800 + \$787.20$.

2. Find the cost of a 45-day draft for \$460.50, exchange being $\frac{1}{4}\%$ discount, and interest at 7%.

3. What is the face of a 90-day draft that can be purchased for $1,175.62, if exchange is at $\frac{1}{3}$% discount, and the rate of interest is 7%? (With grace.)

ANALYSIS. If exchange were at par, a 90-day draft for $1.00 would cost $1.00 – 0.018\frac{1}{12}$. Since exchange is at $\frac{1}{3}$% discount, there should be a further withdrawal of $.003$\frac{1}{3}$; hence, a draft for $1.00 would cost $1.00 – 0.021\frac{5}{12}$ = 0.978\frac{7}{12}$. $1,175.62 will buy a draft whose face is as many times $1.00 as 1,175.62 is times 0.978\frac{7}{12}$.

NOTE. Change divisor and dividend to twelfths for perfect accuracy.

4. How large a draft, payable in 4 months, exchange $\frac{1}{3}$% premium, interest at 8%, can be bought for $1,250?

5. Find the cost of a 30-day draft for $427.50, exchange at $\frac{1}{4}$% discount, interest at 8%.

6. Find the cost of a sight draft for $784.90, at $\frac{1}{8}$% premium.

7. Exchanged the preceding draft for a 45-day draft, $\frac{1}{8}$% discount, interest 6%. What was the face of the time draft?

8. A commission merchant sold 10,000 bushels of wheat at 87 cents, withdrew his commission of 1$\frac{1}{2}$%, and with the remainder bought a 60-day draft, exchange $\frac{1}{4}$% premium, interest at 6%. What was its face?

9. A farmer sold 120 acres of land at $65 an acre, receiving from the purchaser his note for 4 months without interest. He at once discounted the note at a bank at 7%, and with the proceeds bought a time draft on New York, due in 4 months, exchange being 1% discount, interest 6%. If he received the face of the draft, did he gain or lose by the deal? How much?

When drafts are paid in the country in which they are drawn, the exchange is called **Domestic**, or **Inland**. The system is extended also to foreign countries, and constitutes

321. FOREIGN EXCHANGE.

1. Drafts drawn upon banks in foreign countries have their face value expressed in the currency of that country.

2. When a system of exchange is established between two countries it is necessary to be able to express the value of the currency of each in the currency of the other. Such an expression is called the **par of exchange.** The legal value of a pound sterling is $4.8665. If exchange varies from this price, it is above or below par. Bills of exchange on England are called **Sterling Bills.** Their price is quoted at the cost of a pound sterling in United States money.

3. Foreign bills of exchange are sometimes issued in sets of three, called respectively the first, second, and third of exchange. Such bills are transmitted at different times to avoid losses and delays. The one reaching its destination first is paid, and the others become void.

4. The following is the more common form used:

Exchange for

£250. CHICAGO, ILL.; June 1, 1892.

On demand of this Bill of Exchange pay to the order of Dillon Bros. Two Hundred Fifty Pounds sterling, value received, and charge the same, as per advice, to

BROWN BROS.

To Dunn & Co.,
London.

No. 185.

5. Foreign bills may be payable upon presentation, or after a specified time. The value of any foreign exchange is now quoted as that of other commodities, at a specified price per pound sterling, per franc, etc. The following is the Chicago quotation for Feb. 15, 1892:

For	One Pound Sterling	$4.90
	One Franc on France	$.19\frac{1}{2}$
	One Franc on Belgium	$.19\frac{1}{2}$
	One Franc on Switzerland	$.19\frac{1}{2}$
	One German Mark	$.24\frac{1}{8}$
	One Hollandish Florin	$.40\frac{3}{8}$

One Austrian Florin	.42
One Lira Italian	.19¼
One Rouble	.50
One Finnish Mark	.19¾
One Krona on Sweden	.27
One Krone on Denmark	.27
One Krone on Norway	.27

PROBLEMS.

At the quotations given find the cost of each of the following bills of exchange:

1. For 456 German marks.
2. For 874 Austrian florins.
3. For 1,283 roubles.
4. For 697 lire.
5. For 384 pounds sterling.
6. How large a draft on Paris can be bought for $245.70?
7. On Brussels for $174.33?
8. On London for $573.30?
9. What is the rate when a draft on London for £150 costs $732?
10. What is the rate when a draft on Paris for 1200 francs costs $232.50?
11. When a draft on Vienna for 800 florins costs $334?
12. When a draft on Helsingfors for 640 marks costs $121.60?

322. EQUATION OF PAYMENTS.

The **Equation of Payments** is the process of finding a time at which several sums due at different times, and not bearing interest, can be paid, without loss to debtor or creditor, and without the transfer of money for interest.

If the several sums bear interest, all could be paid at any time by discharging the principal and accrued interest, and no one would lose.

The principle upon which the operations are based is very simple. If money is paid before it is due, interest should be allowed upon it; if not paid until after it is due, it should bear interest from maturity until date of payment.

Illustrative Problem. I owe \$500 due in 4 months; \$600 due in seven months; and \$1,000 due in nine months: what is the average term of credit?

These sums do not bear interest until after maturity. If any one of them should be paid before its maturity the debtor would lose the use of the money for the remaining time. If it should not be paid until after maturity, the creditor would lose interest.

Assume money to be worth 6% interest, and the several debts to be paid to-day. On the first the debtor would lose the interest for four months, which is \$10. On the second he would lose interest for seven months, or \$21; and on the third, \$45. Consequently, if the debts were paid in full to-day, the debtor would lose \$76. He may pay them together without loss by keeping them long enough after to-day to earn \$76. The interest on \$2,100 for one day at 6% is 35 cents. $\$76 \div .35 = 217+$. They should be paid 218 days from to-day.

This method of finding the time of payment is called *The Interest Method.* The time sought is called the Equated Time. It may be found also by

THE PRODUCT METHOD.

The interest on \$500 for 4 months equals the interest on \$2,000 for 1 month. The interest on \$600 for 7 months equals the interest on \$4,200 for 1 month. The interest on \$1,000 for 9 months equals the interest on \$9,000 for 1 month. The interest on the several amounts equals the interest on \$15,200 for 1 month. But the interest on \$15,200 for 1 month equals the interest on \$2,100 for as many months as \$15,200 is times \$2,100. \$15,200 is $7\frac{5}{21}$

times $2,100; hence, the equated or average time is 7 months and 8 days.

PROBLEMS.

1. Find the average time for the payment of $325 due in 30 days; $650 due in 40 days; $500 due in 60 days; and $825 due in 90 days; all without interest.

2. Find the equated time for the payment of $275 due in 3 months; $580 due in 5 months; $1,020 due in 7 months; $1,260 due in 10 months; no interest being charged on any account.

3. Find the equated time for the payment of the balance of a debt of $1,250 which was to run one year, without interest, one half of it having been paid at the end of 3 months; one fourth of the remainder at the end of 6 months; and one fourth of that remainder at the end of 9 months.

4. When could the following non-interest-bearing debts be paid at one time without loss?

$840 due May 1; $650 due June 15; $900 due July 8; $1,275 due August 1.

NOTE. Assume May 1 as the date of payment.

5. On March 1, 1888, A owed B $2,500 due December 15, without interest. On June 12 he paid $500, and September 10, $1,000. When should he pay $400 to entitle him to keep the remainder until Dec. 15, 1889?

6. What was the equated time for paying the following non-interest-bearing bills?

March 1, 1889, a bill of $300 for 60 days.
April 15, 1889, " " $400 for 30 days.
June 10, 1889, " " $583.50 for 4 months.
July 1, 1889, " " $962.80 for 3 months.

7. Jan. 1, 1890, A bought a bill of goods amounting to $2,560, on 90 days' time, without interest. On January 16,

he paid $850; on February 1, $725. If he settled the bill at maturity, how much should the balance be discounted, money being worth 6%?

8. A man owes a bill of $1,200 due in 8 months, without interest. How much must he pay at the end of 4 months, to extend the balance 2 months?

9. Find the average time of payment for the following bills, which do not bear interest.

January 1, $400 on 3 months.
February 15, $600 on 30 days.
May 10, $560 on 4 months.
June 12, $800 on 60 days.
June 20, $250 on 20 days.

10. A merchant, on the first of March, bought goods to the amount of $1,000. He agreed to pay $250 cash, $250 on the 3d of May; $250, July 4; and $250, September 15, all without interest. He prefers to pay the whole at one time; when should it be?

11. William Jones of Council Bluffs buys goods of Marshall Field as follows:

1.	May 1,	bill of	$600	on 3 mo.	credit.
2.	May 15,	"	$800	" 4 mo.	"
3.	June 1,	"	$500	" 6 mo.	"
4.	June 9,	"	$900	for cash.	

Marshall Field agrees to take Mr. Jones's note for the whole amount, for 30 days, with interest. When should the note be dated?

NOTE 1. First find the equated time of payment. Assume for this purpose the earliest date on which a payment falls due.

(1) 3 mo. credit from	May 1.	Due	August 1.
(2) 4 mo. " "	May 15.	"	September 15.
(3) 6 mo. " "	June 1.	"	December 1.
(4) Cash payment.		"	June 9.

June 9 to August 1 = 53 days, etc.

NOTE 2. For further work see Appendix.

323. MISCELLANEOUS PROBLEMS.

1. A note for $428.50, dated Aug. 15, 1889, is to run 90 days, without interest. It was discounted October 1, at 8%. What were the proceeds?

NOTE. The following illustration explains the method of finding the date of maturity of a note.

A note is dated May 18, and is to run 72 days:

May	. . . . .	13 days.
June	. . . .	30 days.
July	. . . .	31 days.
Matures, Aug.	. . . .	1 day.
		75 days.

2. Find the difference between the true discount and the bank discount of the above note if discounted August 15, at 6%.

NOTE. True discount is interest on what? Bank discount? What is the difference, then?

3. A merchant bought a bill of goods for $1,875.60, on 90 days' time. Finding that he could get a discount of 5% of the whole amount by paying cash, he borrowed the requisite amount from a bank. For what must he draw a note for 90 days, current rate being 8%, to pay the bill? How much did he gain by the process? (Bank Discount.)

4. A merchant bought a bill of goods for $1,674.20. He was allowed 5% off for cash. What did he pay?

5. A stock train has 29 cars, each containing 19 cattle, whose average weight is 1,450 pounds. They sell for $5.25 a hundred. What do they bring?

6. A man bought a half-section of land at $81.50 an acre. He gave one note for $12,650, a second note for $5,830, paid $3,600 in cash, and gave a third note for the remainder. What was the face of the third note?

7. The above notes were all dated July 1, 1892. The first was due in 2 years and bore 6% interest. Falling heir to

$15,000, he discounted this note on Jan. 15, 1893, at 7% bank discount. What did he pay for it?

8. The second of the above notes was due in 3 years, and bore interest at 6%. On Sept. 25, having sold his wheat crop, he discounted the second note by true discount at 8%. What did he pay for it?

9. The third of the above notes was due in 4 years, and bore annual interest at 5%. He paid no interest until the maturity of the note. What amount was then due?

10. He borrowed the $3,600, in Problem 6, at a bank by giving his note for 4 months at 7%. What was the face of his note which yielded him that amount?

11. Find the amount of the following bill:

24 Arithmetics @ 95¢.
16 Readers @ $1.20.
20 Geographies @ $1.40.
18 Grammars @ 75¢.
42 Spellers @ 22¢.

The dealer made a discount of $33\frac{1}{3}$% from the regular price, and a further discount of 5% of the balance for cash.

12. Bought a bill of $748.80, the seller making a discount of $33\frac{1}{3}$% and 5%. What was the amount to be paid?

13. What is the weight of a stone roller 8 feet long and 8 feet in circumference ($\pi = \frac{22}{7}$), the stone being 2.3 times as heavy as water?

14. What is the weight of a grindstone 4 inches thick and 30 inches in diameter, the hole being 2 inches in diameter, the stone being 2.143 times as heavy as water?

15. What fraction is that which being multiplied by $\frac{2}{3}$ of $\frac{4}{8}$ gives $\frac{3}{4}$ of $\frac{7}{12}$ as a product?

16. Bought 250 yards of broadcloth for £87 10 s. The import duty was 36%. Transportation and other charges amounted to $12.50. It was sold at $3.25 a yard. What

was the per cent of gain, the pound sterling being worth \$4.90? (Approximate.)

17. Bought a piano for \$560 on March 5, 1895. The agreement is that I shall pay 6% interest on the purchase price with the privilege of making payments upon which interest shall be allowed at the same rate. I made the following payments: May 25, \$175; July 18, \$160; Nov. 12, \$125. What was due March 5, 1896?

18. Find the cost of a draft on London to pay for the broadcloth in Problem 16, exchange being \$4.89 for a pound sterling.

19. A man bought a business lot 60′ × 160′ at \$4½ a square foot. He erected a six-story building, costing \$50,000, on the lot. In the first story there are two store-rooms, which rent for \$1,500 each. There are 20 rooms on each of the other floors. Those on the second floor average \$20 a month. There is a reduction of \$3 a month as the stories ascend. If insurance, taxes, and care of the building amount to \$3,500, what interest will he receive on his investment?

20. If 1½% be paid for collecting the rents in the above building, what is the collector's commission per month?

21. What is the compound interest on \$345 for 4 yr. 3 mo. 20 d., at 6%?

22. What time will be required for \$460 to earn \$64.80, at 7%?

23. What principal earns \$56.40 interest in 2 yr. 5 mo. 15 d., at 6%?

24. Divide .46083 by .37.

25. What is the difference between London local time and New York local time? If a cablegram be sent from London at 8 A. M., and one hour be employed in getting it to its destination in New York, at what time will it reach there?

26. Bought 5% school bonds at 102. What rate of interest on the investment do they pay?

27. Bought 40 shares of National Bank stock at 160. They pay a semi-annual dividend of $160. What is the rate of dividend? What rate do they pay on their cost?

28. What is the diameter of your bicycle wheels? How many revolutions do they make in going a mile? How many revolutions do they make for one revolution of a pedal? How many revolutions of the pedals will take the wheel one quarter of a mile?

29. What is the "accurate" interest on $560.40 for 93 days at 6%?

30. What principal will amount to $1,200 in 3 yr. 7 mo 21 d., at 8%?

31. I send to a commission merchant $1,000 to be invested in wheat at 57¾ cents a bushel, after deducting his commission at 2%. How many bushels will my remittance buy?

32. I have a bin 36 feet long, 10 feet wide, and 9 feet high, which is filled with shelled corn. A commission merchant sells it for me at 24½ cents a bushel, charging 1½% commission, and remits to me the balance? What is the amount of his remittance?

33. I own 30 shares of Building and Loan stock upon which I pay 50 cents a share monthly. After paying for 4 years I draw out my investment and receive $936. What rate of interest do I receive?

NOTE. Find average time.

34. A man whose watch shows Chicago local time finds that it is 36 min. 30 sec. faster than the local time of the place in which he is. What is his longitude? (See Table.)

35. In a school of 750 pupils the number of boys is 87½% of the number of girls. How many boys are there in the school?

36. How many prescriptions each weighing 5 dr. 1 sc. 12 gr. can be made from 1 lb. 7 oz. 2 dr. 2 sc. 16 gr.?

37. The tax for State purposes in Illinois is 53 mills on a hundred dollars. What does this amount to on an assessed valuation of $15,624?

38. Find the cost of the following at $17.50 a thousand:

24 studs 2×4, 18 feet long.
32 joists 2×10, 16 feet long.
8 sills 8×10, 14 feet long.
1520 feet common fencing.

39. An auctioneer sold the following articles at a farmer's sale. What is his commission at 2%?

5 horses, averaging $65.
8 cows, averaging $36.50.
2 wagons, one for $18.75, the other for $31.60.
20 tons hay at $9.75.
3 plows at $5.31.
1 harrow at $4.90.
26 hogs at $8.30.

40. At the above sale a discount of 5% was made for cash. What amount would discharge the obligation of the man who bought the horses and the cows?

41. It was a condition of the above sale that purchasers might pay in one year without interest. The man buying the wagons and the hay discounted his note at 8% — true discount — at the end of 4 mo. 18 d. What did he pay?

42. The man buying the hogs borrowed the money from a bank, at 7%, for 90 days, with grace, and paid cash for them. What was the face of his note?

43. Only what common fractions can be changed to pure decimals? Why? Change $\frac{1}{37}$ to a decimal (four places).

44. Find the compound interest on $465.72 for 3 yr. 2 mo. 8 d., at $6\frac{1}{2}\%$.

45. Reduce $\dfrac{3\frac{1}{2} \times 5}{3\frac{1}{2} \div 5} \times \dfrac{\frac{3}{8} + \frac{7}{12}}{\frac{7}{12} - \frac{3}{8}} \div \frac{3}{4}$ of $\frac{5}{8}$ of $\frac{7}{12}$.

46. In what time will a principal of \$864.12 amount to \$1,040 at 7% simple interest?

47. What principal will amount to \$501.83 in 10 yr. 3 mo. 15 d., at 7% compound interest?

48. The time from 9 o'clock to 15 min. 20 sec. past 10 is what part of a day?

49. What is the area of a city lot 50′ × 150′? What is it worth at \$6,000 an acre?

50. Add 21 lb. 8 oz. 12 pwt. 21 gr.; 12 lb. 10 oz. 16 gr.; 5 lb. 18 pwt. 19 gr.; 26 lb. 8 pwt. 22 gr.; 10 oz. 3 pwt. 9 gr.

51. What is the date of your birth? How old are you to-day?

52. Change $\frac{7}{15}$ of a square mile to integers of lower denominations.

53. Change .325 of a mile to integers of lower denominations.

54. An auctioneer's commissions for a year, at $2\frac{1}{2}$%, amounted to \$3,124.80. He was employed 265 days. What was the daily average of his sales?

55. A manufacturer received from his foreign agent 50 bales of wool, 250 pounds each, invoiced at 36 cents per pound, and 36 bales, 300 pounds each, invoiced at 31 cents a pound. What was the duty, at 25% ad valorem?

56. The width of this book is what per cent of its length?

57. The area of this page is what decimal of a square yard?

58. What will it cost to carpet this room with Brussels, $\frac{5}{8}$ of a yard wide, at \$1.12 a yard, if the strips run lengthwise, with no loss for matching? Would the cost be changed if the strips ran crosswise, with no loss for matching?

59. If 15 horses eat $101\frac{1}{4}$ bushels of oats in 12 days, how many bushels will 32 horses eat in 21 days at the same rate?

60. A and B can do a piece of work in 9 days, and A can do it in 16 days. If they receive $32 for the job, and each is to be paid in proportion to what he does, what should B receive?

61. Three boys start from the same point, at the same time, and ride their bicycles around a block in the same direction. The block is $\frac{1}{8}$ of a mile on a side. If the first rides at an average rate of 10 miles an hour, the second 12 miles, and the third 15 miles, how long before they will be together at the starting-point?

62. A man wished to purchase a horse, saddle, and bridle. A dealer offered to sell him a horse for $160, a pony for $\frac{1}{3}$ of what he asked for the horse, saddle, and bridle, or the pony, saddle, and bridle for $\frac{1}{2}$ of what he asked for the horse. What was the price of the saddle and bridle? of the pony?

63. On the Centigrade thermometer the freezing-point is 0° and the boiling-point 100°. How is it on the Fahrenheit thermometer? What is the relation between the two scales? Change 72° Fahrenheit to the Centigrade scale.

SECTION IX.

324. RATIO.

1 3 is what part of 6? 4 is what part of 7? 5 is what part of 6? of 9? of 15? 9 is what part of 12? of 7? of 6?

2. $\frac{1}{2}$ is what part of $\frac{3}{2}$? of $\frac{5}{2}$? of $\frac{2}{3}$? of $\frac{5}{6}$? of $\frac{1}{3}$?

3. \$4 is what part of \$8? of \$10? of \$16? of \$2?

4. 5 feet is what part of 10 feet? of 12 feet? of 7 feet? of 15 feet? of 2 feet?

5. $\frac{2}{3}$ of 1 pound is what part of $\frac{5}{6}$ of 1 pound? of $\frac{7}{8}$ of 1 pound? of $\frac{1}{5}$ of 1 pound?

6. Instead of the forms used above, a briefer form may be used to express the same relation. When a colon is placed between two numbers, it indicates that the first is to be measured by the second.

7. One number is measured by another:

(*a*) By finding what part the first is of the second; or,

(*b*) By finding how many times the second number the first is.

This is done by dividing the first number by the second.

8 Such an expression is called a **Ratio**.

The ratio of one number to another is that relation which is found by dividing the former by the latter.

9. 5 : 8 is a ratio. It is read, "the ratio of 5 to 8." Its value is $\frac{5}{8}$. The expression may also be read "5 is $\frac{5}{8}$ of 8." The form is new, but the idea is familiar.

10. Read the following in both ways: 5 : 10. 6 : 7. $\frac{1}{2}:\frac{2}{3}$. \$8 : \$3. 5 lb. 4 oz. : 6 lb. 13 oz.

11. 3 ft. 9 in. : 8 ft. 4 in. $7\frac{1}{2}:2\frac{1}{5}$. 8.7 : 2.9. 5 A. 12 sq. rd. : 15 A. 36 sq. rd.

12. The first term of a ratio is the **Antecedent**, and the second term the **Consequent**.

13. A ratio is like a fraction, the antecedent corresponding to the numerator, and the consequent to the denominator. The difference lies in the fact that the antecedent must always be thought of as a part of, or some number of times, the consequent.

14. Find the value of each of the following ratios: 7 : 2.1. $\frac{2}{3} \cdot \frac{3}{4}$. $3\frac{1}{2} : 4\frac{2}{3}$. $\$2\frac{1}{5} : \$3\frac{1}{2}$. 3 mi. 20 rd. : 2 mi. 5 rd. .7 : 2.1. 3 lb. 2 oz. : 5 lb. 14 oz.

15. The terms of a ratio must be like numbers. Why?

325. PROPORTION

1. Compare the value of 5 : 6 with that of 10 : 12. Of 4 : 5 with 12 : 15. Of $\frac{1}{3} : \frac{3}{4}$ with $\frac{2}{3} : \frac{3}{2}$.

2. Find a ratio equal to 8 : 12. Equal to \$7 : \$21. $3\frac{1}{2}$ pounds : 2 pounds.

3. Find a number whose ratio to 12 equals 6 : 9. Whose ratio to $\frac{3}{4}$ equals $\frac{1}{10} : \frac{1}{5}$.

4. 6 : 7 = 10 : ? 12 : 15 = 8 : ? $\frac{2}{3}$: ? = 4 : 12.

NOTE. 6 : 7 = 10 : ? is read, "the ratio of 6 to 7 = the ratio of 10 to what number?" or, "6 is the same part of 7 that 10 is of what number?"

5. ? : 15 = 9 : 27. $6\frac{1}{2}$: ? = 9 : 27. 11 : 33 = ? : 19.

6. A double colon is usually used instead of the sign of equality. Thus, 4 : 6 : : 8 : 12. Such an expression is called a **Proportion.**

7. A proportion is an equality of two ratios.

The first ratio is called the **First Couplet**, and the second ratio, the **Second Couplet.**

8. The first and last terms of a proportion are called the **Extreme Terms**; the second and third terms, the **Mean Terms.**

326. PRINCIPLES.

1. The product of the extreme terms equals the product of the mean terms.

PROOF. A ratio may always be expressed in the form of a fraction.

Since the two ratios of a proportion are equal, the fractions to which they are equivalent must be equal.

If two equal fractions are made to have the same denominator, their numerators will be equal; hence, the product of the first numerator and second denominator will equal the product of the first denominator and second numerator.

2. If the product of the extremes be divided by either mean, the quotient will be the other mean.

3. If the product of the means be divided by either extreme, the quotient will be the other extreme.

Prove Principles 2 and 3.

Make rules for finding either extreme or either mean of a proportion.

PROBLEMS.

Find the missing term in each of the following proportions:

1. $6 : 10 = 15 : x$.
2. $12 : 39 = x : 91$.
3. $\frac{1}{3} : x = \frac{2}{7} : \frac{5}{8}$.
4. $x : 125 = 72 : 108$.
5. 26 A. : 36 A. = 13 T. : x.
6. \$625 : \$825 = x : \$33.
7. 3 lb. : x = $\frac{2}{5}$ yd. : $3\frac{3}{10}$ yd.
8. $55 : 77 = x : 42$.
9. $8 : 15 :: x : 24$.
10. $15 : 36 :: 72 : x$.
11. $x : 48 :: 60 : 75$.
12. $30 : x :: \frac{5}{6} : 2\frac{1}{3}$.
13. $\frac{2}{3} : \frac{5}{8} :: 64 : x$.
14. $.36 : x :: .125 : 4$.
15. \$8 : \$$2\frac{1}{2}$ :: \$144 : x.
16. 42 bu. : 36 bu. :: x : 3 pk.
17. 184 mi. : x :: 75 mi. : 525 mi.
18. x : 320 A. :: $\frac{3}{4}$ A. : $\frac{5}{8}$ A.
19. 3,200 lb. : 200 lb. :: 96 lb. : x.
20. 468 rd. : 920 rd. :: x : 2760 rd.

21. $\frac{1}{8} : \frac{1}{12} :: 3 : x$.

22. $2\frac{1}{3} : 7 :: .0084 : x$.

23. 63 gal. : 90 gal. :: x : \$120.

NOTE. Is the above a true proportion?

24. 75 lb. : 40 lb. :: 60 d. : x.

327. 1. The proportion may be used in the solution of problems in which three numbers are given with which to find a fourth, if two of the numbers have the same ratio to each other that the third has to the required number.

2. ***To solve such a problem, state it in the form of a proportion in which the required number is the fourth term, and the two given related numbers are the first couplet, determining their order by the nature of the problem.***

PROBLEMS.

1. If 12 yards of cloth cost \$36, what will 40 yards cost at the same price?

ANALYSIS. 12 yards is the same part of 40 yards that the cost of 12 yards is of the cost of 40 yards; hence, the proportion is stated as follows:

12 yards : 40 yards :: \$36 : x.

To find the 4th term, apply Principle 3.

2. If 50 bushels of corn cost \$20, what will 600 bushels cost at the same price?

3. If a man travel 48 miles in 12 hours, how many miles can he travel in 60 hours at the same rate?

4. If 17 horses cost \$1,360, what will 39 horses cost at the same price?

5. If the cost of 1,360 square yards of plastering be \$340, what will 3,824 yards cost at the same price?

6. If a steeple 124 feet high cast a shadow 93 feet long, how long a shadow will a steeple 216 feet high cast at the same time and place?

7. If a steeple 216 feet high cast a shadow 162 feet long, how long a shadow will be cast by a steeple 124 feet high at the same time and place?

8. Suppose the shadow cast by a 216-foot steeple to be 162 feet long, what is the length of a steeple that casts a 93-foot shadow, the other conditions being equal?

9. If a 124-foot steeple cast a 93-foot shadow, what is the height of a steeple that casts a 162-foot shadow, under the same conditions?

10. If 25 men can do a piece of work in 12 days, in how many days can 10 men do the same work?

NOTE. This proportion differs from those preceding it, in that it involves an "inverse proportion." Since the length of time will increase as the number of men diminishes, the proportion will be $10 : 25 = 12 : x$. Be ready to recognize such proportions.

11. If 10 men can do a piece of work in 30 days, how many men can do the same work in 12 days?

12. If a principal of $325 earn $76.50 in a given time, what will a principal of $648 earn in the same time?

13. A room is 24 feet wide and 28 feet long. How long must a room 18 feet wide be to contain the same area?

14. If 50 acres of land produce 2,800 bushels of corn, how many bushels will 76 acres yield at the same rate?

15. If a man can do a piece of work in 36 days, working 10 hours a day, in how many days could he do the same working 12 hours a day?

16. A railway train runs 429 miles in 8 hours and 15 minutes. How far would it run in 10 hours and 20 minutes at the same rate?

17. If 3 lb. 5 oz. of butter cost $1.06, how much would 8 lb. 12 oz. cost at the same price?

18. A garrison of 320 men is supplied with provisions for 60 days. How long would the remainder of the stock of supplies last the rest of the garrison, if 40 men were withdrawn at the end of 20 days?

19. Two cog wheels are geared together. The larger has 42 cogs and the smaller 16. How many revolutions does the smaller make while the larger makes 24?

20. If 120 bushels of oats be necessary to seed 40 acres of land, how many bushels will seed 195 acres?

21. If $2\frac{5}{8}$ barrels of pork cost $23.625, how much will $8\frac{7}{12}$ barrels cost?

22. If $3\frac{7}{12}$ acres of land yield 172 bushels of wheat, how much will 18.75 acres yield?

23. If the interest on $375.50 for a certain time is $52.57, what is the interest on $680 for the same time?

328. COMPOUND RATIO.

1. The value of a ratio is expressed by making the antecedent the numerator, and the consequent the denominator, of a fraction.

2. Since a fraction may be multiplied by a fraction, a ratio may be multiplied by a ratio.

3. The ratio of two numbers is called a **Simple Ratio.**

4. The indicated product of two or more simple ratios is a **Compound Ratio.**

5. Compound ratios are usually expressed in the following form:

$$\left.\begin{array}{r} 6 : 10 \\ 4 : 5 \\ 2 : 7 \end{array}\right\}$$

This expression is read: the compound ratio of 6 to 10, 4 to 5, and 2 to 7. Its value is $\frac{6}{10}$ of $\frac{4}{5}$ of $\frac{2}{7}$. The value of a compound ratio may be expressed as a compound fraction.

NOTE. Give the rule for multiplying a fraction by a fraction. Make a rule for multiplying a ratio by a ratio.

Find the value of each of the following compound ratios, employing cancellation where possible:

1. $\left.\begin{array}{r} 3:5 \\ 4:7 \end{array}\right\}$ 3. $\left.\begin{array}{r} 5:\ 8 \\ 7:12 \\ 1:\ 6 \end{array}\right\}$ 5. $\left.\begin{array}{l} 7 \text{ pt.} : 12 \text{ pt.} \\ 4 \text{ A.} : 9 \text{ A.} \end{array}\right\}$

2. $\left.\begin{array}{r} \frac{2}{3} : \frac{3}{4} \\ \frac{1}{5} : \frac{2}{7} \end{array}\right\}$ 4. $\left.\begin{array}{l} \$3 : \$8 \\ 4 \text{ ft.} : 7 \text{ ft.} \end{array}\right\}$ 6. $\left.\begin{array}{r} 18:24 \\ 30:\ 9 \end{array}\right\}$

7. $\left.\begin{array}{r} 7\frac{1}{2} : 8\frac{3}{4} \\ 15 : 2\frac{1}{2} \\ \frac{5}{8} : \frac{7}{16} \end{array}\right\}$ 8. $\left.\begin{array}{r} 125 : 600 \\ 2.5 : 75 \\ \frac{2}{3} : 2 \\ 1\frac{1}{2} : 10 \end{array}\right\}$

329. 1. **A Compound Proportion** is a proportion in which there is a compound ratio.

2. The following problem involves a compound proportion:

If a man, working 8 hours a day, build 60 feet of fence in 2 days, how many feet of fence can a man build in 6 days, working 10 hours a day?

NOTE. What is assumed about the two fences? About the men?

ANALYSIS. If the days were of the same length, the proportion would read, 2 days : 6 days :: 60 feet : x feet. Since the 2 days are $\frac{8}{10}$ as long as the 6 days, the work done in 2 days will not be to the work done in 6 days as 2 is to 6, but will be $\frac{8}{10}$ of that ratio, which is found by multiplying 2 : 6 by 8 : 10.

$$\text{The form: } \left.\begin{array}{r} 2 : 6 \\ 8 : 10 \end{array}\right\} :: 60 : x$$

Because there are two elements involved in determining the time that each man worked (the number of days and their length), the relation of the times is expressed by a compound ratio.

RULE.

To solve a problem involving a compound proportion, take for the third term the antecedent of the ratio of which the required term is the consequent. With one pair of the remaining terms for the first ratio, state the proportion according to the conditions of the problem.

Proceed with the remaining pairs in the same way

until all of the conditions are stated. Find the product of the third term and all of the second terms, and divide it by the product of the first terms.

PROBLEMS.

1. If 4 men, in 5 days of 8 hours each, can dig a ditch 120 yards long, 3 feet wide, and 4 feet deep, in how many days of 10 hours each can 12 men dig a ditch 300 yards long, 4 feet wide, and $4\frac{1}{2}$ feet deep?

$$\text{Statement: } \left.\begin{array}{l} 120 \text{ yd.} : 300 \text{ yd.} \\ 3 \text{ ft.} : 4 \text{ ft.} \\ 4 \text{ ft.} : 4\frac{1}{2} \text{ ft.} \\ 12 \text{ men} : 4 \text{ men} \\ 10 \text{ hr.} : 8 \text{ hr.} \end{array}\right\} :: 5 \text{ days} : x.$$

Why is 12 put first in the fourth ratio? Why 10 in the last? What cancellation can be employed? Why?

2. If \$180 be paid for the work of 5 men for 24 days, what should be paid for the work of 17 men for 36 days?

3. If 15 men, in 16 days of 9 hours each, can do a piece of work, how many men will be needed to do the same piece of work in 8 days of 6 hours each?

4. If 15 men, in 16 days of 9 hours each, can do a certain piece of work, in how many days of 6 hours each can 45 men do the same work?

5. If 45 men can do a piece of work in 8 days of 6 hours each, how many hours a day must 15 men work to do the same work in 16 days?

6. If 50 tons of coal are required to run 4 engines 15 hours a day for 6 days, how many tons will be required to run 7 engines 18 hours a day for 11 days, with 3 times as heavy a load?

7. If it cost \$50 to make a walk 8 feet wide and 60 feet long, what will it cost to build a walk $7\frac{3}{8}$ feet wide and 72 feet long?

8. If it cost $50 to make a walk 8 feet wide and 60 feet long, what is the width of a walk that is 72 feet long, and costing $57.50?

9. If a walk that is 72 feet long and $7\frac{2}{3}$ feet wide cost $57.50, how long a walk that is 8 feet wide can be built for $50?

10. If 83 horses eat 933 bushels 3 pecks of oats in 30 days, how many bushels will 125 horses eat in 45 days?

11. If 44,640 bricks, 4 inches by 8 inches, will pave a court-yard, how many tiles 8 inches wide and 15 inches long will pave it?

NOTE. Solve these problems by straight-line analysis. Additional problems may be formed from those given, as illustrated in 7 and 8.

Remark. No problem can be solved by proportion that cannot be more easily solved by straight-line analysis.

330. PARTNERSHIP.

1. A and B went into business together, A investing $5,000, and B $7,000. They agreed to share gains and losses in proportion to their investments. The net gain was $2400. What was each one's share?

(*a*) Solve by analysis.

What was the whole investment? What part of it did A invest? What part of it did B invest? What was A's share? B's?

(*b*) Solve by proportion.

Explain these proportions.

(1) $12000 : $5000 : : $2400 : A's share.

(2) $12000 : $7000 : : $2400 : B's share.

2. The action of A and B is called "the formation of a partnership." The amount invested is called the **Capital.** A and B are called **Partners.** The agreement into which they enter is called the **Conditions of Partnership.**

3. A **Partnership** is an association of persons for the prosecution of business on joint account.

PROBLEMS.

1. A, B, and C formed a two-year partnership, agreeing to share gains and losses in proportion to their investments. A put in \$5,000; B, \$6,000; C, \$7,000. Their net gain was \$8,000. Find the share of each by analysis and by proportion.

2. If their net loss had been \$2,160, what would have been the loss of each?

3. A, B, and C formed a partnership, C being a silent partner. A invested \$6,000: B, \$8,500; C, \$10,000. By the conditions of partnership, A was to receive a salary of \$1,000, and B, \$700. The net profits were to be divided in proportion to investments. At the end of the first year the profits, exclusive of all expenses but salaries, were \$4,150. What was the share of each?

4. A, B, and C formed a partnership for 3 years. They were to draw equal amounts as salaries, and were to share the net profits equitably. A invested \$5,000 at the beginning, added \$3,000 to it at the end of the first year, and \$2,000 more at the end of the second. B invested \$6,500 at the beginning, withdrew \$2,000 at the end of the first year, and \$1,500 at the end of the second. C invested \$5,000 at the beginning, and did not change it. At the end of the time their net profits were \$4,850. What was the share of each?

5. The investments of three partners are in the ratio of 3, 4, and 5. If they gain \$3,600, what is the share of each?

6. A, B, and C owned a mill valued at \$18,000. A owned $\frac{1}{3}$ of it; B, $\frac{2}{5}$ of it; and C, the rest. It was insured for $\frac{4}{5}$ of its value. If it should be destroyed by fire what would each partner lose?

7. A, B, C, and D constructed a street railroad costing \$135,000. A furnished $\frac{1}{3}$ of the capital; B, $\frac{1}{2}$ of it; and C and D each furnished $\frac{1}{2}$ of the remainder. The company

sold to E $\frac{1}{15}$ of the road for $9,000; what part of this amount should each receive? What part of the stock would each of the original partners own after the sale?

8. A, B, and C took a contract for excavating a railroad cut. A furnished 60 men for 25 days; B, 50 men for 48 days; C, 75 men for 56 days. They received $20,250 for the work. What was the share of each?

9. A, B, and C engaged in business for one year, agreeing to share the profits in proportion to their investments. On January 1, A put in $3,000; B, $3,500; and C, $2,500. On March 1, A increased his share $500, B diminished his $500, and C increased his $250. On July 1, A withdrew $1,000, B put in $800, and C increased his $1,000. On October 1, A put in $600, B withdrew $400, and C withdrew $750. They gained $3,000. What was the share of each?

10. A, B, and C engaged in business for 2 years, with a capital of $16,000. A furnished $\frac{3}{8}$ and B $\frac{5}{8}$ of the capital. C conducted the business for one half the net profits. The gross earnings were $4,800. The expenses were $12\frac{1}{2}$%. What was A's share? B's?

11. What would have been A's share, if at the end of the first year he had transferred to B one third of his interest?

SECTION X.

331. INVOLUTION.

1. Multiply each of the following numbers by itself: 7, 9, 13, 24, 48, 69. The result in each of these cases is a Square.

2. The **Square** of a number is the product arising from multiplying that number by itself. The result is also called the **Second Power** of the number.

3. Learn the squares of all whole numbers from 1 to 25.

4. The expression 8^2 indicates that 8 is to be used twice as a factor. $8^2 = 8 \times 8 = 64$. The 2 as here used is called an Exponent.

5. An **Exponent** is an expression placed at the right of and above a number to indicate how many times it is to be used as a factor.

6. $12^2 = ?$ $18^2 = ?$ $125^2 = ?$ $(\frac{3}{4})^2 = ?$ $2\frac{1}{2}^2 = ?$ $.015^2 = ?$

7. $3 \times 3 \times 3 = ?$ This result is the cube, or third power, of 3. What is the cube of 4? of 5? of 8? of 10?

8. The **Cube** of a number is the product arising from using that number three times as a factor.

9. Make a definition for the Fourth Power of a number; for the Fifth Power; for the Eighth Power.

10. $5^3 = ?$ $8^3 = ?$ $(\frac{3}{4})^3 = ?$ $7^3 = ?$ $2^4 = ?$ $3^5 = ?$ $(\frac{1}{4})^4 = ?$ $(\frac{2}{3})^5 = ?$ $.05^4 = ?$

11. $25^2 = ?$ $16^3 = ?$ $(\frac{5}{6})^3 = ?$ $32^4 = ?$ $2.5^4 = ?$ $.02^5 = ?$ $(\frac{1}{5})^4 = ?$

12. Learn the cubes of all integers from 1 to 10.

13. Recite the following rapidly: 19^2, 16^2, 12^2, 5^3, 24^2, 7^3, 22^2, 4^3, 18^2, 21^2, 10^3, 6^3, 23^2, 13^2, 8^3, 17^2, 15^2, 9^3, 20^2, 14^2, 25^2.

332. EVOLUTION.

1. Name one of the two equal factors whose product is 4, 9, 25, 49, 81, 169.

2. Name one of the three equal factors whose product is 8, 27, 64, 125, 343, 729.

3. Name one of the four equal factors whose product is 16, 81, 256, 625.

4. Each of the preceding results is a Root.

5. A **Root** of a number is one of the equal factors whose product is the number.

6. The **Square Root** of a number is one of the two equal factors whose product is the number.

7. Define the Cube Root of a number; the Fourth Root; the Sixth Root.

8. What is the square root of 49? of 81? of 144? of 324? of 441? of 625?

9. **Evolution** is the process of finding the root of a number.

10. $\sqrt{4} = 2$. $\sqrt{25} = 5$. $\sqrt{144} = 12$. The sign placed before these numbers is called the **Radical Sign.** It indicates that a root of the number is to be found.

11. $\sqrt[3]{8} = 2$. $\sqrt[3]{64} = 4$. $\sqrt[4]{81} = 3$. $\sqrt[5]{32} = 2$. The expression placed above the sign is called the **Index.** The radical sign, when used alone before a number, indicates that its square root is to be extracted.

12 The index is read as its corresponding ordinal number. $\sqrt[3]{8}$ is read, "the third root of 8," or "the cube root of 8." Read $\sqrt[4]{16}$, $\sqrt[5]{32}$, $\sqrt{\frac{1}{4}}$, $\sqrt{2\frac{1}{4}}$.

13. $\sqrt{324}$, $\sqrt{\frac{4}{25}}$, $\sqrt{12\frac{1}{4}}$, $\sqrt{.25}$, $\sqrt{121}$, $\sqrt{1.21}$, $\sqrt{.0016}$.

14. Instead of the radical sign, a fractional exponent may be employed. $9^{\frac{1}{2}} = \sqrt{9}$. $27^{\frac{1}{3}} = \sqrt[3]{27}$. $64^{\frac{1}{4}} = \sqrt[4]{64}$.

15. To understand how numbers may be separated into equal factors, let us study their composition.

16. Since the square of 1 is 1, and of 9 is 81, it is clear that the square of a one-place number cannot be more than a two-place number. The square of 10, the smallest two-place number, is 100, a three-place number. The square of 99, the largest two-place number, is 9,801, a four-place number.

17. Similarly it may be shown that the square of any integral number contains twice as many places as the number, or one less than twice as many. Conversely, if any integral number be separated into periods of two places each, beginning with units, the number of periods thus formed will be the same as the number of places in the square root. The left-hand period may have but one place.

18. What is the square of .1? of .9? of .01? of .09? The square of any decimal fraction will contain how many times as many decimal places as the fraction? Why?

19. To find the number of places in the square root of a decimal fraction:

a. If the fraction has an odd number of places, annex one zero. Why?

b. Beginning with tenths' order, separate the expression at the right of the decimal point into periods of two places each.

20. Study the form of the square of 64.

$$\begin{array}{r}
64 \\
64 \\
\hline
4 \times 6 + 4 \times 4 \\
6 \times 6 + 6 \times 4 \\
\hline
6 \times 6 + 2 \times 4 \times 6 + 4 \times 4
\end{array}$$

It is clear that the square of any two-place number must take this form: The square of the tens + twice the product of the tens and units + the square of the units. Remem-

bering this fact, the square root of a four-place number may be found as follows:

Illustrative Problem. **21.** $\sqrt{4489} = ?$

Since 4489 is a four-place number, its square root must be a two-place number. Since the square of tens is hundreds, it must be found in 44 hundreds. The largest square in 44 hundreds is 36 hundreds. Its square root is 6 tens. Subtracting 36 hundreds, the remainder is 8 hundreds. The entire remainder is 889. If the original number is a square, 889 is the sum of twice the tens of the root by the units, and the square of the units. The tens' term is 6. Twice the tens is 12 tens. The product of the 12 tens and the units is tens; hence, it must be in 88 tens. 7 is probably the units' term of the root. 7×12 tens is 84 tens. 88 tens $-$ 84 tens $=$ 4 tens. 4 tens $+ 9 = 49$. 49 is the square of 7; hence, the square root of 4489 is 67.

NOTE. If the number should be a pure or mixed decimal, it may be considered an integer, and the result may be corrected.

FORM.

$$\begin{array}{r|l} & 4489 \;\big|\; \underline{67} \\ & 36 \\ \underline{12} & 88 \\ & 84 \\ \underline{7} & 49 \\ & 49 \end{array}$$

NOTE. Explain the following form, showing that it is briefer than the former.

$$\begin{array}{r|l} & 4489 \;\big|\; 67 \\ & 36 \\ \underline{127} & 889 \\ & 889 \end{array}$$

22. How can this plan be extended to larger numbers?

Illustrative Problem. $\sqrt{288369} = ?$

By examining the number it is found that its root is a three-place number. We may first deal with 2883.

FORM.

```
       2883 | 53
       25
  103 | 383
        309
         74
```

53 may now be regarded as the tens' term of the root. To the remainder, 74, the remainder 69 may be annexed, and the work continued as before.

FORM.

```
       288369 | 537
       25
  103 | 383
        309
 1067 | 7469
        7469
```

333. RULE FOR THE EXTRACTION OF THE SQUARE ROOT OF A NUMBER.

1. ***Beginning at the decimal point, group the figures into periods of two orders each.***

2. ***Find the largest square in the left-hand period and place its root at the right, as the first term of the root.***

3. ***Subtract the square from the left-hand period, and to the remainder annex the next period.***

4. ***Double the root already found, and using it as a trial divisor find how many times it is contained in the new dividend exclusive of its right-hand term. Place the quotient as the second term of the root, and also annex it to the trial divisor. Multiply the complete divisor by the second term of the root, subtract the product from the partial dividend, and proceed as before.***

5. ***If the trial dividend will not contain the trial divisor, annex a zero to the root and to the trial divisor, annex a new period to the trial dividend, and proceed as before.***

6. ***If there is a remainder after the last operation, to continue the work reduce the remainder to hundredths, ten-thousandths, etc., continuing the work as before.***

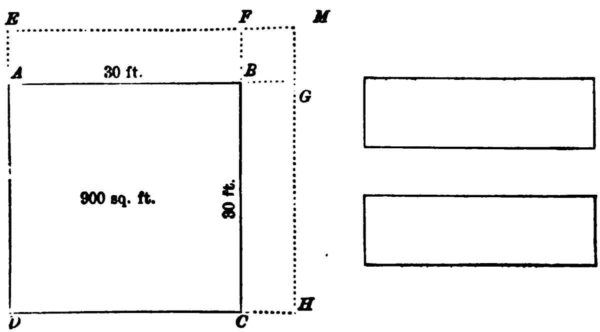
7. ***To extract the square root of a common fraction where the terms are squares, extract the square root of each term. If only the denominator is a square, extract the approximate root of the numerator and divide it by the root of the denominator.***

If the denominator is not a square, change the fraction to a decimal and extract its approximate root.

334. 1. Since the number of units in the area of a square is the square of the number of units in one side, the square root of the number of units of area is the number of units in a side; hence, if the area of a square be given, a side may be found by applying the preceding rule. The process of applying the rule may also be illustrated by diagrams.

2. What is one side of a square whose area is 1225 square feet?

First, confining our attention to the left-hand period, 12, it appears that this square contains at least 900 square feet. Each side of such a square is 30 feet.



There yet remain 325 square feet with which to make additions to this figure. They must be made so as to retain the form of a square; hence, the length and width must be equally increased. If a rectangle a foot wide were to be added to the sides A B and B C, 2 × 30 square feet (twice the tens) would be needed. The remainder is large enough to make these additions 5 feet wide, and leave 25 square feet with which to fill out the space F M G B.

NOTE. Use the figure to illustrate the extraction of the square root of 541696.

PROBLEMS.

Find the square root of each of the following numbers:

1. 4489.	8. 143641.	15. 40640625.
2. 7921.	9. 214369.	16. 9036036.
3. 9216.	10. 450241.	17. 23261329.
4. 15625.	11. 466489.	18. .6889.
5. 42436.	12. 519841.	19. .355216.
6. 82369.	13. 567009.	20. 76.3876.
7. 93025.	14. 622521.	21. $\frac{49}{64}$.

Find the root to hundredths.

22. $\frac{25}{81}$.	26. 28.	30. 3.6.
23. $\frac{121}{169}$.	27. 6.	31. 4.9.
24. $\frac{196}{625}$.	28. 2.	32. 8.1.
25. $\frac{225}{441}$.	29. .7.	33. 12.1.

See Art. **333**, 7.

34. $\frac{18}{25}$.	38. $\frac{2}{3}$.	42. $12\frac{1}{4}$.
35. $\frac{26}{81}$.	39. $\frac{7}{8}$.	43. $8\frac{1}{4}$.
36. $\frac{7}{9}$.	40. $\frac{11}{24}$.	44. $6\frac{2}{3}$.
37. $\frac{35}{144}$.	41. $2\frac{1}{4}$.	45. $16\frac{1}{4}$.

335. 1. Find the length of one side of a square field containing 17 A. 89 sq. rd.

2. The entire surface of a cubical block contains $22\frac{1}{24}$ square feet. What is the length of one edge?

3. A square contains 900 square inches. What are the width and length of an equivalent rectangle whose width is to its length as 1 to 4?

4. Supply a mean proportional in each of the following proportions:

a. $12 : x :: x : 48.$

b. $192 : x :: x : 27.$

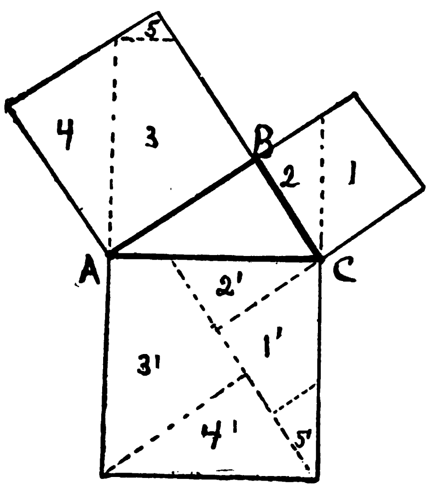
c. $48 : x :: x : 432.$

NOTE. A mean proportional is a number that is the second and third term of a proportion.

5. How many rods of fence will enclose a square field containing 10 acres?

6. A body of 7,921 soldiers is arranged so that there are as many in rank as there are in file. How many are there in each?

7. A rectangular surface whose width is $\frac{5}{6}$ of its length contains 1,470 square feet. Find its width and length.



336. THE RIGHT TRIANGLE.

1. A triangle, one of whose angles is a right angle, is called a **right triangle.** The side opposite the right angle is the **hypotenuse.** The other sides are the **base** and **altitude.** The sides forming the right angle are called the **arms.**

2. Draw a right triangle, A B C, on a sheet of pasteboard.

Draw the squares on the three sides and subdivide as shown in this figure, by extending the sides of the largest square through the smaller squares, and drawing a line at right angles to the longer extension. With a sharp knife cut out the five pieces and place them in the positions 1′, 2′, 3′, etc. We thus see that the square of the hypotenuse is equal to the sum of the squares of the arms.

PROBLEMS.

	Base.	Altitude.	Hypotenuse.
1.	8 inches.	?	10 inches.
2.	20 feet.	15 feet.	?
3.	224 yards.	?	260 yards.
4.	?	272 miles.	353 miles.
5.	192 rods.	144 rods.	?

6. The top of a ladder that is 30 feet long rests against a telegraph pole 24 feet from the ground; how far is the foot of the ladder from the foot of the pole?

7. A and B start from the same point at the same time. A travels north and B east, the former traveling at the rate of four miles an hour and the latter three. How many feet apart are they in 15 minutes?

8. A rope is attached to the top of a 96-foot pole. It touches the ground 28 feet from the foot of the pole. What is its length?

9. What is the diagonal of a rectangle whose dimensions are 6 yards and 8 yards?

Note. The diagonal of a rectangle is the straight line joining opposite vertices.

10. A 13-foot ladder rests with its top against a window sill 12 feet from the ground. How many feet from the wall to the foot of the ladder?

11. Against the top of a pole 15 feet high are braced in opposite directions a 17-foot ladder and a 25-foot ladder. How far apart are the feet of the ladders. Make diagram.

12. A 25-foot ladder stands erect against the wall of a building. The foot is pulled out until the top is lowered one foot. How far from foot of ladder to wall?

13. If the foot of the ladder is pulled out 8 feet farther, how much more is the top lowered?

14. What is the area of a rectangle whose length and diagonal are respectively 15 and 17 rods?

15. What is the length of the longest wire that can be stretched straight in a room $20' \times 15' \times 12'$?

16. What is the diagonal of an inch cube?

17. What is the shortest line that can be traced on the surface of a cube, joining the extremities of a diagonal?

18. What is the diameter of a circle that will just enclose four silver dollars arranged in the form of a square?

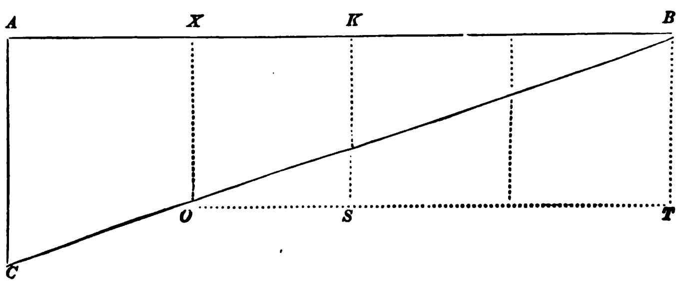
19. How much is saved by going diagonally across a section instead of along the boundary?

20. How many acres in a square field whose diagonal exceeds its side by 16.568 rods?

NOTE. The values of $\sqrt{2}$ and $\sqrt{3}$ should be memorized.

21. A rectangular field three times as long as wide contains 87 acres. What is its length?

22. A triangular field, A B C, is 60 rods long and 20 rods wide. At what distance from B may a fence be built dividing the field into two equal areas?



QUERIES. A C is what part of A B? X O is what part of X B? What is the area of rectangle O X B T? What is the area of the square O X K S? What is the length of O X? of X B?

23.* At what point shall a triangular board, 12 feet long and 12 inches wide, be sawn into two equivalent pieces?

24. At what point, if the 12-foot board be a trapezoid, 15 inches wide at one end, 3 inches at the other?

25. Show that 128 stakes a foot apart can be driven on a ten-foot square.

26. The sides of an equilateral triangle are six feet each. What is the altitude?

27. What is the area of an equilateral triangle, each side of which is 4 feet?

28. Draw the three altitudes of an equilateral triangle from the vertices to the opposite sides. They meet at a common point. Join this point with the vertices forming three equal triangles. Show that the altitude of each is $\frac{1}{3}$ the altitude of the equilateral triangle.

29. What is the diameter of the circle that will just enclose three silver dollars arranged in the form of a triangle?

30. What is the diameter of a circle whose area is 314.16 square feet?

31. With what length of rope shall a horse be tethered to graze over one fourth of an acre?

32. A road two rods wide about a square field contains one acre. What is the area of the field?

33. A road two rods wide about a circular field contains one acre. What is the area of the circle?

34. About a circular field 80 rods in diameter is a road of uniform width containing six acres. Width of the road?

35. How much must the diameter of a 36-inch grindstone be reduced, to reduce the weight one fourth? (No allowance for the opening.)

* A right triangle. Use figure, page 326. A C is $\frac{1}{12}$ of A B. Assume X dividing point. X O is $\frac{1}{12}$ of X B. Divide X B into 12 parts. Area of X B O? Of X B T O? Of each of the 12 squares? Length of X O? Of X B?

337. CUBE ROOT.

1. What is $5 \times 5 \times 5$? $9 \times 9 \times 9$? $23 \times 23 \times 23$? $86 \times 86 \times 86$? $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$? $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$? $.8 \times .8 \times .8$? $.04 \times .04 \times .04$? The product arising from each of these indicated multiplications is a **Cube.**

2. The cube of a number is the product arising from using that number three times as a factor.

3. What is the cube of 79? 93? 207? 300? $\frac{3}{4}$? $\frac{7}{12}$? $1\frac{1}{2}$? 1.6? .24? $10^3 = ?$ $\frac{2}{5}^3 = ?$ $(\frac{1}{7})^3 = ?$ $.06^3 = ?$

4. What is one of the three equal factors whose product is 8? 27? 125? 729? $\frac{64}{125}$? $\frac{1}{8}$? .216? .000512? $3\frac{3}{8}$?

5. The **Cube Root** of a number is one of the three equal factors whose product is the number.

6. $\sqrt[3]{8} = ?$ $\sqrt[3]{27} = ?$ $\sqrt[3]{\frac{125}{512}} = ?$ $\sqrt[3]{.729} = ?$

7. Define Exponent, Radical Sign, Index, Fractional Exponent.

338. EXTRACTION OF THE CUBE ROOT OF NUMBERS.

1. The method of extracting the cube root of any number above 1000 will be ascertained by studying the form of the cube of a two-place number.

2. $46^3 = 46 \times 46 \times 46 = (4t + 6)(4t + 6)(4t + 6)$.

$$\begin{array}{r}
4t + 6 \\
\underline{4t + 6} \\
4t \times 6 + 6^2 \\
\underline{(4t)^2 + 4t \times 6 } \\
(4t)^2 + 2 \times 4t \times 6 + 6^2 \\
\underline{4t + 6 } \\
1 \times (4t)^2 \times 6 + 2 \times 4t \times 6^2 + 6^3 \\
\underline{(4t)^3 + 2 \times (4t)^2 \times 6 + 1 \times 4t \times 6^2 } \\
(4t)^3 + 3 \times (4t)^2 \times 6 + 3 \times 4t \times 6^2 + 6^3
\end{array}$$

3. Stating the above result in words, we have the following:

The cube of a two-place number consists of (1) the cube of the tens, (2) plus three times the square of the tens by the units, (3) plus three times the tens by the square of the units, (4) plus the cube of the units.

NOTE. Verify the above statement and fix it in the memory.

4. *Illustrative Example.* The cube of $35 = 30^3$ or 27000 $+ 3 \times 30^2 \times 5$, or $13500 + 3 \times 30 \times 5^2$, or 2250, $+ 5^3$, or 125.

Give the several parts of the cube of 24; of 32; of 41; of 66; of 87; of 93.

5. The cube of 1 is 1. The cube of 9 is 729. Therefore, the cube of a one-place number is not more than a three-place number.

6. The cube of 10 is 1000. The cube of 99 is 970299. Therefore, the cube of a two-place number cannot be less than a four-place, nor more than a six-place, number.

7. Similarly, it may be shown that the cube of any integral number contains three times as many orders as the number, or three times as many less one or two.

8. The number of orders in the root of an integer may be ascertained by beginning at the decimal point, and, so far as possible, grouping the figures into periods of three orders each. The left-hand period may contain only one or two figures.

339. *Illustrative Example.* $\sqrt[3]{103823} = ?$

Since this is a six-place number, the cube root of the largest cube in it is a two-place number. The root, therefore, consists of some number of tens plus some number of units.

I first withdraw from the number the largest cube in 103 thousands. This is 64 thousands. Its cube root is 4 tens. $103823 - 64000 = 39823$. We have seen that the second part of the cube is "three times the square of the tens by

the units." Three times the square of the tens is 48 hundreds. If this were multiplied by the units of the root, the product would be hundreds; hence, would not be of a lower denomination than 398 hundreds. The 398 hundreds, then, may be used as a trial dividend with which to find the units' term of the root. The trial dividend seems large enough to indicate that the units' term is 8, but it is to be remembered that there are two more parts to the cube. Trying 7 as the units' term, "three times the square of the tens by the units" is 336 hundreds. "Three times the tens by the square of the units" is 588 tens. "The cube of the units" is 343. The sum of these numbers is 39823; hence, 103823 is a cube, and its root is 47.

FORM.

$$
\begin{array}{r|l}
 & 103823 \;\underline{|\,47} \\
 & 64 \\
3 \times (4\,t^2) = 48 \text{ hundreds} & 398 \\
 & \underline{336} = 7 \times 48 \text{ hundreds.} \\
 & 622 \\
 & \underline{588} = 3 \times 4\,t \times 7^2 \\
 & 343 \\
 & \underline{343} = 7^3
\end{array}
$$

Since "three times the square of the tens" is to be multiplied by the units, and since "three times the tens by the square of the units" may be found by multiplying three times the tens by the units, and that product by the units, and since "the cube of the units" equals the square of the units by the units, it is clear that the units' term is a factor of each of these parts of the cube. The three parts may be reduced to one by arranging them as follows: $(3\,t^2 + 3\,t\,u + u^2)\,u$. Here $3\,t^2$ is the trial divisor, and $(3\,t^2 + 3\,t\,u + u^2)$ is the complete divisor.

FORM.

$$\begin{array}{rr|l}
 & & t\ u \\
 & 103823 & \underline{4\ 7} \\
 & 64 & \\
3 \times t^2 = 4800 & 39823 & \\
3\,t\,u \ = \ 840 & & \\
u^2 \ = \ 49 & & \\
\hline
5689 & 39823 &
\end{array}$$

NOTE. $3\,t\,u$ is one order lower than $3\,t^2$. Why? u^2 is one order lower than $3\,t\,u$. Why? Why are the two zeros placed at the right of 48?

PROBLEMS.

1. $\sqrt[3]{79507} = ?$
2. $\sqrt[3]{157464} = ?$
3. $\sqrt[3]{314432} = ?$
4. $\sqrt[3]{357911} = ?$
5. $\sqrt[3]{551368} = ?$
6. $\sqrt[3]{658503} = ?$
7. $\sqrt[3]{804357} = ?$
8. $\sqrt[3]{884736} = ?$
9. $\sqrt[3]{970299} = ?$
10. $\sqrt[3]{912673} = ?$

NOTE. Pupils should work on similar problems until they can be solved easily at the rate of one a minute.

340. DECIMAL FRACTIONS.

1. The cube of .1 is .001. The cube of .9 is .729. The cube of .01 is .000001. The cube of .09 is .000729. The cube of tenths is thousandths; of hundredths is millionths; of thousandths is billionths, etc.

2. ***To find the number of orders in the cube root of a decimal fraction, begin at the decimal point and group the figures into periods of three orders each.***

If the right-hand period is not full, annex zeros. (Why?)

PROBLEMS.

Proceed as with whole numbers.

1. $\sqrt[3]{.032768} = ?$
2. $\sqrt[3]{.079507} = ?$
3. $\sqrt[3]{.000195112} = ?$
4. $\sqrt[3]{.000000493039} = ?$

341. COMMON FRACTIONS.

1. How is the cube of a common fraction found? How, then, may the cube root of a common fraction be found? $\sqrt[3]{\frac{8}{27}} = ?$ $\sqrt[3]{\frac{125}{512}} = ?$ $\sqrt[3]{\frac{79507}{157464}} = ?$

Make a rule for the extraction of the cube root of a common fraction.

2. If the denominator is not a cube, change the common fraction to a decimal fraction, and proceed as in Art. **340**.

342. $\sqrt[3]{101847563} = ?$

OPERATION.

$$
\begin{array}{r r | l l}
 & & 101847563 & \underline{|\,467} \\
 & & 64 & \\
\cline{3-3}
3 \times 4^2 = & 4800 & 37847 & \\
3 \times 4 \times 6 = & 720 & & \\
6 \times 6 = & 36 & & \\
\cline{2-2}
 & 5556 & 33336 & \\
\hline
3 \times 46^2 = & 634800 & 4511563 & \\
3 \times 46 \times 7 = & 9660 & & \\
7 \times 7 = & 49 & & \\
\hline
 & 614509 & 4511563 & \\
\cline{3-3}
\end{array}
$$

First consider the first two periods only, and proceed as in Art. **339**. Having found the first two terms of the root, consider them as the tens' term, and proceed as before.

343. RULE FOR FINDING THE CUBE ROOT OF A NUMBER.

1. ***Point off the number into periods of three places each, beginning at the decimal point and counting to the left for integers and to the right for decimals.***

2. ***Find the largest cube in the left-hand period and place its root at the right. Subtract the cube from the left-hand period and annex the second period to the remainder.***

3. ***Find three times the square of the first term of the root, annex two zeros, and place it at the left as a trial divisor. Compare it with the second dividend and place the quotient as the second term of the root.***

4. ***Find three times the product of the first and second terms of the root, annex one zero, and write it under the trial divisor. Square the second term of the root and write the result under the preceding product. Find the sum of these three results and multiply it by the second term of the root. Subtract the product thus found from the partial dividend and to the remainder annex the next period.***

5. ***Find three times the square of the root already found, annex two zeros, and write it at the left as a trial divisor. Find the third term of the root and complete the divisor as before.***

NOTES. (1) If the number is not a cube, the work may be continued to any extent by reducing the last remainder to thousandths, millionths, etc., by annexing periods of three zeros each.

(2) If the denominator of a common fraction is not a cube, both terms may be multiplied by some number that will make the denominator a cube. Why is it more important to have the denominator a cube than the numerator?

(3) If the partial dividend at any time will not contain the trial divisor, write a zero in the root, annex two zeros to the preceding divisor as a new divisor, and proceed as before. Show the reason for annexing these zeros by going through the work with zero as a term of the root.

(4) The rules for squaring numbers from 25 to 100 will be found convenient in forming the trial divisor for the third term of the root.

(5) Mixed numbers should usually be changed to improper fractions or to mixed decimals.

344. The following rules will be found convenient:

RULES FOR SQUARING NUMBERS FROM 25 TO 100.

1. ***To square numbers from 25 to 50.***

(a) ***Subtract 25 from the number.***

(b) ***Subtract the remainder from 25.***

(c) ***Square the last result, and***

(d) ***Add it to the first result considered as hundreds.***

Illustrative Example. $42^2 = ?$

(*a*) 17. (*b*) 8. (*c*) 64. (*d*) 1764.

2. ***To square numbers from 50 to 75.***
(a) ***Subtract 50 from the number.***
(b) ***Add the result to 25.***
(c) ***Square the first result, and***
(d) ***Add it to the second result considered as hundreds.***

Illustrative Example. $69^2 = ?$

(*a*) 19. (*b*) 44. (*c*) 361. (*d*) 4761.

3. ***To square numbers from 75 to 100.***
(a) ***Subtract the number from 100.***
(b) ***Subtract the first result from the number.***
(c) ***Square the first result, and***
(d) ***Add it to the second result considered as hundreds.***

Illustrative Example. $88^2 = ?$

(*a*) 12. (*b*) 76. (*c*) 144. (*d*) 7744.

NOTE. Inquisitive students will desire to find the reasons for these rules.

The first is an application of this formula: $(a - b)^2 = a^2 - 2\,a\,b + b^2$. a here represents 50, and b the difference between 50 and the given number.

The second is an application of $(a + b)^2 = a^2 + 2\,a\,b + b^2$; the letters being used as in the first.

The third employs $(a - b)^2$; a representing 100, and b the difference between 100 and the given number.

345. PROBLEMS.

1. $\sqrt[3]{86004573} = ?$ What is the remainder?
2. $\sqrt[3]{.0685} = ?$ Carry root to thousandths.
3. $\sqrt[3]{2} = ?$ Carry root to hundredths.
4. $\sqrt[3]{.5} = ?$
5. $\sqrt[3]{3698.400375} = ?$
6. $15.625^{\frac{1}{3}} = ?$
7. $\sqrt[3]{44\frac{3}{8}} = ?$

8. $\sqrt[3]{244\frac{9}{64}} = ?$

9. $\sqrt[3]{9\frac{305}{512}} = ?$

10. $\sqrt[3]{27189441343} = ?$

11. $84328125^{\frac{1}{3}} = ?$

12. $194104539^{\frac{1}{3}} = ?$

13. $223648543^{\frac{1}{3}} = ?$

14. $736314327^{\frac{1}{3}} = ?$

15. $\sqrt[3]{1003003001} = ?$

16. $\sqrt[3]{\frac{2}{3}} = ?$

17. $\sqrt[3]{2\frac{1}{2}} = ?$

18. $\sqrt[3]{.27} = ?$

19. $\sqrt[3]{6.4} = ?$

20. $\sqrt[3]{.0125} = ?$

21. $\sqrt[3]{\frac{5}{27}} = ?$

22. $9^{\frac{1}{3}} = ?$

23. $\sqrt[3]{\frac{3}{7}} = ?$

NOTE. Teachers should dictate many problems until pupils can work rapidly and accurately.

24. A cubical block contains 96 feet of lumber. How many inches long is each edge?

25. A cubical cistern holding 3,600 gallons is how deep?

26. Find the dimensions of a cubical bin whose capacity is 2,000 bushels.

346. 1. The method of extracting the cube root of a number may be illustrated by the use of blocks.

2. $\sqrt[3]{50653} = ?$

A cubical figure contains 50,653 cubic inches. What is the length of one edge?

3. The number of inches in the edge is a two-place number. The largest cube in 50,000 is 27,000. 27,000 inch-cubes will form a figure each of whose edges is 30 inches. This 27,000 is "the cube of the tens," the first part of the expansion shown in Art. 338, 2. 23,653 inch-cubes remain with which to enlarge the figure. The size must be increased in such a way as to retain the form of a cube. Since the length, breadth, and thickness are equal, the additions must be made to three adjacent faces, and must be equal. If a layer of cubes were placed upon one face, $30 \times 30 = 900$ cubes would be required. Since three such additions are to be made, 2,700 cubes would be needed to make the additions one inch thick on each face. We thus find the illustration of "three times the square of the tens as a trial divisor." It is called a "trial divisor" because we wish to ascertain how many such additions may be made to each face with the remaining blocks, and yet leave enough to fill out the figure.

The indications are that 7 such layers may be added to each face. This would use up $3 \times 30^2 \times 7$ blocks. The expression is "three times the square of the tens by the units," or the second part of the expansion shown in Art **338**, 2.

$3 \times 30^2 \times 7 = 18{,}900$. $23{,}653 - 18{,}900 = 4{,}753$, the number of inch-cubes remaining.

The figure is now 37 inches wide, 37 inches long, and 37 inches high, but is not a cube, since additions are still to be made along three edges and on one corner. Each of these additions to the edges must be 7 inches by 7 inches and 30 inches high. These will require $3 \times 30 \times 7^2 = 4{,}410$ blocks. This expression is "three times the tens by the square of the units," or the third part of the expansion shown in Art. **338**, 2.

$4{,}753 - 4{,}410 = 343$, the remaining blocks.

An unfilled corner, 7 inches by 7 inches by 7 inches, remains. Since 343 blocks are needed to make the figure a cube, it is clear that 50,653 is the cube of 37. The 343 is "the cube of the units," or the last part of the expansion shown in Art **338**, 2.

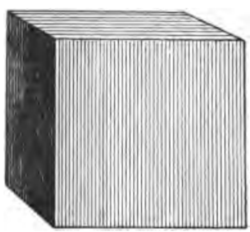


FIG. 1. A Cube.

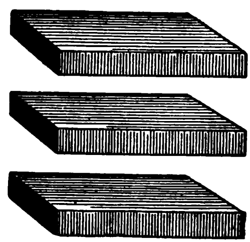


FIG. 2. Additions for Cube.

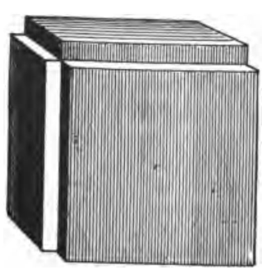


FIG. 3. Additions made to Cube.

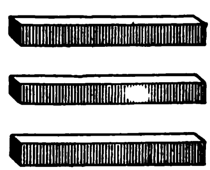


FIG. 4. Three Corners.

FIG. 5. Small Cube for Corner.

EXPLANATION OF THE FIGURES.

Fig. 1 represents the cube formed from 27,000 blocks.

Fig. 2 represents the three additions made to the faces of Fig. 1.

Fig. 3 represents the figure resulting from the three additions to the faces of Fig. 1.

Fig. 4 represents the three additions made to the edges, and

Fig. 5 the final addition made to the corner to complete the cube.

Apply this method of illustration to Problems 1–6, Art. **339**.

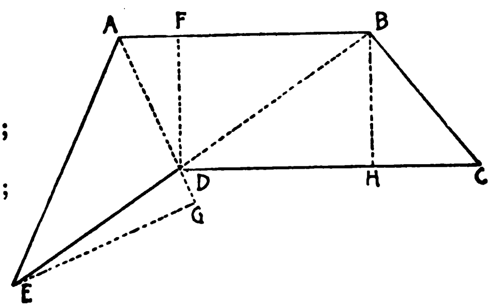
347. MISCELLANEOUS PROBLEMS.

1. What is the area of a piece of ground arranged in the form of a triangle, the length of one side being 124 rods, and the distance to the vertex of the opposite angle being 86 rods?

2. How many feet of lumber are there in a two-inch plank in the form of an isosceles triangle, 18 feet long and 14 inches wide at the base?

3. Find the area in acres of the irregular field $ABCDE$.

Distance:
$AD = 40$ rd.;
$EG = 46$ rd.;
$AB = 46\frac{1}{2}$ rd.;
$DF = 37$ rd.;
$BH = 38\frac{1}{2}$ rd.;
$DC = 52$ rd.

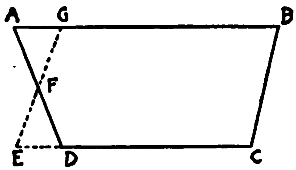


4. A board in the form of a rhombus is 19 inches on each side and 10 inches high. Draw the figure. What is its area? Compare it with a square that is 19 inches on each side.

Note. A Rhombus is an oblique-angled equilateral parallelogram.

5. A field in the form of a trapezoid contains $23\frac{1}{3}$ acres. One of its parallel sides is 95 rods and the other 65 rods long. What is its width?

NOTE 1. A Trapezoid is a quadrilateral only two of whose sides are parallel. Its area is equal to the product of its height and half the sum of its parallel sides.



NOTE 2. Drawing GE parallel to BC makes $GBCE$ a parallelogram. $AGF = EFD$; hence $ABCD$ is equivalent to $GBCE$, and

$$EC = \tfrac{1}{2}(AB + CD).$$

6. What is the area of a circular pond whose diameter is 15 rods?

7. What is the circumference of a circus ring whose area is $872\frac{2}{3}$ square yards?

8. What is the area of a field in the form of a trapezoid, the parallel sides being 42 rods and 88 rods respectively, and the distance between these sides being 36 rd. 3 yd.?

9. A cubical box contains 79,507 cubic inches. What is the length of each edge?

10. A cubical tank has a capacity of $760\frac{1}{4}$ gallons. Find the length of one side. (Approximate.)

11. A cubical cistern is 9 feet deep. What will it cost to construct it at 75 cents a barrel ($31\frac{1}{2}$ gallons)?

12. A corn-bin whose width equals its height is 4 times as long as it is high. If it will hold 3,200 bushels of shelled corn, what is its length?

13. A building is 36 feet wide. If the attic is 9 feet high, what is the length of the rafters, allowing for a projection of 18 inches?

14. If the area of an equilateral triangle is 12 square feet, what is the area of a similar triangle each of whose sides is twice as long? Four times as long? $7\frac{1}{2}$ times as long?

NOTE. Similar plane figures are those having the same shape; e. g., a square is similar to a square, a circle to a circle.

Two similar surfaces are to each other as the squares of like lines.

15. A circle whose diameter is 4 feet is what part of a circle whose radius is 6 feet? 8 feet? 9 feet? $10\frac{1}{2}$ feet?

16. A circle whose diameter is 6 feet is how many times a circle whose diameter is 6 inches? 9 inches? $1\frac{1}{2}$ feet? 4 feet?

17. A cube 2 feet high is how many times one that is 1 foot high?

NOTE. Similar solids are those having the same shape; e. g., a cube is similar to a cube, a sphere to a sphere.

Similar solids are to each other as the cubes of like lines.

18. A sphere whose diameter is 1 inch is what part of a sphere whose diameter is 2 inches? 4 inches? 7 inches? 1 foot?

19. How many 2-inch spheres contain as much volume as a 4-inch sphere? an 8-inch sphere? a foot sphere?

20. Compare the volumes of earth and moon, the diameter of the former being about 8,000 miles, and of the latter about 4,000 miles. Compare their surfaces.

21. Compare the volumes of sun and earth, the diameter of the former being about 880,000 miles. Compare their surfaces.

22. If a cannon-ball weighs 36 pounds, what will one weigh whose diameter is 3 times as great?

23. Height of cylinder 6 feet; diameter of base $2\frac{1}{2}$ feet. Find convex surface. Find entire surface. Find volume.

24. The volume of a cylinder is 72 cubic feet. The diameter is 4 feet. What is the convex surface? Entire surface?

25. The volume of a cylinder is 196.35 cubic feet. Its height is 10 feet. What is the area of the base of a similar cylinder whose volume is 27 times as great?

Note. The surface of a sphere is 3.1416 times the square of its diameter.

26. What is the surface of a sphere whose diameter is 4 inches? $2\frac{1}{2}$ feet? 1 yard?

27. What is the area of the earth's surface, counting the diameter 7,912 miles?

28. The surface of a sphere that is 1 inch in diameter is what part of the surface of a sphere whose diameter is 2 inches? 4 inches? $6\frac{1}{2}$ inches?

29. What is the diameter of a sphere whose surface is 31,416 square miles? 201.0624 square feet? 337 square inches?

30. Compare the surfaces of two spheres whose diameters are as 3 to 7. As 2 to 5. As $2\frac{1}{2}$ to $3\frac{1}{3}$.

31. The volume of a sphere is 3.1416 times $\frac{1}{6}$ of the cube of the diameter. What is the volume of a sphere whose diameter is 6 inches?

32. What is the approximate interior diameter of a sphere that will hold a gallon?

33. What is the weight of a sphere of gold 2 inches in diameter?

34. What is the weight of a sphere of stone whose diameter is 30 inches, the stone weighing three times as much as water?

35. Steel weighs about 7.84 times as much as water. What is the weight of a hollow steel sphere whose interior diameter is 12 inches, the shell being $\frac{1}{4}$ of an inch thick?

36. The State in which you live is what part of the surface of the earth?

37. What is the diameter of a circle whose area is equal to the area of the State in which you live?

38. What is the surface of a sphere whose diameter is one foot?

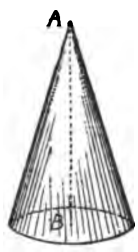
39. If the above sphere represents the world, for how many square miles does a square inch of its surface stand? What, then, is the scale?

40. What is the diameter of a circle that will represent the area of the State in which you live, on the above globe?

41. What is the weight of a cast-iron street-roller, 8 feet long and 5 feet in diameter, the shell being 2 inches thick, cast-iron being 7.2 times as heavy as water?

348. THE CONE.

1. A Cone is a solid formed by the revolution of a right triangle upon its base or perpendicular as an axis.



2. A cone is one third of a cylinder having the same base and altitude (height).

3. The convex surface of a cone is equal to one half the product of the circumference of the base and the slant height. Why?

PROBLEMS.

1. Height, 20 inches; radius, 15 inches. Find convex surface and volume.

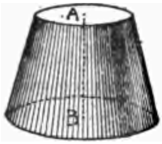
2. Slant height, 10 inches; height, 8 inches. Find volume and entire surface.

3. Circumference of base, 75.3984 feet; slant height, 20 feet. Find volume and entire surface.

4. The radius of a cone is 7 inches and its altitude 15 inches. What is the convex surface of a similar cone whose radius is 10 inches? What is its volume?

5. What is the weight of a steel paper-weight in the form of a cone, the diameter of the base being 3 inches and the height 4 inches, steel being 7.83 times as heavy as water?

349. 1. The Frustum of a cone is that portion of the cone included between the base and a plane passing through the cone parallel to the base.



2. The convex surface of a frustum of a cone is a Trapezoid. How may its area be found?

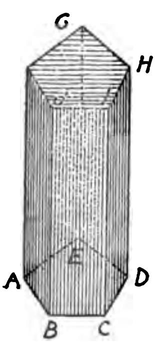
3. The frustum of a cone is equivalent to three cones having the altitude of the frustum, and whose bases are the upper base of the frustum, the lower base, and a mean proportional between them.

4. ***To find the volume of a frustum of a cone, find the sum of the upper base, the lower base, and the square root of their product, and multiply the result by one third of the altitude (distance between the bases).***

PROBLEM.

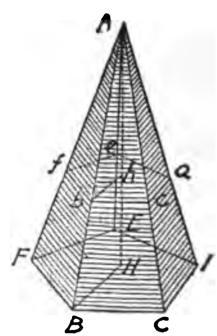
Find convex surface of a frustum of a cone, the radius of whose upper base is 10 inches, of the lower base 15 inches, and whose altitude is 21 inches. Find the volume.

350. PRISMS AND PYRAMIDS.



1. A Right Prism is a solid two of whose faces are equal and parallel polygons, and the rest of whose faces are rectangles.

2. The volume of a right prism is found in the same way as the volume of a right parallelopiped.



3. A pyramid is a solid that is bounded by a polygon called the base, and three or more triangles that meet at a point.

4. The volume and convex surface of a pyramid are found in the same way as the volume and convex surface of the cone.

5. Define a Frustum of a Pyramid. Art. **349**, 1.

Tell how to find the volume and convex surface of a frustum of a pyramid. Art. **349**, 2, 3.

Describe the above figures. Find a frustum of a pyramid.

PROBLEMS.

1. What is the volume of a prism whose base is $7\frac{1}{2}$ square feet and height 10 feet? Of a pyramid of the same dimensions?

2. What is the volume of a prism whose altitude is 6 feet and whose base is a triangle each of whose sides is 2 feet? of a pyramid of the same dimensions?

3. What are the volume and convex surface of a pyramid whose altitude is $5\frac{1}{4}$ feet, and whose base is a square the diagonal of which is 8 feet?

4. Find the convex surface and volume of a frustum of a square pyramid, a side of the lower base being $3\frac{1}{2}$ feet, of the upper base $2\frac{1}{2}$ feet, and whose height is $4\frac{1}{2}$ feet.

5. Find the volume of a pyramid whose base is an equilateral triangle, each side measuring 6 feet, and whose altitude is 24 feet.

6. Compare the volume of this pyramid with that of a pyramid of the same height, but whose base is 6 feet square.

351. GENERAL REVIEWS.

I.

1. Define Multiplication, Denominator, Decimal Fraction, Interest, Sphere.

2. What common fractions can be changed to pure decimals? Why?

3. $824^3 = ?$ $\sqrt[3]{80964.73807} = ?$

4. Simplify $\frac{1.18}{.152} \times \frac{3.64}{2.95}$.

5. Derive a rule for dividing by a fraction.

6. Find the interest on \$824.60, at 7%, from June 12, 1887, to Sept. 5, 1890.

7. How many gallons of water will a cylindrical cistern hold whose diameter is $7\frac{1}{4}$ feet and depth $9\frac{1}{2}$ feet?

8. Change $\frac{2}{3}$ of a square mile to integers of lower denominations.

9. What is the cost of the lumber and posts to fence the N. W. $\frac{1}{4}$ of the S. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ of a section with a four-board fence, posts 8 feet apart and costing 23 cents each, fencing at \$18.50 per M.?

10. What is the cost of a 40-day draft for \$580, exchange being $\frac{1}{2}$% premium, and interest at 6%?

11. What is the difference between seven hundred and two thousandths and seven hundred two thousandths?

II.

1. Divide 3,744 into three parts in the proportion of $\frac{2}{3}$, $\frac{3}{8}$, $\frac{5}{7}$.

2. Define Ratio, Proportion, Commission, Notation, Division.

3. How many square feet of sheet-iron $\frac{1}{4}$ of an inch thick can be made from a cylindrical shaft 20 feet long and 4 inches in diameter?

4. If 52 men can dig a canal 355 feet long, 60 feet wide, and 8 feet deep in 15 days, how long will a canal be that is 45 feet wide and 10 feet deep which 45 men can dig in 25 days?

5. What is the selling price of an article bought at a discount of $33\frac{1}{3}\%$, and sold at an advance of $12\frac{1}{3}\%$, yielding a gain of \$8?

6. How many gallons of liquid will a hollow sphere hold whose inner diameter is 22 inches?

7. A 45-foot ladder placed between two poles reaches one of them 24 feet from the ground, and the other 28 feet. How far apart are they?

8. What is the difference of time between two places whose difference of longitude is 46° 18′ 46″?

9. If $\frac{7}{15}$ of an acre of land cost \$$33\frac{2}{3}$, what will $36\frac{5}{8}$ acres cost?

10. A, B, C, and D together own a tract of land 2 miles square. A owns $\frac{5}{6}$ as much as B; B $\frac{2}{3}$ as much as C; C $\frac{3}{4}$ as much as D. How many acres has each?

III.

1. Define Antecedent, Consequent, Mean Proportional, Complex Fraction, Compound Interest.

2. What is the average time for the payment of \$600 due in 3 months, \$480 due in 5 months, \$390 due in 8 months, and \$850 due in 14 months?

3. Find the l. c. m. of 174, 485, 14,065.

4. Give the rule for "pointing" the quotient in division of decimals, and give the explanation of it.

5. Change 18 mi. 124 rd. 4 yd. to feet, and use three forms of analysis in the reductions.

6. A public square is surrounded by a walk 2 rods wide. The area of the walk is an acre. What is the area of the square? Make a figure.

7. For what amount shall a 90-day note be made that the proceeds shall be $358.60, interest at 7%? (Bank Discount.)

8. State the three general problems of percentage.

9. $\frac{2}{11}$ is what per cent of $\frac{8}{15}$? Give an analysis.

10. A, B, and C form a partnership, and make a gross gain of $16,440. A invests $5,000 for 12 months; B, $9,000 for 16 months; C, $7,100 for 6 months. The total expenses were $4,110, which they agreed to share equally. What was each partner's share of the net gain?

IV.

1. Define Subtraction, Minuend, Subtrahend, Remainder, Partition.

2. Divide 83,600 by 37½, and give the analysis. Multiply 564 by 83⅓, and give the analysis.

3. Give tests of divisibility by 3, 4, 8, 9, 11.

4. If a school-room is 15 feet high, how many square feet of floor must it have to furnish 60 persons 300 cubic feet of air? If the length is to the breadth as 4 to 3, what will each be?

5. A cubic bin with a square bottom holds 164,025 cubic inches. Depth is to width as 9 to 5. What is the depth? The width?

6. 40 men agree to do a piece of work in 50 days, but after working 9 hours a day for 30 days only half the work is completed. How many additional men must be employed to finish the work on time by putting in 10 hours a day?

7. What is the present worth of a non-interest-bearing debt of $728.40, due in 3 years, 7 months, and 19 days, money being worth 7%?

8. $\sqrt{860473.02986} = ?$

9. Find the value of the following lumber at \$21 a thousand:

4	6 × 8 sills,	16 feet long.	
26	2 × 8 joists,	18 "	"
30	2 × 4 studs,	22 "	"
18	2 × 6 rafters,	20 "	"

10. Find the premiums on the following policies of insurance:

\$2,100, at $1\frac{1}{4}$%.   \$2,800, at $2\frac{1}{7}$%.   \$3,150, at $\frac{5}{8}$%.

V.

1. When do you conclude that a number is prime? Why?

2. Give the demonstration of the test of divisibility by 9.

3. The proceeds of a note for \$265.50, discounted on June 12, 1891, at 7 %, were \$263. When was the note due?

4. The proceeds of a 90-day note for \$480 were \$470.08. What was the rate of discount? (With grace.)

5. Bought goods for \$729. What must they be marked that the merchant may fall 10%, lose 10% on bad debts, and still gain 10%?

6. A circular piece of land 16 feet in diameter is to be divided into 3 equal parts, the inner part being a circle, and the second and third parts being circular strips. What is the diameter of the inner circle? What is the width of each of the circular strips?

NOTE. What is the area of the whole circle? What is the diameter of the inner circle?

7. Give two ways of changing a common fraction to a decimal. Change $\frac{3}{125}$ to a decimal, and explain each step.

8. A can do a piece of work in $2\frac{2}{5}$ days; B, in $3\frac{4}{5}$ days; and C, in $4\frac{1}{5}$ days. How long would it take them to complete the job working together? If \$6 is paid for the whole work, what is the share of each?

9. A note of $1,200, dated April 1, 1886, and bearing interest at 8%, had the following indorsements: Sept. 12, 1886, $130. March 20, 1887, $240. Aug. 24, 1888, $325. What was due April 1, 1890?

10. When it is noon in Boston, what time is it at San Francisco?

11. At what rate must 4% bonds be purchased to yield $5\frac{1}{2}$% on the investment?

12. Write the tables of long, square, and cubic measures; also of Troy, Apothecaries', and Avoirdupois weights.

VI.

1. A man travelled at the rate of 3 mi. 165 rd. 4 yd. 2 ft. an hour. How far did he go in 36 hours?

2. How many revolutions will a wheel, whose diameter is $4\frac{1}{2}$ feet, make in rolling 3 miles?

3. A hollow brass sphere, whose diameter is 4 inches, weighs $\frac{1}{8}$ as much as a solid sphere of the same size and material. How thick is the shell?

4. Express the ratio of a pound Troy to a pound Avoirdupois; of an ounce; of a cubic foot to a bushel; of a quart liquid measure to a quart dry measure.

5. Solve the following by compound proportion: If 15 men in 12 days of 10 hours each can dig a ditch 180 rods long, 6 feet wide, and 4 feet deep, how many hours a day must 10 men work to dig a ditch 200 rods long, 8 feet wide, and 2 feet deep in 10 days?

6. Add:

5 A.	120 sq. rd.	21 sq. yd.	6 sq. ft.
12 "	96 " "	18 " "	7 " "
22 "	83 " "	25 " "	4 " "
17 "	74 " "	28 " "	8 " "

7. Find the interest on $1,580, at $7\frac{1}{2}$%, from Dec. 18, 1889, to May 1, 1891.

8. What is the cost of a 90-day draft on London for £850, exchange being \$4.86, and interest 5%? (With grace.)

9. What is the value of a pile of wood 360 feet long, 12 feet wide, and 6 feet high, at \$3.20 a cord?

VII.

1. Put the following items into the form of a receipted bill:

R. D. Smith bought of Cole Bros., Newark, N. J., on June 1, 1892, 16 yards silk, @ \$1.85. June 12, 56 yards cotton cloth, @ 9 cents. June 15, 8 yards broadcloth, @ \$2.25. June 20, 24 yards carpet, @ 96 cents. July 1, 31 yards matting, @ 40 cents. July 10, 5 sets curtains, @ \$3.85.

2. Find the g. c. d. of 1127 and 6581.

3. A man bought a house and lot for \$5,088. $\frac{3}{7}$ of the cost of the house was $\frac{5}{6}$ of the cost of the lot. What was the cost of each?

4. At \$4.75 a cord, what is the cost of the following piles of cord wood:

a.	18	feet long,	6	feet	high.
b.	23	" "	$5\frac{1}{2}$	"	"
c.	17	" "	7	"	"

5. Change 2 rd. 4 yd. 2 ft. to the decimal of a mile.

6. What must be the rate of taxation in a town to yield a net return of \$16,660, if the real estate is assessed at \$531,000, the personal property at \$200,182.80, 7 % of the tax being uncollectible, and the collector's commission being 2%?

7. In what time will \$469.50 yield \$36.80, at 7%?

8. What is the volume of a sphere whose diameter is $7\frac{1}{2}$ inches?

9. If a block of stone 18 inches long, 4 inches wide, and 2 inches thick, weighs 12 lb. 15 oz., what is the weight of a

block of the same material $2\frac{1}{2}$ feet long, 2 feet wide, and 9 inches thick?

10. What is the volume of the frustum of a cone the diameters of whose bases are respectively 28 inches and 16 inches, and whose height is 30 inches?

VIII.

1. Analyze each of the following:

a. 2 is what part of $5\frac{1}{3}$? *b.* $\frac{2}{7}$ is what part of $\frac{11}{18}$? *c.* $.015\frac{1}{3}$ is what part of $.6\frac{2}{3}$?

2. Add $\frac{7}{25}$, $\frac{2}{3}$ of $5\frac{1}{8}$, $\frac{2}{5} \div \frac{3}{8}$, $\frac{2}{7} \times 3\frac{1}{2}$.

3. Give the rule for "pointing" the product in Multiplication of Decimals, and explain it.

4. Change 61,368 seconds to integers of higher denominations, and give the analysis for two reductions.

5. $\frac{5}{7}$ of an inch is what part of a rod? Analyze.

6. What number multiplied by $\dfrac{2\frac{3}{5}}{4\frac{1}{2}}$ will give 2 for a product?

7. Change $\frac{7}{18}$ of a mile to integers of lower denominations.

8. What will it cost at 24 cents a square yard to plaster a hall $46\frac{1}{2}$ feet wide, 82 feet long, and 24 feet high, no allowance being made for openings?

9. Bought 42 shares of stock at $105\frac{1}{2}$. Received a 4% dividend, and sold the stock at $103\frac{1}{8}$. The gain was what per cent of the investment?

10. What principal will amount to $1,690 from Jan. 12, 1890, to June 17, 1892, at 7% interest?

11. A railroad train moves a mile in 65 seconds. What is its rate per hour?

IX.

1. What is the area of an equilateral triangle each of whose sides is 42 rods?

2. Find the number of acres in the following tracts of land:

a. N. $\frac{1}{2}$ of S. W. $\frac{1}{4}$ of a section.

b. S. $\frac{1}{2}$ of N. W. $\frac{1}{4}$ of S. E. $\frac{1}{4}$.

c. S. W. $\frac{1}{4}$ of S. E. $\frac{1}{4}$ of N. W. $\frac{1}{4}$. Make a figure showing each tract.

3. A is 26 rd. 4 yd. north of C. B is 13 rd. $3\frac{1}{2}$ yd. east of C. What is the distance from A to B?

4. From $\frac{12\frac{3}{4}}{3\frac{7}{8}}$ take the sum of $\frac{2}{3} \times \frac{3\frac{1}{3}}{5\frac{5}{6}}$ and $\frac{5}{6} \times \frac{13\frac{1}{5}}{7\frac{7}{10}}$, and divide the result by $2\frac{16}{21}$.

5. How many 40-gallon barrels will a cylindrical cistern hold, the diameter of which is $9\frac{1}{2}$ feet, and whose depth is 10 feet?

6. Which is the better investment, 5% bonds at 98, or 4% bonds at 95? Give the rate per cent of interest on each investment.

7. What is the per cent of gain if $\frac{7}{9}$ of an article is sold for $\frac{10}{11}$ of its cost?

8. Explain the rule for "pointing" a number to ascertain the number of terms in its cube root.

9. Bought 15 railroad bonds at $1\frac{1}{2}$% discount, brokerage $1\frac{3}{8}$%. For what must a 90-day note be drawn, interest at 8%, to obtain the amount of the purchase at a bank? (With grace.)

10. Find the cube root of 3 to within .001.

X.

1. Define a ratio. Define each term. What is the difference between a ratio and a fraction? Define a proportion, and each term. By what principle is any term of a proportion found when three are given?

2. A can do a piece of work in 6 days; B, in 7; and C, in 8. In what time can they do it working together?

3. What is the capacity of a hollow sphere whose outside diameter is 15 inches, and whose walls are $\frac{1}{4}$ of an inch thick?

4. A man bought four articles for \$896.45. For the first he gave \$21 more than for the second; for the second, \$62.50 more than for the third; for the third, \$81.75 more than for the fourth? What did each cost?

5. Sold 160 acres of land at \$87.50, commission $2\frac{1}{2}\%$. Directed the agent to invest the proceeds in 5% bonds at 98, reserving his commission at 2%, and returning the surplus of less than \$100. How many bonds did he purchase, and how much did he return?

6. $\sqrt{\frac{2}{3}} = ?$ $\sqrt[3]{\frac{4}{5}} = ?$

7. The interest on two sums of money for 4 years and 8 months at 6% was \$256. $\frac{2}{3}$ of the first sum equalled the second. What was each?

8. A steamer can sail 10 miles an hour with the current, and 5 miles an hour against it. What is the rapidity of the current? How long a trip up stream and down can it make in 6 hours?

9. Find the volume of a cone whose altitude is 15 inches, and radius of base $3\frac{1}{2}$ inches.

10. A commission merchant sold goods at 2% brokerage. He invested the proceeds at 2%, reserving his commission. His commissions amounted to \$149. What was the amount of the first sale?

XI.

1. Give the laws of the Roman Notation.

2. Explain the philosophy of "pointing" for the extraction of the cube root.

3. How many rolls of paper are needed to cover the walls and ceiling of a room 16 feet by 18 feet, and 11 feet high, deductions being made for three windows 3 ft. 2 in. by

7 ft. 4 in., 2 doors 3 ft. by 8 ft. 2 in., and a 10-inch base board?

4. Find the compound interest on $483.96 for 4 yr. 5 mo. 13 d., at 6%, compounded semi-annually.

5. A room is 15 feet by 18 feet, and 10 feet high. What is the length of a line extending from an upper corner diagonally through the room to an opposite lower corner?

6. What is the diameter of a circle containing 20 acres of land? What is the area of a strip 18 feet wide, lying next to the circumference and reaching around the field on the outside?

7. $\dfrac{\frac{2}{3} + \frac{5}{8}}{\frac{2}{3} - \frac{5}{8}} \div \dfrac{\frac{7}{12} \times \frac{6}{7}}{\frac{7}{12} \div \frac{6}{7}} \times \dfrac{.06\frac{1}{4}}{3\frac{1}{8}} = ?$

8. On a certain farm the barn cost $\frac{2}{5}$ as much as the house, and the house $\frac{2}{11}$ as much as the land. The tenant raised 3,600 bushels of corn and 3,000 bushels of oats. The landlord received $\frac{2}{5}$ of the corn and $\frac{1}{3}$ of the oats for rent. Corn sells for 44 cents, and oats for $31\frac{1}{2}$ cents. The landlord's income was $9\frac{132}{161}\%$ of his investment. Find cost of barn, house, and farm.

9. How many bushels of corn in the ear will a crib hold that is 46 feet long, 8 feet wide, and 10 feet high, counting the bushel at $\frac{2}{3}$ of true capacity?

XII.

1. What is the face of a 60-day note the proceeds of which are $2,654.38 when discounted at a bank at 7%? (With grace.)

2. A and B can do a piece of work in 24 days. A can do $\frac{2}{3}$ as much as B. In how many days can each do it alone?

3. A rectangular field contains $12\frac{1}{2}$ A. Its width is $\frac{4}{5}$ of its length; what is the distance around it?

4. A, B, and C were partners in business. A's capital was $\frac{2}{3}$ of B's, and B's was $\frac{4}{7}$ of C's. A's capital was in 8 months; B's, 9 months; C's, 10 months. Their net gains were $2,674; what was the share of each?

5. An agent sold a house and lot for his principal. After reserving his commission of 2% for selling and 2% for buying, he invested the remainder in corn at 51 cents a bushel. His total commissions were $400; how many bushels of corn did he buy?

6. How many gallons of water will a hollow sphere hold whose interior diameter is 3½ ft.?

7. A piece of land in the form of a trapezoid is 120 rd. between its parallel sides, one of which is 45 rd. long, and the other 60 rd. long. What is the land worth at $62.50 an acre?

8. A merchant sold a customer 7 pieces of cloth, each containing 50 yd. He made a reduction of 20% from the retail price, and a further reduction of 5% for cash. The retail price was 40% above cost. He received $532. What was the retail price per yard?

9. A certain note for $600 is dated June 1, 1890. It is due two years from date, and bears 6% interest. What should be paid for it Sept. 1, 1890, in order that the investment shall yield 10% per annum?

XIII.

1. Define Division. Define a Common Fraction. Effect of multiplying numerator and denominator by the same number? Explain fully.

2. Give and fully explain the rule for the multiplication of Decimal Fractions.

3. What sum of money put at interest for 3 years 7 months 18 days, at 6%, will amount to $648.36?

4. A tank can be filled by keeping one pipe open 4 hours, or by keeping a second pipe open for 5 hours. The tank has a pipe by means of which it can be emptied in 2½ hours. In what time will the tank be filled if the three pipes be left open?

5. Change $\frac{3}{17}$ of a mile to integers of lower denominations.

6. A farmer has a 40-acre field in the form of a square. He has it planted in corn, the rows being 3 feet 6 inches apart. The first row is 3 feet from the line. How far does he walk in plowing it once, taking a row at a time?

7. A, B, and C form a partnership for one year. A put in $3,000 for the first six months, when he withdrew $1,000. B put in $3,000 at first, and when A withdrew he made the deficit in their joint capital good. C put in $5,000 for the year. Their net gain was $2,750. What was each one's share?

8. I spent 25% of my money, 33⅓% of the remainder, and 8⅓% of the remainder. I then had $550. How much did I have at first?

9. A man was paying rent at the rate of $15 a month. He borrowed an amount from a Building Association which enabled him to build a house as good. He made a monthly payment of $18 for six years, when his house was paid for. How much more than his rent did the house cost him, counting interest on his money at 6%?

10. A house is 30′ by 40′. The cistern connected with its roof is cylindrical in shape, 9 feet deep, and has an average diameter of 10 feet. At the end of a rain the cistern was found to be half full. How many inches of rain had fallen?

11. A man left $10,000, to be invested for his three sons, aged 12, 15, and 18, at 4% compound interest. He directed that the money should be so divided that the children would receive equal amounts when 21 years of age. How much was set aside for each one?

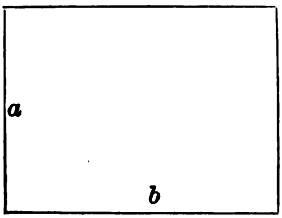
12. What is the rate of interest received on an investment in 4½ bonds purchased at 106?

13. At what rate must 3½% bonds be bought to yield 4% on the investment?

352. ALGEBRAIC QUESTIONS.

Find what number x stands for, if

1. $x + 7 = 10$
2. $9 + x = 16$
3. $x - 3 = 12$
4. $12 - x = 2$
5. $7x = 21$
6. $3x + 5 = 29$
7. $7 + 2x = 17$
8. $17 - 2x = 13$
9. $3x + 5x = 32$
10. $9x - 2x = 35$
11. $7x + 3x - 5x = 30$
12. $5x - 2x + 11 = 17$



13. If the base of this rectangle is b units, the altitude a units, express the sum of the base and altitude.

14. Express the difference of base and altitude.

15. Express the length of the perimeter (entire boundary).

16. Express the area.

17. Express what part the altitude is of the base.

18. Express what part the base is of the perimeter.

353. THE LITERAL NOTATION.

1. Numbers are often represented by letters, as in the questions above. Any letter may stand for any number, but in any particular problem a letter must stand for the same number throughout.

2. A letter, or a combination of letters, standing for a number is called an **algebraic expression.**

3. The numbers expressed by algebraic expressions may be sums, differences, products, quotients, powers, or roots.

A **power** is the product of equal factors; the **degree** of the power is determined by the number of equal factors, and is indicated by a small figure or letter (written above and to the right), called the **exponent** or **index.**

Thus, $2^3 = 2 \times 2 \times 2 = 8$. Eight is the third power of two.

A root of a number is one of the equal factors whose product is the given number. 2 is the 3d root of 8; 5 is the 2d root of 25. Roots are indicated by the radical sign, $\sqrt{}$ The index of the root is a numeral written above the radical sign.

$\sqrt[4]{16} = 2$. The fourth root of 16 is 2.

$\sqrt[3]{d^3} = d$. The third root of d third power is d

2d powers and 3d powers are called squares and cubes.
2d roots and 3d roots are called square roots and cube roots.
The index of square roots is usually not written.

$\sqrt{9} = 3$. The square root of 9 is 3.

4. In reading algebraic expressions involving more than one letter, it is often best to name the kind of number before reading it; thus,

(1) $a + 3b$, the sum of a and $3b$, or a plus $3b$.

(2) $6a - 3c$, the difference of $6a$ and $3c$, or $6a$ minus $3c$.

NOTE. Such expressions for sums and differences are called binomials. The expressions connected by the signs + or − are called terms. $7a$ and $5c$ are the terms of the binomial $7a - 5c$. An expression of three terms is a trinomial, as $3a + 2c - 5x$. Any expression of two or more terms may be called a polynomial, although the name is usually applied to expressions of more than three terms. A monomial contains one term only. Polynomials are sometimes called compound expressions. Compare compound numbers with polynomials.

(3) $a \times (b - c)$, or $a(b - c)$, the product of a and the binomial b minus c, or a times the binomial b minus c.

(4) $(3x - a) \div c$, the quotient of the binomial $3x$ minus a divided by c.

NOTE. A quotient written $\frac{3x - a}{c}$ is called a fraction.

(5) $a - (x - y)^3$, a minus the cube of the binomial x minus y.

(6) $a - (x - y^3)$, a minus the binomial x minus y cubed.

(7) $\sqrt[3]{a^2} - 2c$, the cube root of a squared minus $2c$.

(8) $(\sqrt[3]{a})^2 - 2c$, the square of the cube root of a minus $2c$.

(9) $\sqrt[3]{a^2 - 2c}$, the cube root of the binomial a squared minus $2c$.

(10) $\dfrac{\sqrt{(3a - b)(2x + y)}}{a + x - y}$, the fraction, the square root of the product of the binomials $3a$ minus b and $2x$ plus y divided by the trinomial a plus x minus y.

354. EXERCISES IN LITERAL NOTATION.

Write as algebraic expressions:

1. The sum of a squared and the square root of c.
2. Five times the binomial the fourth root of x cubed minus $7ab$.
3. Five times the fourth root of x cubed minus $7ab$.
4. Five times the fourth root of the binomial x cubed minus $7ab$.
5. a plus the fraction b divided by c plus d.
6. a plus the fraction b divided by the binomial c plus d.
7. The fraction a plus b divided by the binomial c plus d.
8. The fraction a plus b divided by c plus d.
9. The square of the fraction a minus c divided by x.
10. The product of 3 times the sum of x and y by the sum of $3x$ and y.

355. EVALUATION OF ALGEBRAIC EXPRESSIONS.

To evaluate an algebraic expression is to find its numerical value; thus:

If $a = 1$, $b = 2$, $c = 3$, $x = 10$, what is the value of $\left(\dfrac{5abc - 2a + bx}{3x - 2c}\right)^3$

FORM.

(1) $$\left(\frac{5abc - 2a + bx}{3x - 2c}\right)^3 =$$

(2) $$\left(\frac{5 \cdot 1 \cdot 2 \cdot 3 - 2 \cdot 1 + 2 \cdot 10}{3 \cdot 10 - 2 \cdot 3}\right)^3 =$$

(3) $$\left(\frac{30 - 2 + 20}{30 - 6}\right)^3 =$$

(4) $$\left(\frac{48}{24}\right)^3 =$$

(5) $$(2)^3 =$$

(6) $$8$$

DESCRIPTION. Substituting in expression (1) the numerical values of the letters we have exp. (2). Performing indicated multiplications we have exp. (3). Performing additions and subtractions we have exp. (4). Dividing we have exp. (5). Cubing the quotient, as indicated, we obtain 8.

356. PROBLEMS IN EVALUATION.

$a = 1,\ b = 2,\ c = 3,\ d = 10,\ x = 0.$

1. $3ab + cx + 4d$
2. $\sqrt{bc + d}$
3. $(2d - bc)^2$
4. $(3a + b)\,x + 2acd$
5. $\dfrac{ax + bx + cd}{a + 2b + d}$
6. $10c - (bd - 4ac)$
7. $10c - bd - 4ac$
8. $\sqrt[3]{4b} + 2d - a$
9. $\sqrt[3]{4b + 2d - a}$
10. $(d - b)^3$
11. $(d^2 - b^2)$

357. EQUATIONS.

Problems. 1. A boy has four times as many white marbles as brown ones. Of both he has 30. How many of each?

Let $x =$ number of brown marbles.
Then, since, etc. $4x =$ number of white marbles.
And $x + 4x =$ number of both.
But $30 =$ number of both.
Therefore $x + 4x = 30.$

$$5x = 30.$$

$x = 6$, no. of brown marbles.
$4x = 24$, no. of white marbles.

358. DEFINITIONS.

(1) An **equation** is a statement in mathematical symbols that two expressions stand for the same number.

$x + 4x = 30$ in the problem above is an equation. $x + 4x$ and 30 each stand for the total number of marbles.

(2) The **numbers** of an equation are the two equivalent expressions, $x + 4x$ and 30, in the equation above.

(3) Equations are used to find the value of unknown numbers represented by x, y, z, etc.

(4) To **solve** an equation is to find the value of the unknown number involved.

2. In a school of 45 pupils there are 7 more girls than boys. How many of each?

Let $x =$ number of boys.
Then, since, etc., $x + 7 =$ number of girls.
And $x + x + 7 =$ number of pupils in school.
But $45 =$ number of pupils in school.
Therefore $x + x + 7 = 45.$

$$2x + 7 = 45.$$

$$2x = 38.$$

$x = 19$, number of boys.
$x + 7 = 26$, number of girls.

Show that in the above solution we have proceeded in accordance with the following truths:

359. Axioms.

(1) Things equal to the same thing are equal to each other.

(2) If equals be added to equals, the sums are equal.

(3) If equals be subtracted from equals, the remainders are equal.

(4) If equals be multiplied by equals, the products are equal.

(5) If equals be divided by equals, the quotients are equal.

3. Six times John's age exceeds four times his age by 22 years. How old is he?

NOTE. Let $x =$ the *number of years* in John's age. If we say let $x =$ John's age, we treat x as a mere quantity of time, not as a *number* of time-units.

4. $21,000 is divided among three children so that the first receives twice as much as the second, the second twice as much as the third. What is the share of each?

NOTE. Let $x =$ no. of dollars in the share of the third.

5. Thomas, Richard, and Henry have 72 marbles. Thomas has twice as many as Richard. Henry has twice as many as both the others. How many has each?

6. How old am I, if three times my age four years ago exceeds twice my present age by 27 years?

7. Equal weights of sugar and flour were bought for 63 cents. The sugar cost 5 cents per pound, the flour 2 cents. How many pounds of each?

8. The perimeter of a rectangular field 80 rods long is 280 rods. What is its width?

9. The perimeter of a rectangular field, twice as long as wide, is 180 rods. What is its length?

10. It takes 70 feet of border to enclose a square room. What are its dimensions?

11. A room 27 feet wide and x feet long requires 99 square yards of matting. What is the value of x?

12. A Sunday-school collection in dimes, nickels, and cents amounted to 200 cents. There were three times as many nickels as dimes and five times as many cents as nickels. How many of each?

13. Grace is 5 years older than May. May is two years older than Ethel. The sum of their ages is 42 years. What is the age of each?

14. A father is four times as old as his son. Five years ago, he was seven times as old. What is the father's age?

Let	$x =$	no. of years in son's age.
Then (why?)	$4x =$	no. of years in father's age.
" "	$x - 5 =$	no. of years in son's age 5 years ago.
" "	$7(x - 5) =$	no. of years in father's age 5 years ago.
" "	$4x - 5 =$	no. of years in father's age 5 years ago.
Hence (1)	$7(x - 5) = 4x - 5.$	
(2)	$7x - 35 = 4x - 5.$	
(3)	$7x = 4x - 5 + 35.$	
(4)	$7x - 4x = 35 - 5.$	
(5)	$3x = 30.$	
(6)	$x = 10.$	
(7)	$4x = 40,$	no. of years in father's age.

What was added to each member of (2)? What was subtracted from each member of (3)?

NOTE. If a term contain a "numerical" factor, and one or more literal factors, the numerical factor is called the **coefficient** of the term. Terms containing the same literal factors are called **like terms**. In the terms $7x$, $3a^2b$, 7 and 3 are the coefficients. $3ax^2$ and $5ax^2$ are like terms. $3a^2b$ and $7ab^2$ are unlike terms. Why?

In describing the solution of the equation in Problem 14 we may say:

Performing indicated multiplication in Eq. (1) we have Eq. (2). Adding 35 to each member of Eq. (2) we obtain Eq. (3). Subtracting $4x$ from each member of Eq. (3) we obtain Eq. (4). Collecting like terms in (4), we obtain Eq. (5). Dividing each member of Eq. (5) by the coefficient of

x, 3, we obtain Eq. 6. Multiplying each member of (6) by 4 we get Eq. (7). (Adding 35 to, and subtracting $4x$ from, each member of Eq. 2 we obtain Eq. 5.)

15. A man of 35 is 7 times as old as his son. In how many years will he be twice as old?

Let x = no. of years hence when the father's age will equal twice the son's age.

Then $5 + x$ = no. of years in son's age at required time;

and $2(5 + x)$ = no. of years in father's age at required time.

But $35 + x$ = no. of years in father's age at required time.

Hence (1) $2(5 + x) = 35 + x$.

(2) $10 + 2x = 35 + x$.

(3) $2x - x = 35 - 10$.

NOTE. In subtracting 10 and x from each member of Eq. (2), we cause each of these terms to pass to the other member of the equation with change of sign. This transfer of a term to the opposite member with change of sign is called **transposition**.

16. A has 8 dollars more than B. After paying B 12 dollars, A has only $\frac{1}{3}$ as many as B. How much had each at first?

17. John has 40 marbles more than Fred. After giving Fred 50, John has only $\frac{1}{7}$ as many as Fred.

18. A debt of \$102 is paid with an equal number of ten-dollar, five-dollar, and two-dollar bills. How many bills were paid in all?

19. In paying 27 cents for an article, I tendered some dimes and received an equal number of cents as change. How many dimes did I tender?

20. Harry and Walter, 62 miles apart, ride towards each other. Harry, starting at 9 A. M., rides 2 miles per hour faster than Walter, who started at 8 A. M. They meet at noon. What is the rate of each?

21. Take some number, double it, add 20, divide by 2, take away the first number; you have 10 left. Why is this? If you had added 30, instead of 20, how many would you have left?

22. Take some number, multiply by 6, add 30, divide by 3, subtract 4, divide by 2, take away the first number; you have 3 left. Explain.

360. EQUATIONS CONTAINING FRACTIONS.

1. $\frac{1}{3}$ of what number $= 4$?

2. $\frac{1}{5}$ of what number $= 3$?

3. $\frac{3}{7}$ of what number $= 6$?

4. $\frac{5}{11}$ of what number $= 10$?

5. $\frac{1}{3}$ of a number $+ \frac{1}{4}$ of the same number $= 21$. What is the number?

6. $\frac{5}{8}$ of a number $- \frac{1}{3}$ of the same number $= 10$. What is the number?

7. $\frac{2}{3}$ of a number $= 7$. What is twice the number?

8. $\frac{4}{5}$ of a number $= 11$. What is four times the number?

9. $\frac{x}{3} = 5$. Find value of x.

By what number must we multiply each member to obtain the value of x?

10. $\frac{4x}{5} = 36$.

By what number must we multiply each member to obtain the value of $4x$? Will 10 do? What other multipliers? What axiom is involved?

11. $\frac{x}{3} + \frac{4x}{5} = 17$.

ANALYSIS. To make the first fraction integral we must multiply it by 3, or some multiple of 3; to make the second fraction integral we must multiply it by 5, or some multiple of 5; to preserve the equality of the members we must multiply both by the same multiplier. 15 is the least common multiple of 3 and 5. Multiplying the first fraction by 15 (first by 3 to suppress the denominator, and then by 5) we have $5x$. Multiply-

ing the second fraction by 15 (first by 5, and then by 3) we have $12x$. Multiplying the second member by 15, we have 255. Our equation now stands:

$$5x + 12x = 255.$$

NOTE. This process of transforming a fractional equation into an integral equation is called **clearing of fractions.**

Solve:

12. $\frac{4x}{5} + \frac{3x}{4} = 31.$

13. $\frac{2x}{3} + 10 = \frac{5x}{2} - 1.$

14. $\frac{3x}{4} - \frac{2x}{3} = 1.$

15. $\frac{x}{2} - 6 = \frac{x}{4} - 2.$

16. $\frac{4y}{7} + 5 = \frac{3y}{7} + 8.$

17. $\frac{z}{2} + \frac{z}{3} + \frac{z}{4} = 39.$

18. $\frac{x}{4} + \frac{x}{2} - \frac{x}{5} = 22.$

19. $\frac{3z}{4} - \frac{33z}{50} = 81.$

20. $\frac{2y}{3} - \frac{y}{2} + \frac{y}{5} = 22.$

21. $\frac{3x+4}{5} + \frac{2x-8}{3} = x.$

22. $\frac{2x}{3} + \frac{3x+4}{5} = x + \frac{3x-4}{8}.$

23. $\frac{3}{x} + \frac{4}{x} + \frac{5}{x} = 1.$

24. $\frac{1}{3}x + \frac{1}{4}x - \frac{1}{6}x = 25.$

25. $\frac{2}{3}x + \frac{2}{5}x - \frac{2}{7}x = 82.$

26. $\frac{3y}{7} - \frac{5y}{14} + \frac{6y}{35} = 34.$

27. $\frac{5y}{3} + \frac{7y}{6} - \frac{9y}{8} = \frac{95}{48}.$

28. $2\frac{5}{8}x = 105.$

29. $3\frac{1}{5}x = 48.$

30. $3\frac{1}{2}x - 2\frac{3}{4}x = 45.$

31. $5\frac{1}{2}x - 3\frac{2}{3}x = 44.$

32. $5\frac{1}{2}(x-3) - \frac{2x}{7} = 0.$

33. $x + 2x + \frac{3x}{5} = 36.$

34. $\frac{4x}{5} + \frac{7-x}{20} = 5\frac{3}{5}.$

35. $\frac{19x}{5} + \frac{x-3}{2} = 20$

361. DEFINITIONS.

1. If the members of an equation are alike in form, or if they are reducible to the same form, the equation is called an **identical equation**, or an **identity**. Thus, $9 = 9$, $5 + 2 - 4 = 3$, and $5x - 7 = 3x - 7 + 2x$ are identities.

2. A solution is verified by substituting for x in the given equation the value of x, as found in the solution, and performing all indicated operations in each member. If the equation reduces to an identity, the solution is correct.

Illustration:

$$\frac{3 + 2x}{5} + \frac{7x + 6}{11} = \frac{18x}{10}$$

$$66 + 44x + 70x + 60 = 198x$$

$$114x + 126 = 198x$$

$$126 = 84x$$

$$\tfrac{3}{2} = x.$$

Verification:

$$\frac{3 + 2 \cdot \frac{3}{2}}{5} + \frac{7 \cdot \frac{3}{2} + 6}{11} = \frac{18 \cdot \frac{3}{2}}{10}$$

$$\frac{3 + 3}{5} + \frac{10\frac{1}{2} + 6}{11} = \frac{27}{10}$$

$$\frac{6}{5} + \frac{16\frac{1}{2}}{11} = \frac{27}{10}$$

$$\frac{6}{5} + \frac{3}{2} = \frac{27}{10}$$

$$\frac{27}{10} = \frac{27}{10}$$

3. If upon substituting for x its supposed value the equation becomes an identity, the value of x is said to **satisfy** the equation.

362. PROBLEMS.

Solve and verify:

1. Divide 90 into two such parts that one shall be $3\frac{1}{2}$ times the other.

2. Divide 100 into two such parts that one shall be $2\frac{1}{2}$ times the other.

3. A horse was sold for \$80, at a gain of $\frac{1}{4}$ of the cost. What was the cost?

4. A is 12 years older than B. $\frac{1}{8}$ of A's age $=$ $\frac{1}{5}$ of B's. What is the age of each?

5. If to John's age there be added its half, its third, and its fourth, the sum is 25 years. What is his age?

6. If to Mary's age there be added its half, its third, and its fifth, the sum is $2\frac{1}{30}$ times her age. What is her age?

QUERY. What is the matter with the foregoing problem?

7. If to A's age there be added its double, its half, and its third, the sum lacks 7 years of four times his age. What is his age?

8. A, B, and C received \$162 for digging a ditch. A dug 4 rods to B's 3 rods and C's 2 rods. What pay should each receive?

9. Two barrels contain respectively 42 and 50 gallons of oil. After drawing the same amount from each, the first contained $\frac{2}{3}$ as much as the second. How much was drawn from each?

10. A campaign pole 84 feet high broke at such a point that the top was $\frac{3}{4}$ of the stump. What was the height of the stump?

11. A campaign pole 100 feet broke at such a point that the top was 6 feet longer than the stump. What was the length of the stump?

12. The sum of two numbers is s, the difference d. What are the numbers?

Let $x =$ the smaller number.

Then $x + d =$ the greater number,

and $x + x + d = s$

$$2x + d = s$$

$$2x = s - d$$

$$x = \frac{s - d}{2}, \text{ smaller number.}$$

$$x + d = \frac{s - d}{2} + \frac{2d}{2} = \frac{s + d}{2}, \text{ greater number.}$$

NOTE. In solving the preceding problem we have solved every problem in which the sum and difference of two numbers are given to find the numbers; for s and d are any numbers.

By substituting the values of s and d in any particular problem of this type, we avoid a formal solution. Thus:

13. The sum of two numbers is 42, their difference 12. What are the numbers?

$$\text{Greater number} = \frac{s + d}{2} = \frac{42 + 12}{2} = \frac{54}{2} = 27.$$

$$\text{Smaller number} = \frac{s - d}{2} = \frac{42 - 12}{2} = \frac{30}{2} = 15.$$

14. Sight problems:

	Sum.	Difference.		Sum.	Difference.
(1)	30	20	(7)	88	12
(2)	30	6	(8)	33	3
(3)	22	8	(9)	52	12
(4)	55	5	(10)	26	6
(5)	44	6	(11)	13	9
(6)	23	7	(12)	51	19

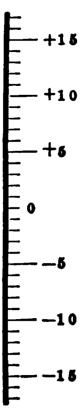
15. A boat runs 6 miles per hour up-stream, and 16 miles per hour down-stream. What is the rate of the current?

16. John had six marbles more than James. Harry had none. After receiving $\frac{1}{4}$ of James's marbles, and $\frac{2}{5}$ of John's, Harry had as many as John or James. How many had each?

363. POSITIVE AND NEGATIVE QUANTITIES.

1. Quantities are sometimes so related that one tends to neutralize or destroy the other. Thus, a rise in temperature counteracts an equal fall; debts are opposed to assets; traveling southward cancels traveling northward; the force of the current destroys an equal force propelling the boat up stream. To these quantities, opposite in kind, the names **positive** and **negative** are applied. Either of a pair of opposites may be called the positive quantity; usually the more familiar of the two is so named.

2. The signs + and −, when used as signs of operation, are read "plus" and "minus;" when placed before numbers to show their character, they are read "positive" and "negative."



FORM.

$$\begin{array}{r} +\ 3 \\ +12 \\ -19 \\ +\ 7 \\ -15 \\ +22 \\ -\ 9 \\ \hline +44 \\ -43 \\ \hline +\ 1 \end{array}$$

3. At 4 A. M., Monday, the thermometer stood at 3° above zero. In the succeeding 12-hour periods it (1) rose 12°, (2) fell 19°, (3) rose 7°, (4) fell 15°, (5) rose 22°, (6) fell 9°. What was the sum, or resulting temperature, Thursday morning?

Explanation. The initial temperature, 3° above zero, is positive. Adding to it the various positive numbers (rises), we obtain +44°. The sum of the negative numbers (falls) is −43°. This sum cancels +43 of the positive sum, leaving +1° as the total sum, or final temperature.

364. PROBLEMS.

1. Seven boys pull at a rope; three pull northward, exerting respectively forces of 75, 85, and 63 pounds; four

pull southward with forces of 52, 57, 59, and 43 pounds. The rope is moved in what direction and with what force?

2. In the left scale-pan of a balance are two 8-ounce weights, three 4-ounce, and five 2-ounce. In the right pan are two 16-ounce weights and three 2-ounce. What must be added to the left pan to produce equilibrium?

3. A letter-carrier has walked north 7 blocks, east 3 blocks, south 5 blocks, east 2 blocks, north 4 blocks, west 11 blocks, south 8 blocks, east 2 blocks. He is now how far east of his starting-point? How far south?

NOTE. Mark eastings and northings +, westings and southings −.

4. A surveyor has measured N. 6.22 ch., E. 4.17 ch., N. 2.12 ch., W. 6.96 ch., N. 3.16 ch., E. 11.25 ch., S. 12.58 ch., W. 8.62 ch., N. 2.00 ch. He is now how far north and east of his starting-point?

5. Add $6a, -3a, -7a, -5a, +3a, +6a, -3a$.

6. Add $2ax, -4ax, +5ax, +6ax, -7ax, +3ax$.

7. Simplify $3ab - 6ab + 4ab - 5ab - 7ab + 12ab$.

8. Simplify $5xy - 7xy - 22xy - xy + 25xy + 20xy - 10xy$.

9. Add $3a + 5b$, $9a + 3b - 2c$, $6a - 4c$, $12c - 18a$.

FORM.

$$\begin{array}{rrr}
3a & +5b & \\
9a & +3b & -\ 2c \\
6a & & -\ 4c \\
-18a & & +12c \\
\hline
 & 8b & +\ 6c
\end{array}$$

10. Add $5a - 3x$, $9x - 5y + 2a$, $6x - 2y - 7a$, $5a + 9y$.

11. Add $3ab - 2c + d$, $7ab - 6d$, $5c - 10ab - 4d$, $10d - 3c$.

365. Subtraction.

1. What is the change in temperature if the thermometer reading changes from $+75°$ to $+90°$? from $+33°$ to $+56°$? from $-7°$ to $+8°$? from $-2°$ to $+11°$? from $-4°$ to $-22°$? from $+3°$ to $-7°$? from $+10°$ to $-1°$?

2. What must be added to $+3$ to make $+10$? to -3 to make $+10$? to $+10$ to make $+3$? to $+10$ to make -3? to -3 to make -10? to -7 to make -2?

3. Since the sum of the subtrahend and difference equals the minuend, we may define subtraction as the process of finding from two given numbers called subtrahend and minuend a third number called difference, which added to the subtrahend produces the minuend.

4. The minuend contains the subtrahend and the difference. If we can destroy the subtrahend in the minuend, only the difference will remain. To destroy, or cancel, a number, we add an equal number with opposite sign. Hence, we add to the minuend the subtrahend with changed sign and obtain the difference.

5. From $5a - 7b + 4c$ take $2a - 2b - 2c$.

If we write $5a - 7b + 4c - 2a$, we have taken $2a$ from the minuend; but we were required to take away, not $2a$, but $2a$ diminished by $2b$ and $2c$. We have therefore taken away too much, and must add $2b$ and $2c$ to obtain the true difference. Hence,

$$5a - 7b + 4c - (2a - 2b - 2c) = 5a - 7b + 4c - 2a + 2b + 2c.$$

Here we see, as in **4**, that to find the difference, we must change the signs of the subtrahend and add to the minuend.

366. PROBLEMS.

1. From $7ax - 3bx - 4cy$ take $3ax - 2bx - cy$.
2. From $5m + 3n + 12p$ take $8m - 2n - 7p$.
3. From $4a - 3b - 5c$ take $6a + 5c - 4d$.

4. $a + b + (4a - 3b) = ?$

5. $9a - 7b - 6c - (3a + 6b - 9c) = ?$

6. $17a - 12am - 4c^2 - (-4a - 14am^2 + 2c^2) = ?$

7. $3a + 4b - 5c - [2a + 3b - (2a - 6c)] = ?$

8. $5a - (3x - 4y) - [5ay - 3a - (2x + 3y)] = ?$

9. $7a^2 - (7b^2 - 3c^2) - 2c^2 - [4b^2 + (2a^2 - 3c^2)] = ?$

367. Multiplication.

1. If a street-car ride costs m cents, what is the cost of 8 rides? 7 rides? 22 rides? a rides? x rides?

2. A dime is tendered in payment for car-fare; b cents are returned as change. What is the cost of the ride?

3. In paying 4 such fares, how many dimes are tendered? How many cents change are returned? In paying m such fares?

NOTE. In the expression $m(10 - b) = 10m - bm$, we see that the signs of the product are the same as in the corresponding terms of the multiplicand.

4. A conductor starts on a trip with two dollars, collects 12 such fares, and refunds 3 fares. The number of cents he now has is expressed

$$200 + 12(10 - b) - 3(10 - b).$$

The sign before the multiplier shows what is to be done with the product.

In receiving 12 fares, he receives 12×10 cents and pays out $12 \times b$, or $12b$, cents. In refunding 3 fares, he pays out 3×10 cents and receives $3b$ cents. Therefore he now has

$$200 + 12 \cdot 10 - 12b - 3 \cdot 10 + 3b.$$

NOTE. The last four terms are products. The positive terms, $+12 \cdot 10$ and $+3b$, are the products of factors with like signs; the negative terms, $-12b$ and $-3 \cdot 10$, are the products of factors of unlike signs. Would the signs be as they are if other numbers than 12, b, 10, and 3 had been used?

RULE.

Two factors of like sign give a positive product; two factors of unlike sign, a negative product.

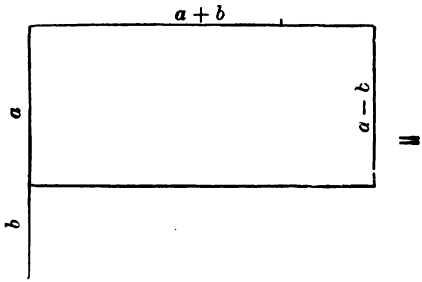
368. PROBLEMS.

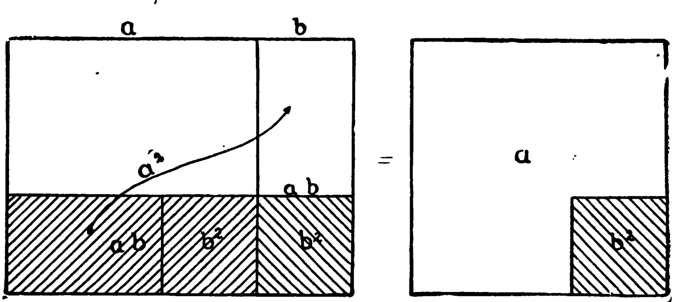
1. A conductor starting with c cents collects $a - b$ fares of $m - x$ cents each. How many cents has he at the end of the trip? What do $-b$ and $-x$ signify in this problem?

2. A butcher sold m guaranteed hams at $a - b$ cents each. x hams were returned as spoiled. What were the net receipts?

3. A rectangle is $m + n$ units long, $a + x$ units wide. What is its area?

Explain these diagrams:





4. Construct diagrams for $(a + b)^2$ and $(a - b)^2$.

5. What is the area of a square $a + b$ units on each side?

6. What is the volume of a rectangular solid b feet long, a feet wide, and h feet high? How many cubic feet in the bottom layer?

7. What is the volume of a cube $a + b$ feet on each edge?

8. What is the volume of a rectangular solid $a + b$ feet long, $m + n$ feet wide, $x + y$ feet high?

9. What is the area of a rectangle $a + b$ feet long, $a - b$ feet wide?

10. What is the area of a rectangle $(3a - x)$ by $(4a - 3x)$?

11. $(3a - 4x + 7b)(9x - 3a + 2b) = ?$

12. $(4a - 3ab)(3a - 5b)(a + b) = ?$

13. $(a - x)(a - 2x)(a + x) = ?$

14. $(7a - 6ax)(3a + 4ax - 5x) = ?$

15. $3a - 7(a + b) + 4(a - 2b) = ?$

16. $5(x^2 - xy) + 3x(4x - 5y) - 5y(2x + 3y = ?$

17. $3(9ax - 2by) - 5a(5x + 4y) + 2y(3b + 10a) = ?$

369. Division.

1. $3b$ dollars are paid for b cords of wood. What is the price per cord?

2. axy trees are planted in a equal rows. How many trees in each row?

3. The area of a rectangle is $ax + ay$. It is a units long. What is its width?

4. The area of a rectangle is $4ax - 8xy$. It is $4x$ units long. What is its width?

5. The area of a rectangle is $6x^2 + 5x - 6$ square feet. Its length is $3x - 2$ feet. What is its width?

A rectangle $3x - 2$ feet long and 1 foot wide contains how many square feet? A rectangle $3x - 2$ feet long and $2x$ feet wide? How many more square feet in the given rectangle? How many more "rows" of square feet do they make?

FORM.

$$
\begin{array}{r}
3x-2\,)\,6x^2+5x-6\,(\,2x+3 \\
\underline{6x^2-4x\phantom{{}-6\,(\,2x+3}} \\
9x-6 \\
9x-6
\end{array}
$$

Note. **The first term of the quotient is found by dividing the first term of the dividend by the first term of the divisor. The work proceeds as in ordinary long division.**

6. Find the width of the following rectangles:

$5a + 3$

$$10a^2 + 41a + 21$$

$7y + 2$

$$28y^2 + 50y + 12$$

$4x + y$

$$12x^2 + 11xy + 2y^2$$

370. LAW OF SIGNS IN DIVISION.

Remembering that the dividend is the product of the divisor and quotient, and that a positive product is the product of two factors of like sign, write the quotients required.

1. $\frac{+12}{+3} =$

2. $\frac{+12}{-3} =$

3. $\frac{-12}{+3} =$

4. $\frac{-12}{-3} =$

5. $\frac{-ab}{+b} =$

6. $\frac{-bx}{+b} =$

7. $\frac{-bc}{-c} =$

8. $\frac{+12ax}{+6x} =$

9. $\frac{+21a^2x}{-7a} =$

10. $\frac{-36ab^2c}{+12} =$

11. $\frac{ab-ax}{+a} =$

12. $\frac{bx-xy}{-x} =$

From the examination of the first four problems we conclude:

If dividend and divisor are of like sign, the quotient is positive.

If dividend and divisor are of unlike sign the quotient is negative.

371. Perform divisions as indicated:

1. $\frac{a^2+2ab+b^2}{a+b}$.

2. $\frac{a^2-2ab+b^2}{a-b}$.

3. $\frac{a^2-b^2}{a-b}$.

4. $\frac{x^3-y^3}{x-y}$.

5. $\frac{x^3+y^3}{x+y}$.

6. $\frac{x^4+x^2y^2+y^4}{x^2+xy+y^2}$.

7. $\frac{a^4-x^4}{a-x}$.

8. $\frac{a^4-x^4}{a+x}$.

9. $\frac{a^4-x^4}{a^2-x^2}$

10. $\frac{x^2+x-12}{x-3}$.

11. $\frac{x^2-7x+12}{x-3}$.

12. $\frac{a^3+3a^2b+3ab^2+b^3}{a+b}$.

13. $\frac{(x^2-3x+2)(x-3)}{x^2-5x+6}$.

14. $\frac{(x^2-x-12)(x+5)}{x-4}$

15. $\dfrac{(a^2+x^2)\,(a^2-x^2)}{a+x}$.

16. $\dfrac{(x-y)^2+2xy}{x^2+y^2}$.

17. $\dfrac{(x-y)^2\,(x+y)}{(x^2-y^2)}$.

18. $\dfrac{(x-y)(x+y)(x^2+y^2)}{x^2-y^2}$

19. $\dfrac{(x^2+y^2)^2-x^2y^2}{x^2+xy+y^2}$

20. $\dfrac{a^3-b^3}{a^2+ab+b^2}$.

APPENDIX.

372. GREATEST COMMON DIVISOR.

1. **A divisor of a number is one of the integral numbers which being multiplied together will produce that number.**

2. Name all of the divisors of each of the following numbers:

4, 6, 8, 12, 15, 24, 36, 39, 40, 48, 56, 64, 72, 96.

3. What number will divide 4 and 6? 9 and 12? 10 and 15? 6, 9, and 12? 15, 18, and 21? 14, 21, 28, and 35?

4. What do you call a number that will divide each of two or more numbers?

5. **A common divisor of two or more numbers is a number that is a divisor of each of them.**

6. Name all of the common divisors of 6 and 12. Which is the greatest? Of 8, 16, and 24, which is the greatest? Of 16, 24, and 32, which is the greatest? What is such a number called? Define it.

7. The greatest common divisor of two or more numbers is the greatest number that is a divisor of each of them.

8. Examine these groups of numbers and find of what the greatest common divisor is the product in each case.

9. **The greatest common divisor of two or more numbers is the product of their common prime factors.**

Prove the preceding statement.

373. PRINCIPLES.

1. Any number is divisible by each of its prime factors and by the product of any number of them.

2. The product of any of the common prime factors of two or more numbers is a common divisor of the numbers.

3. The product of all of the common prime factors of two or more numbers is their greatest common divisor.

Find the g. c. d. of 21, 42, and 63.

FORM.

$$21 = 3 \times 7$$
$$42 = 2 \times 3 \times 7$$
$$63 = 3 \times 3 \times 7$$

Explanation. 3 is a prime factor of each of these numbers. 7 is also a prime factor of each of these numbers. Hence, 3×7 will divide each of them. As they have no other common prime factors, 21 is their g. c. d.

Name all of the common divisors of these numbers. Which is the greatest? Of what is it the product?

EXAMPLES.

Find the g. c. d. of the following:

1. 24, 28, 36.
2. 60, 84, 96.
3. 45, 60, 75.
4. 48, 64, 96.
5. 28, 42, 56, 98.
6. 39, 65, 91.
7. 112, 140, 168.
8. 210, 294, 462.
9. 195, 273, 429, 507.
10. 204, 255, 357, 459.
11. 342, 399, 513, 627.
12. 295, 413, 531.
13. 414, 690, 966, 1242.
14. 780, 234, 312, 390.

RULE I.

For finding the greatest common divisor.

Separate the numbers into their prime factors, and find the product of those that are common.

The factoring method may be employed satisfactorily with any numbers, but the process may be shortened when the numbers are large, by devices that render some of the factoring unnecessary.

374. FINDING THE G. C. D. BY AN EXAMINATION OF DIFFERENCE.

Illustrative Example. Find the g. c. d. of 2002, 2366, 3367.

A divisor of two numbers is also a divisor of their difference; hence, the g. c. d. of these numbers must also divide the difference between 2002 and 2366. This difference is 364. Its prime factors are 2, 2, 7, and 13. The g. c. d. of 2366 and 2002 is also the g. c. d. of 364 and 2002.

FORM.

$$\begin{array}{r} 2366 \\ 2002 \\ \hline 364 = 2 \times 2 \times 7 \times 13. \end{array}$$

(Why?) Hence, we need to compare only these numbers. The prime factors of 364 are 2, 2, 7, 13. By examining 2002, I find that only three of the prime factors of 364 will divide 2002; viz., 2, 7, 13. The product of these three factors is consequently the g. c. d. of 364 and 2002, and hence, of 2002 and 2366 also.

If these factors are also found in 3367, their product is the g. c. d. of the three numbers. By trial I find that 2 is not a factor of 3367; 7 and 13 are prime factors of 3367; hence, 91 is the g. c. d. of the three numbers.

EXAMPLES.

(Solve by the above method.)

1. 59449, and 61659.

ANALYSIS. The difference is 2210. Its prime factors are 2, 5, 13, 17. Only 13 and 17 are factors of 59449; hence 13×17 is the g. c. d.

Why need we pay no attention to 61659?

2. 83971 and 79463.

The difference is 4508. Its prime factors are 2, 2, 7, 7, 23. None of these are factors of 79463, hence the g. c. d. is 1.

3. 387 and 2754.

Multiply 387 by 7. Is the g. c. d. sought a divisor of this product? Why? 2754 — 2709 = 45. Will the g. c. d. divide 45? Why? What are the prime factors of 45? Which of these are prime factors of 387? What, then, is the g. c. d. of 387 and 2754? How do you know?

375. RULE II.

1. ***Find the difference between two of the numbers. Find its prime factors. Determine which of them are prime factors of the smaller of the two numbers. Their product is the g. c. d. of the two numbers.***

2. ***Compare this product with a third number, proceeding as before, and so continue until all of the numbers have been disposed of.***

376. FINDING G. C. D. BY DIVISION.

Illustrative Example. Find the g. c. d. of 91 and 325.

FORM.

```
91 ) 325 ( 3
     273
     ---
      52 ) 91 ( 1
           52
           --
           39 ) 52 ( 1
                39
                --
                13 ) 39 ( 3
                     39
                     --
```

Explanation. The g. c. d. of these numbers cannot be greater than 91. If 91 will divide 325, it is the g. c. d. of 91 and 325. The quotient is 3, and the remainder 52; hence, 91 is not their g. c. d. Since a divisor of a number is a

divisor of any of its multiples, the g. c. d. of these numbers must be a divisor of 273. Since a divisor of two numbers is a divisor of their difference, the g. c. d. must divide 52; hence, it cannot be greater than 52. Since 52 is a divisor of itself, if it will divide 91, it will divide 273, by Principle 1, and 325, by Principle 2. The quotient is 1, and the remainder 39; hence, 52 is not the g. c. d. sought. Since the g. c. d. of 91 and 325 must divide 52 and 91, it must divide 39, by Principle 3; hence, it cannot exceed 39. If 39 will divide 52, it will divide 91, by Principle 2; 273, by Principle 1; and 325, by Principle 2. The quotient is 1, and the remainder 13; hence, 39 is not the g. c. d. sought. Since the g. c. d. must divide 39 and 52, it must divide 13, by Principle 3. Since 13 will divide itself and 39, it will divide 52, by Principle 2; 91, by Principle 2; 273, by Principle 1; and 325, by Principle 2; hence, 13 is the g. c. d. of 91 and 325.

377. RULE III.

Select two of the numbers and divide the greater by the less, and the less by the remainder, if there is one. Continue the process until there is no remainder. The last divisor will be the g. c. d. sought.

Compare this divisor with a third number, proceeding in the same manner, and thus continue until all of the numbers are disposed of.

NOTE. Observe that this method discovers numbers that are smaller than the given numbers, and yet that have the same g. c. d.

EXAMPLES.

Find the g. c. d. of the following:

1. 340 and 578.
2. 333 and 703.
3. 533, 697, and 779.
4. 1265, 1870, and 8613.
5. 7944, 12247, and 13902.

378. APPLICATIONS OF G. C. D.

1. What is the greatest width that a carpet can be to prevent waste in covering the floors of four rooms that are, respectively, 15, 18, 21, and 24 feet in width?

2. What is the greatest length of flooring that can be used, without cutting, for three halls that are, respectively, 24, 36, and 60 feet in length?

3. What is the length of the longest paving-stones that, without cutting, may be used to build 4 walks, 144 feet, 180 feet, 204 feet, and 300 feet long?

4. What is the capacity of the largest box that will be filled an integral number of times in measuring 160 bushels of oats, 304 bushels of wheat, and 400 bushels of rye?

379. LEAST COMMON MULTIPLE.

The Method for Large Numbers.

In finding the l. c. m. of large numbers that are not easily factored, the work may be simplified by employing the g. c. d.

Observe that *the l. c. m. of two or more numbers is the product of their g. c. d. and their uncommon prime factors.*

Illustrative Example. Find the l. c. m. of 6837, 7353, 7869.

(*a*) Find the g. c. d. of 6837 and 7353 by the third method.

(*b*) Divide 7353 by it.

(*c*) Multiply 6837 by the quotient.

(*d*) Proceed in a similar manner with the third number and the result thus obtained.

Employ the method with smaller numbers until the process is familiar.

Form a rule from the solution of the illustrative problem.

PROBLEMS.

Find the l. c. m. of the following:

1. 629, 703, 851.
2. 338, 364, 448.
3. 1496, 1768, 2312.
4. 990, 1305, 1188.
5. What is the smallest sum of money that may be expended by using only 3-cent pieces, 5-cent pieces, 10-cent pieces, or 25-cent pieces?
6. What is the shortest distance that will exactly contain an 8-foot measure, a 12-foot measure, a 15-foot measure, or an 18-foot measure?
7. What is the smallest quantity of oats that will fill, an integral number of times, a 5-bushel box, a 9-bushel box, a 15-bushel box, or a 21-bushel box?
8. What is the product of the l. c. m. of 12, 15, 18, and 24, and their g. c. d.?
9. Divide the l. c. m. of 7, 12, 21, 9, 10, and 252, by the g. c. d. of 80, 120, 840, and 960.
10. What is the difference between the l. c. m. of 10, 45, 75, and 90, and the l. c. m. of 7, 15, 25, and 35?
11. What is the shortest cord that could be cut into pieces of 9, 12, 15, 18, or 45 inches?

380. Stone and Brick Work.

1. Stone Work is usually measured by the **Perch**, although in many localities it is estimated by the cubic foot.
2. A Perch of Stone contains $24\frac{3}{4}$ cubic feet. It is 1 rod long, $1\frac{1}{2}$ feet wide, and 1 foot thick.
3. In estimating the labor of laying stone and brick the corners are usually counted twice, because of the extra care needed.
4. For 8-inch walls it is customary to count 15 bricks to the foot; for 12-inch walls, 21 bricks are counted for a foot.

381. Grain.

A cubic foot is about .8 of a bushel. The capacity of a wagon-box or a bin may be found approximately by finding the number of cubic feet which it contains, and multiplying this result by .8.

382. Ear Corn.

A bushel of ear corn contains about $2\frac{1}{4}$ cubic feet. To find the number of bushels a bin will contain, find the number of cubic feet in the bin and take $\frac{4}{9}$ of it.

383. Hay.

Hay measurement is only approximately correct. About 350 cubic feet of well-settled timothy hay will weigh a ton. If only partially settled, at least 450 cubic feet must be allowed. Clover hay is much lighter. Allow about 550 cubic feet to the ton.

384. Coal.

Lehigh stove-coal and Schuylkill white-ash stove-coal contain about 35 cubic feet to the ton. Other varieties vary but slightly from this estimate.

385. MARINERS' MEASURE.

6	feet	= 1 fathom.
120	fathoms	= 1 cable length.
$7\frac{1}{3}$	cable lengths	= 1 mile.

386. AVERAGE OF ACCOUNTS.

A problem in Average of Accounts is a double problem in Equation of Payments.

The two sides of the account may be compared by assuming a common day of settlement.

Illustrative Problem.

Dr. A. B. SMITH in acct. with BROWN BROS. Cr.

1891.			1891.			
Jan. 1.	To mdse.,	$368.25	Feb. 15.	By cash,		$450.00
28.	" "	225.00	Apr. 12.	" mdse.,		400.00
Mar. 12.	" "	186.50	May 24.	" produce,		158.00

ANALYSIS. Let us assume that A. B. S. paid his account in full on May 24. He would then have had a credit of 4 mo. 23 d. on the first, 3 mo. 26 d. on the second, and 2 mo. 12 d. on the third. If interest were paid at 6%, the charge on the first would be $8.78; on the second, $4.35; on the third, $2.24. Their sum is $15.37. The sum of the items is $779.75. If interest were charged, then, on May 24, A. B. S. would pay to Brown Bros. $779.75, and $15.37 as interest.

If Brown Bros. were to pay A. B. S. on May 24, they would have a credit of 3 mo. 9 d. on the first, and 1 mo. 12 d. on the second. If interest were paid at 6%, the charge on the first would be $7.43; on the second, $2.80. Their sum is $10.23. The sum of the items is $1,008. If Brown Bros. settled with A. B. S., then, on May 24, they would pay him $1,008, and $10.23 as interest. But Brown Bros. owe A. B. S. $228.25 more than he owes them. If there were no interest charge, they could settle by paying him that balance. Since interest is to be considered, they would lose $15.37, and A. B. S. would lose $10.23. Their loss would exceed his by $5.14. To prevent this loss, they should retain the $228.25 until it would earn them $5.14 at 6%. The interest on $228.25 is about $.038 per day. It will take 136 days for $228.25 to gain $5.14; hence, Brown Bros. should retain the balance until Oct. 7.

387. ORIGIN OF UNITS.

1. The yard is the standard from which nearly all of our units of measure are derived. It was definitely fixed by what is known as the "Pendulum experiment."

2. A pendulum was found which, at London, at the sea level, vibrated once in a second. It was divided into 391,393 equal parts, of which 360,000 were about equal to the yard then most commonly used. By law this became the unit of linear measure, under the name of the yard. It was divided

into three equal parts called feet, and each of these was divided into twelve equal parts called inches.

3. Since a square foot is a square, each of whose sides is a linear foot, it is seen that the units of square measure are derived from the same experiment.

4. Since a cubic foot is a cube each of whose edges is a linear foot, the units of cubic measure have the same origin.

5. A gallon is 231 cubic inches. The bushel is similarly derived. The standard of weight is the Troy pound. Its weight is equal to the weight of about 22.8 cubic inches of distilled water.

388. THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

1. On account of the great variety of scales employed in the common system of weights and measures in use in this country, an effort has been made to introduce the French system.

2. The standard unit is the **Meter**, which was supposed to equal one ten-millionth of a quadrant of the meridian passing through Paris. It is about $\frac{6}{5}$ of a yard; but those using this system must learn to think in its units.

3. Decimal parts of the meter are indicated by prefixing to *meter*, *milli*, meaning one-thousandth, *centi*, meaning one-hundredth, and *deci*, meaning one-tenth.

4. Decimal multiples of the meter are expressed by prefixing to *meter*, *deka*, meaning ten, *hecto*, meaning one hundred, *kilo*, meaning one thousand, and *myria*, meaning ten thousand.

NOTE.—The first three prefixes are Latin, and the others are Greek

The abbreviations for names containing the Latin prefixes are printed in small letters, and those containing the Greek, in capitals.

389. Table of Long Measure.

10 millimeters (mm.)	equal	1 *centimeter*	(cm.)	equal	.3937+ in.
10 centimeters	"	1 decimeter	(dm.)	"	3.937+ in.
10 decimeters	"	1 *meter*	(m.)	"	39.37+ in.
10 meters	"	1 dekameter	(Dm.)	"	32.8+ ft.
10 dekameters	"	1 hektometer	(Hm.)	"	19.927+ rd.
10 hektometers	"	1 *kilometer*	(Km.)	"	.621+ mi.
10 kilometers	"	1 myriameter	(Mm.)	"	6.213+ mi.

NOTE.—The units most commonly used are printed in italic.

1. Long distances are measured in kilometers. This unit is about $\frac{5}{8}$ of a mile.

2. The symbol denoting the denomination may be placed after the integral part of the expression; thus 24 m. 56, or after the entire expression, 24.56 m.

390. Surface Measure.

The surface units are squares each of whose sides is a linear unit. It follows that 100 of each order make one of the next higher.

TABLE.

100 sq. mm.	equal	1 sq. cm.	
100 sq. cm.	"	1 sq. dm.	
100 sq. dm.	"	1 sq. m.	= 1.196 sq. yd.
100 sq. m.	"	1 sq. Dm.	= 119.6 + sq. yd.
100 sq. Dm.	"	1 sq. Hm.	= 2.47 + A.
100 sq. Hm.	"	1 sq. Km.	= 247.114 A.

1. The square meter is used in the measurement of small surfaces as the square yard is used in the ordinary system. When used to measure land it is called a centare (cn).

2. The sq. Dm. is called an *are* when used as a land measure, and the sq. Hm. a *hectare* when so used.

391. Measures of Volume.

The volume units are cubes, each of whose edges is a linear unit. In the following table 1000 of each denomination make one of the next higher.

TABLE.

1000 cu. mm.	equal	1 cu. cm.
1000 cu. cm.	"	1 cu. dm.
1000 cu. dm.	"	1 cu. m. = 35.316 cu. ft.

1. The cu. m. is the unit most commonly used.

2. When the cubic meter is used in measuring wood it is called a *stere*. One tenth of a stere is a *decistere*. 10 steres make a *dekaster* (Dst.)

3. The stere is a little more than a quarter of a cord.

392. Tables of Liquid and Dry Measure.

1 milliliter (ml.)	= 1 cu. cm.
10 ml.	= 1 centiliter (cl.)
10 cl.	= 1 deciliter (dl.)
10 dl.	= 1 liter (l.) = 1 cu. dm.
10 l.	= 1 decaliter (Dl.).
10 Dl.	= 1 hectoliter (Hl.)
10 Hl.	= 1 kiloliter (Kl.) = 1 cu. m.
10 Kl.	= 1 myrialiter.

	Dry.	Liquid.
1 liter (l.)	= .908 qt.	= 1.057 qt.
1 decaliter (Dl.)	= 1.135 pk.	= 2.642 gal.
1 hectoliter (Hl.)	= 2.837 bu.	= 26.417 gal.
1 kiloliter (Kl.)	= 28.37 bu.	

The liter, which is very nearly the liquid quart, and the hectoliter — about $2\frac{5}{8}$ bushels — are the units most commonly used.

393. Weights.

The unit of weight is the *gram*, which is the weight of one cu. cm. of pure water at the temperature of greatest density.

TABLE.

10 milligrams (mg.) = 1 centigram (cg.)
10 cg. = 1 decigram (dg.).
10 dg. = 1 gram (g.)
10 g. = 1 decagram (Dg.).
10 Dg. = 1 hectogram (Hg.)
10 Hg. = 1 kilogram (Kg.) = wt. 1 cu. dm. of water.
10 K. = 1 myriagram (Mg.).
10 Mg. = 1 quintal (Q.)
10 Q. = 1 tonneau (T.) = wt. 1 cu. m. of water.

1 gram (g.). = 15.432 + grains
1 kilogram (Kg.) = 2.204 + lb. av.
1 tonneau (T.) = 2204.621 + lb. av.

1. The units most commonly used are the the centigram, the gram, the kilogram, and the tonneau.

2. The kilogram is called the kilo, for brevity.

394. Table of Approximate Equivalents.

1 decimeter	= 4 in.	nearly.
1 meter	= $1\frac{1}{8}$ yd.	"
1 decameter	= 2 rd.	"
1 kilometer	= $\frac{5}{8}$ of 1 mi.	"
1 are	= $\frac{1}{40}$ of an A.	"
1 stere	= $\frac{1}{4}$ of a cord.	"
1 liter	= 1 liq. qt.	"
1 hectoliter	= 3 bushels	"
1 gram	= $15\frac{1}{2}$ grains	"
1 kilo	= $2\frac{1}{5}$ lb. av.	"
1 tonneau	= $1\frac{1}{10}$ tons.	"

395. Exercises on the Metric Tables.

1. Reduce 2864 m. to mm.; to dm.; to Hm.; to Mm.

2. Reduce 24685 sq. dm. to sq. cm.; to sq. Dm.; to sq. Km.

3. Reduce 25709853 cu. dm. to cu. cm.; to cu. m.; to decisteres; to dekasters.

4. Reduce 47073 l. to cl.; to Dl.; to Kl.; to dl.

5. Reduce 279436 Dg. to g.; to mg.; to Mg.; to Q.

6. How many sq. m. in the floor of a room 12 m. long and 10 m. wide?

7. Find the length of your school-room in meters. Find the width of your desk in decimeters. What is the area of the top of your desk in square decimeters? It is what part of a sq. m.?

8. Find the number of square centimeters in a pane of glass in a window of your school-room. What part of a sq. m. is it?

9. Lay off in your school-yard a square a dekameter on a side. What is this unit used for? How many square meters does it contain? What is its name? Calling its area 120 square yards, what part of an acre is it?

10. Find the volume of your school-room in cubic meters. If it were filled with wood how many steres would it contain? About how many cords?

11. What would it cost to lath and plaster the ceiling of your school-room at 28 cents a sq. m.?

12. If your school-room were used for a grain bin, how many hectoliters of shelled corn could be put into it? How many bushels? If it were a tank how many decaliters of water would it hold? How many gallons?

13. How many grains are there in a gram? How many grains in an ounce avoirdupois? A gram is about what part of an ounce? Of a pound?

14. Compare the kilo with the pound; the tonneau with the ton.

15. Find the cost of 10 kilos of coffee at 3 cents an Hg.

16. A field is 1 kilometer in length and 5 hectometers in width. What is it worth at $2.25 an are?

17. What is the weight of a liter of pure water?

18. The atmospheric pressure under ordinary conditions is about 15 pounds to the square inch. What is it in tonneaus per square meter?

19. How many bushels in a quintal of wheat? (Wheat 60 lbs. to bu.)

20. What will a liter of mercury (specific gravity, 13½) weigh? Find weight also in pounds.

21. What is the cost of a pile of wood 12 m. long, 2 m. wide, and 2 m. high, at $1.25 a stere?

22. A bin is 10 m. long, 3 m. wide, and 3 m. high. It is filled with wheat. What is the value of the wheat at 210 cents an Hl.?

23. Find the weight in quintals of the above wheat, counting it ¾ as heavy as water.

24. Find the capacity in Hl. of a cylindrical cistern whose diameter is 3.2 m., and depth 4.1 m.

NOTE.—The hectoliter is about ⅘ of a barrel.

Cook, John W.
The new advanced arithmetic.

81133

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